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Chapter 2
FLUID STATICS
by
Amat Sairin Demun

Learning Outcomes

Upon completing this chapter, the students are expected to be able to:
1. *Calculate the pressure in pipes by using piezometers and manometers.*
2. *Calculate the hydrostatic pressure force for submerged plane surfaces (magnitude, location and direction and the related reaction force) – inclined or vertical position.*
3. *Calculate the hydrostatic pressure force for submerged curved surfaces (magnitude, location and direction and the related reaction force).*

- Study on non-moving fluid – fluid at rest.

2.1) Pressure – Density – Height Relationship

Pressure unit = N/m^2 or Pa or Bar
$1\ kPa = 1\ kN/m^2 = 1000\ Pa = 1000\ N/m^2$
$1\ Bar = 100\ kPa$

$$P = \rho g h$$

h = vertical height downward from the fluid surface (m).

If the fluid is gas or air, the pressure at all places are the same – the pressure NOT depends on height h.

If someone says the pressure is 5 cm Mercury, means that the pressure is:

$$P = \rho g h = (13.57 \times 1000)(9.81)(0.05) = 6.66\ kPa$$

2.2) Relationship Between Absolute and Gauge Pressure

$$P_{abs} = P_{gauge} + P_{atm}$$

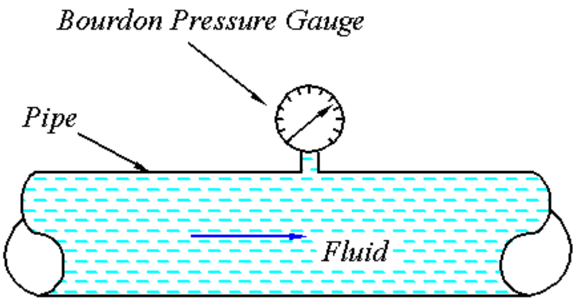
atmospheric pressure = 101.3 kPa = 760 mm Hg

2.3) Pressure Measurements

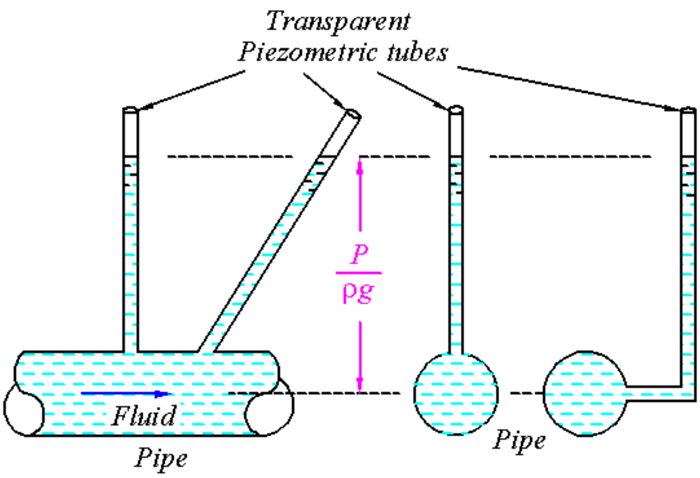
Various types of equipments used to measure pressure.

1. Barometer – to measure atmospheric pressure.
2. Aneroid – to measure air pressure in tyres.
3. Bourdon – to measure fluid pressure.

Bourdon pressure gauge to measure pressure of fluid in a pipe.

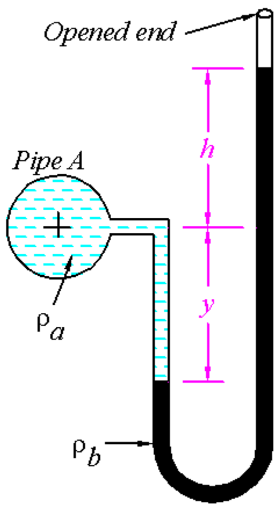


4. Piezometer – to measure fluid pressure in pipes.

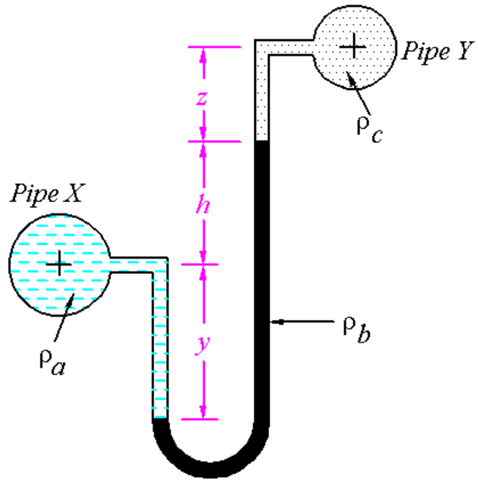


5. Manometers – to measure fluid pressure in pipes, if piezometric tube is not practical (pressure too high)
   The following figures show examples of manometers.

SIMPLE MANOMETER



DIFFERENTIAL MANOMETER



Calculation Procedures:

1. Divide the manometer into sections according to the known vertical height.

2. Draw an arrow pointing DOWNWARD for each of the section.

3. For each drawn arrow, mark (+) or (–) sign.

   (+) for the arrow that the direction LEADS to the point where the pressure is to be known (reference point).
   (–) for the arrow that the direction LEAVES the reference point.
   If the sign is (+), the pressure for the arrow is $+\rho gh$ and if the sign is (–), the pressure for the arrow is $-\rho gh$; where h is the vertical height of the section containing the arrow while ρ is the fluid density in the manometer where the arrow is located.

4. Cancel two pressure which have the same magnitude but opposite direction. Same magnitude means that they have the same height and the same fluid density, ρ.

5. Sum up all of the pressures taking the pressure at the reference point as the heading.
   If the manometer end is open, the pressure at the end is zero.
   If the manometer end is another pipe (for differential manometer), add (+) the pressure value.

2.4) Hydrostatic Pressure Force on Submerged Plane Surfaces

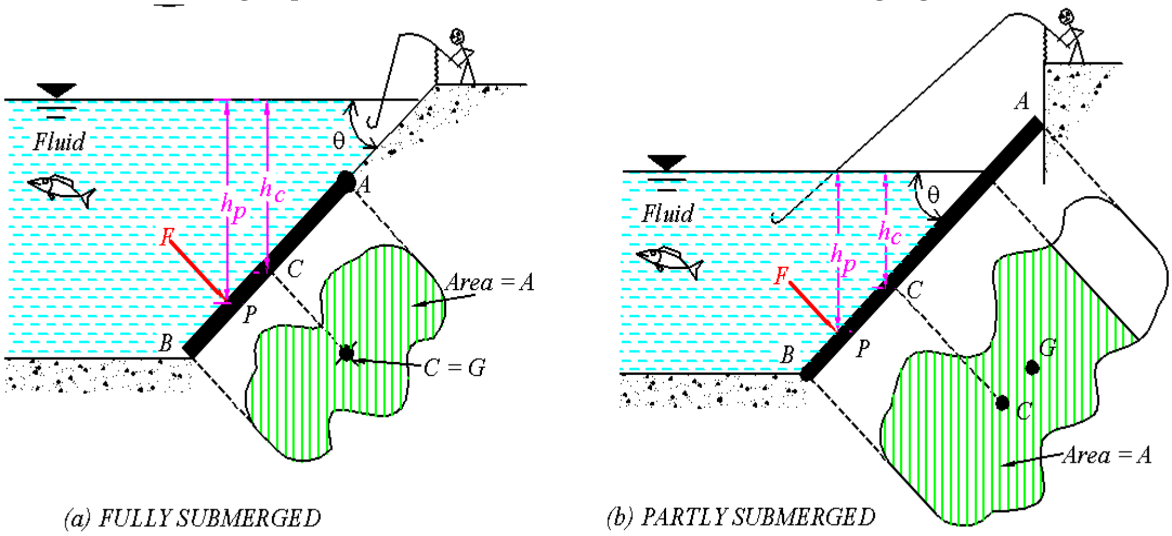
Hydrostatic pressure is the force due to the fluid acting on the submerged plane surface.

Force = Pressure x Area
$F = PA = \rho g h A$
unit = Newton (N) or kg.m/s^2

Force is a vector that must have 3 items:

1. Magnitude
2. Location of act
3. Direction

An inclined submerged plane surface, AB is as shown in the following figure.



(a) FULLY SUBMERGED (b) PARTLY SUBMERGED

Note:-

C	=	Center of gravity of the **Submerged** Surface
G	=	Center of gravity of the whole Plane Surface
P	=	Center of Pressure (the place where the pressure force acts)

Magnitude: $F = \rho g h_c A$

A = Area of the submerged surface (m^2).
h_c = Vertical height from the fluid surface to the center of gravity of the submerged plane surface (**C**) (m).

Location: F is acting on the center of pressure that is at a vertical distance h_p from the fluid surface where:

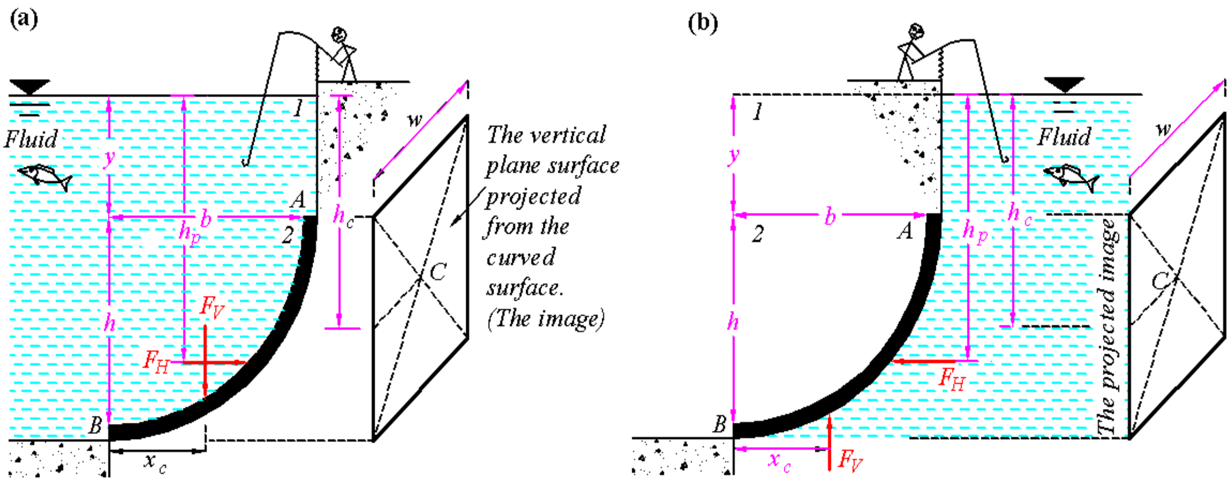
$$h_p = \frac{I_c \sin^2 \theta}{A h_c} + h_c \qquad ?$$

where: θ = the inclined angle (see figure)
I_c = Moment of inertia about the **horizontal line** through the center of gravity of the submerged surface (m^4) – ***see Table 2.1***

Direction: F acts at the plane surface **perpendicular (90^0)** to the wetted surface.

2.5) Hydrostatic Pressure Force on Submerged Curved Surfaces

Curved surfaces AB submerged in a fluid are as shown in the following figures. Compare both figures and spot the differences.



Two force components: Horizontal force and Vertical force.

1) **Horizontal Component, F_H**

(a) **Magnitude:** F_H is the force equivalent to the force acting on the vertical plane surface projected from the curved surface (the rectangular image).

$$F_H = \rho g h_c A$$

A = area of the vertical plane surface projected from the curved surface (area of the rectangular image).

h_c = vertical height from the fluid surface to the center of gravity of the vertical plane surface projected from the curved surface (the image).

(b) **Location:** F_H acts horizontally at the curved surface through the center of pressure of the vertical plane surface projected from the curved surface, that is at the vertical height h_p from the fluid surface, where:

$$h_p = \frac{I_c}{A h_c} + h_c$$

because sin 90^0 = 1 (vertical plane surface)

I_c = Moment of inertia about the **horizontal** line through the center of the plane surface projected from the curved surface. (m^4) – ***See Table 2.1***.

A and h_c is as (a) above.

If you're dealing with horizontal force, remember the Rectangular Image.

2) Vertical Component, F_V

If you're dealing with Vertical Force, remember the Fluid Above the Curve up to the fluid surface.

(a) **<u>Magnitude:</u>** F_V is the force equivalent to the **weight of the fluid <u>above</u>** the curved surface.

$$F_V = \rho g V$$

where, V = fluid volume **<u>above</u>** the curved surface (m^3) up to the fluid surface line.

(b) **<u>Location:</u>** F_V acts vertically at the curved surface through the center of gravity of the fluid volume above the curved surface (up to the fluid surface), that is at the horizontal distance x_c from a reference vertical line, where:

$$x_c = \frac{A_1 x_1 \pm A_2 x_2}{A_1 \pm A_2}$$

A_1 = Area of shape 1
A_2 = Area of shape 2

x_1 = horizontal distance from the center of gravity of area 1 to the reference vertical line.
x_2 = horizontal distance from the center of gravity of area 2 to the reference vertical line.

3) Resultant Force, F

The resultant force of components F_H and F_V is:

Magnitude: $$F = \sqrt{F_H^2 + F_V^2}$$

Location: F acts at the curved surface through the intersection point of F_H and F_V lines.

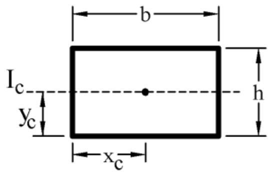
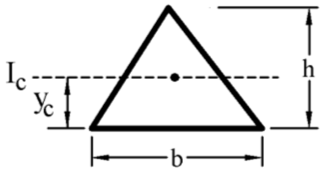
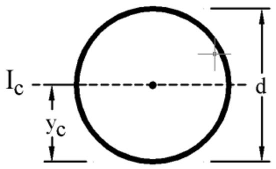
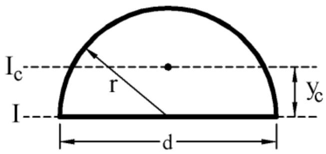
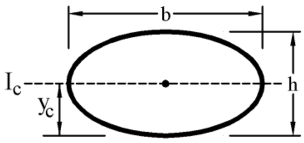
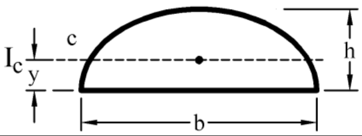
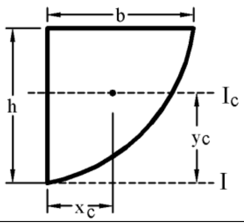
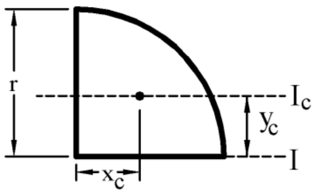
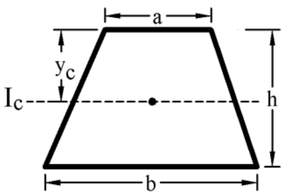
Direction: F acts at an angle α from horizontal;

$$\alpha = \tan^{-1}\left(\frac{F_V}{F_H}\right)$$

2.6 The HSTATIC Computer Program

HSTATIC stands for HydroSTATIC is a medium sized executable computer program to solve any hydrostatic pressure force problem. It is developed by Mr. Amat Sairin Demun using MS DOS based Fortran programming language. It is able to calculate the pressure force and the reaction force of fluid acting on a submerged plane and curved surfaces. The students can copy the file from Mr. Amat Sairin Demun at no cost. To run the computer program, the students will have to double click the HSTATIC file and just follow the instructions appear on the computer screen. If you have difficulties running the computer program, please feel free to contact Mr. Amat Sairin Demun.

TABLE 2.1: Geometric Properties of Plane Surfaces $I = I_c + Ay_c^2$

Shape	Sketch	Area, A	Loc.of Centroid	M. of Inertia, I_c or I
Rectangle		$A = bh$	$y_c = \frac{h}{2}$ $x_c = \frac{b}{2}$	$I_c = \frac{bh^3}{12}$
Triangle		$A = \frac{bh}{2}$	$y_c = \frac{h}{3}$	$I_c = \frac{bh^3}{36}$
Circle		$A = \frac{\pi d^2}{4}$	$y_c = \frac{d}{2}$	$I_c = \frac{\pi d^4}{64}$
Semicircle		$A = \frac{\pi d^2}{8}$	$y_c = \frac{4r}{3\pi}$	$I = \frac{\pi d^4}{128}$ $I_c = \left(6.86 \times 10^{-3}\right) d^4$
Ellipse		$A = \frac{\pi bh}{4}$	$y_c = \frac{h}{2}$	$I_c = \frac{\pi bh^3}{64}$
Semi Ellipse		$A = \frac{\pi bh}{4}$	$y_c = \frac{4h}{3\pi}$	$I_c = \frac{\pi bh^3}{16}$
Parabola		$A = \frac{2bh}{3}$	$x_c = \frac{3b}{8}$ $y_c = \frac{3h}{5}$	$I = \frac{2bh^3}{7}$ $I_c = \frac{8bh^3}{175}$
Quadrant		$A = \frac{\pi d^2}{16}$	$y_c = \frac{4r}{3\pi}$ $x_c = \frac{4r}{3\pi}$	$I = \frac{\pi d^4}{256}$ $I_c = \left(3.43 \times 10^{-3}\right) d^4$
Trapezoid		$A = \frac{h(a+b)}{2}$	$y_c = \frac{h(a+2b)}{3(a+b)}$	$I_c = \frac{h^3\left(a^2+4ab+b^2\right)}{36(a+b)}$