SKF4153- PLANT DESIGN

PROCESS OPTIMIZATION

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Definition of Optimization

Optimization is the science and the art of determining the <u>best solutions</u> to problems

Why OPTIMIZE?

Budget/Financial reason

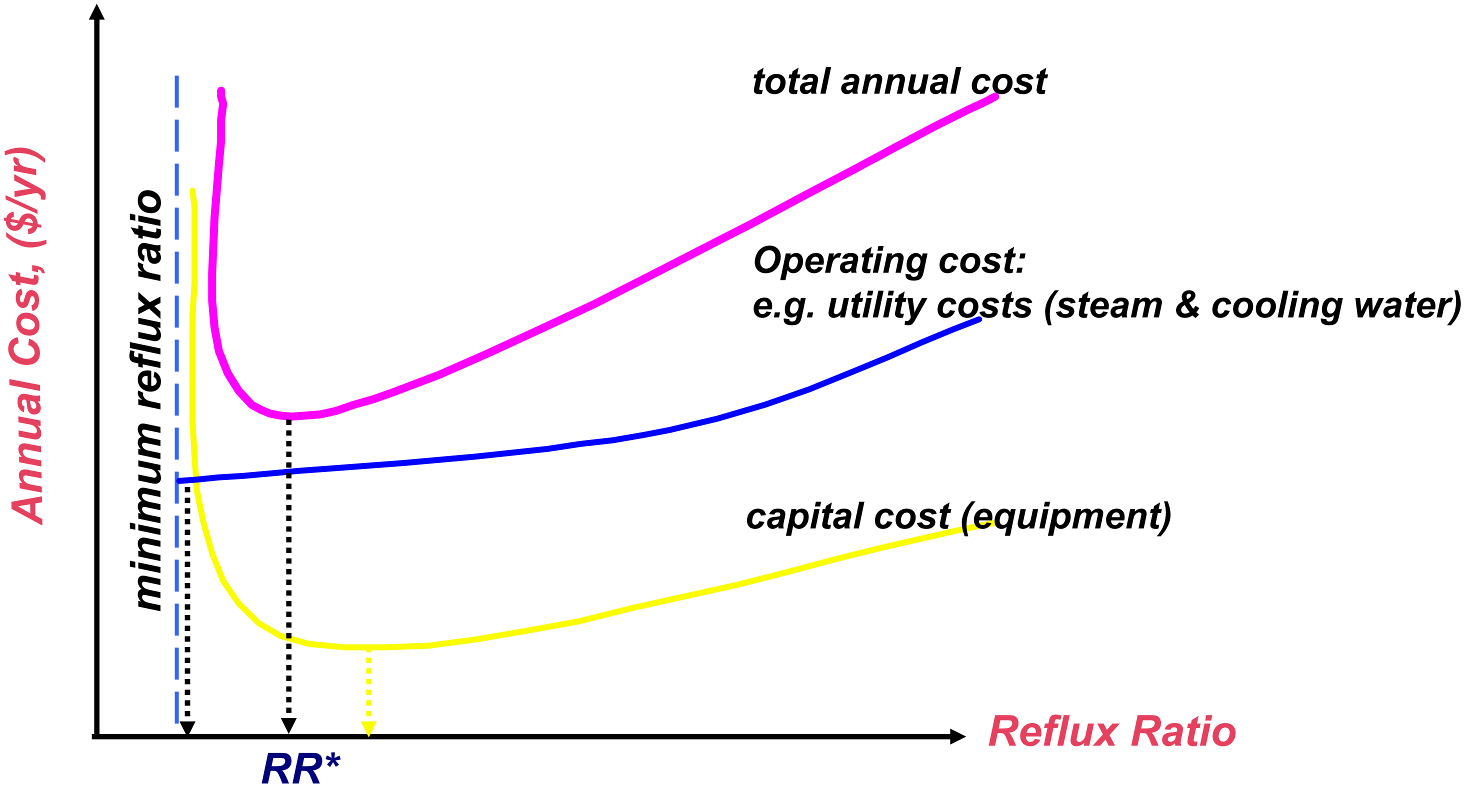
- ➢ Largest production
- ➢ Greatest profit
- ➢ Minimum cost
- ➢ Least energy usage

Plant performance

- ❖ Improved yields of valuable products
- ❖ Reduced energy consumption
- ❖ Higher processing rates
- ❖ Longer time between shutdowns
- ❖ Reduced maintenance costs

Optimization of a Distillation Column at Design Stage

- The relationship between the number of stages and the reflux ratio is linked to capital cost and the operating cost.
- The more stages, the higher the capital cost and the lower the operating cost.
- Choose No. of stages and reflux ratio to minimize total cost while meeting specifications.


Annual Cost, ($/yr)
minimum reflux ratio
total annual cost
Operating cost:
e.g. utility costs (steam & cooling water)
capital cost (equipment)
Reflux Ratio
RR*

Features of Optimization Problem

objective function to be minimised (or maximised)

$\min f(x_i)$

x_i

vector of decision variables to be chosen to find the min. of f(x)

$i = 1, 2, \ldots i$

equality constraints

Subject to

$c(x_i) = 0$

$g(x_i) \geq 0$

in-equality constraints

Terminology

Objective function

Function that you wish to minimize or maximize, usually an economic value eg. maximize profit, production rate, throughput, minimize energy utilisation, cost

Decision variables

Variables that you can adjust (manipulate) or chose in order to minimize or maximize the objective function

Variables can be:

- ✓ Real eg. flow rates, concentration, temperature, pressure
- ✓ Integers eg. No. of trays
- ✓ Binary eg. Select/not select - plant process flowsheet

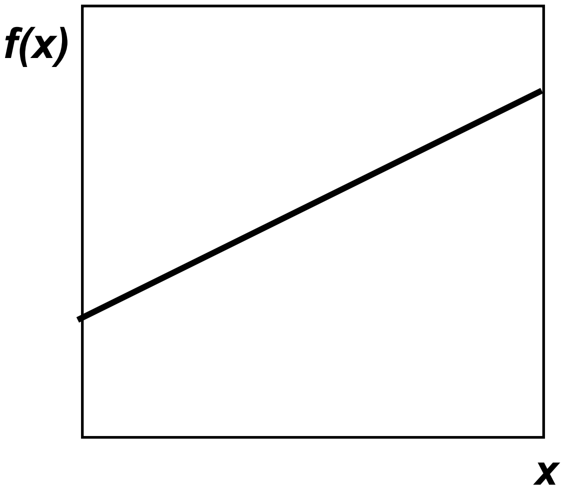
Constraints

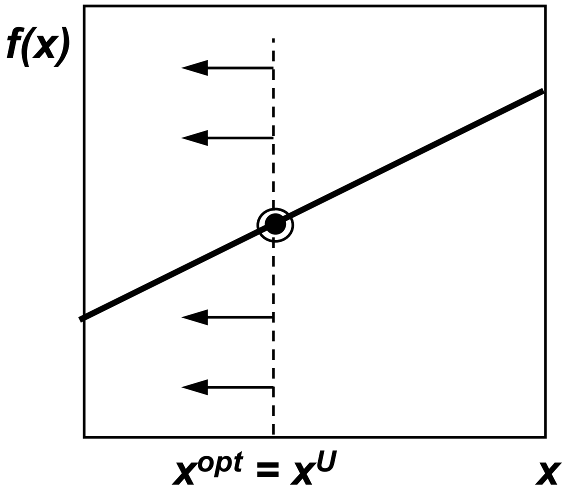
Limitations on the free adjustment of decision variables. They can be physical, economic, policy, or environmental

Constraints can be explicitly defined in terms of the decision variables

Linear Objective Function

Maximum objective function is desired.



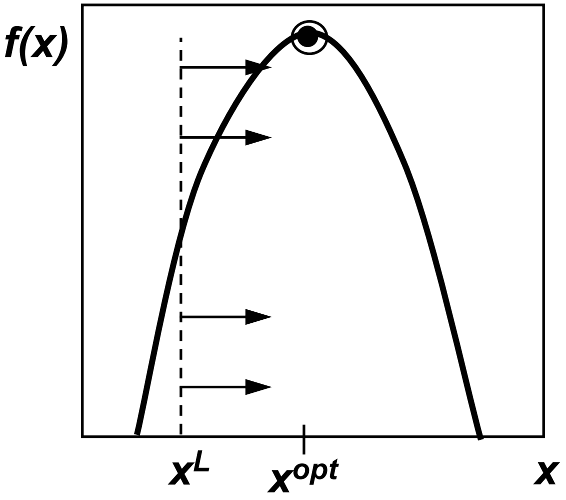


Unconstrained
No equality and inequality constraints, no solution exists

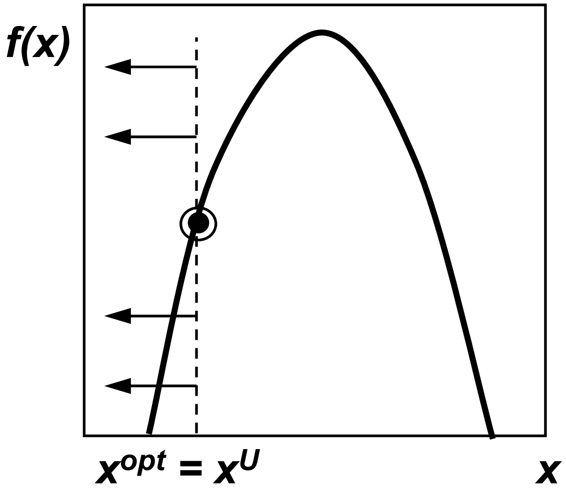
Constrained
subject to inequality constraint. $x \le x^U$. Optimal solution is at the bound

Non-Linear Objective Function

Maximum objective function is desired.



Subject to slack constraint (constraint is at a lower limit of x, or $x \geq x^L$. The ***optimal value is at the maximum value*** *of the objective function)*



Subject to binding constrained ($x \leq x^U$, bound is below the value of maximum objective function)

LINEAR PROGRAMMING (LP) MODEL
Example 1

A farmer is preparing to plant a crop in the spring and needs to fertilize a field. There are two brands of fertilizer to choose from, Super-gro and Crop-quick. Each brand yields a specific amount of nitrogen and phosphate per bag, as follows:

Product	Chemical Contribution	
	Nitrogen (lb/bag)	Phosphate (lb/bag)
Super-gro	2	4
Crop-quick	4	3

The farmer requires at least 16 pounds of nitrogen and 24 pounds of phosphate. Super-gro costs $6/bag, and Crop-quick costs $3/bag. The farmer wants to know how many bags of each brand to be purchased in order to **MINIMIZE** the total cost of fertilizing.

Solution for Example 1:

Step 1: Define the decision variables

x_1 = bags of Super-gro
x_2 = bags of Crop quick

Step 2: Define the objective function

min $f(x) = 6x_1 + 3x_2$

Step 3: Define the equality and inequality constraints

- Equality constraint

  None except if the farmer said that the phosphate requirement must be exacly 24 pound, the constraint should be written

  $4x_1 + 3x_2 = 24$ (phosphate constraint)

- Inequality constraint

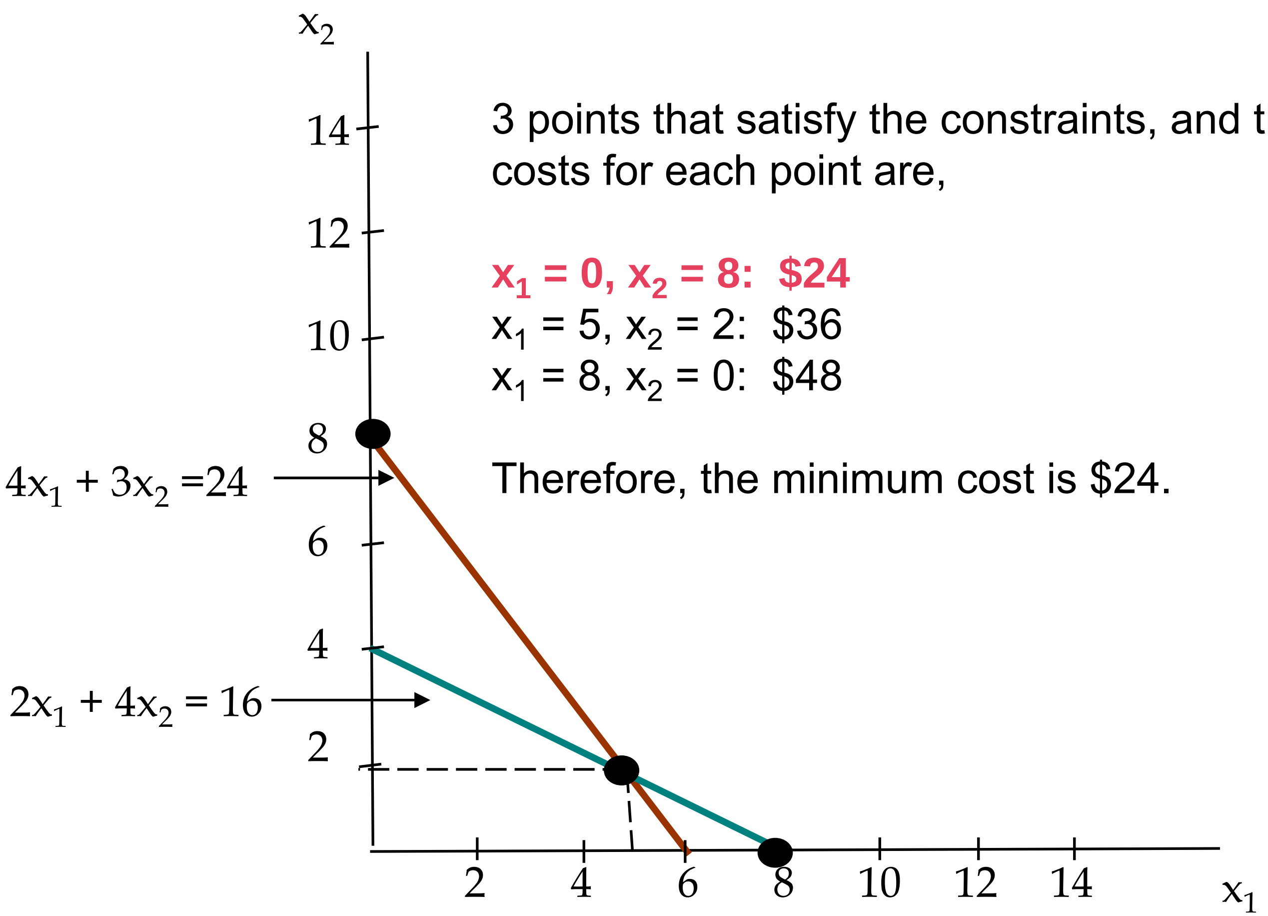
  $2x_1 + 4x_2 \geq 16$ (nitrogen constraint)
  $4x_1 + 3x_2 \geq 24$ (phosphate constraint)

- Non-negativity constraints – to indicate that negative bags of fertilizer cannot be purchased

  $x_1, x_2 \geq 0$

Step 4: Simplification – not necessary

Step 5: Apply suitable optimization technique



3 points that satisfy the constraints, and the respective costs for each point are,

$x_1 = 0, x_2 = 8$: \$24
$x_1 = 5, x_2 = 2$: \$36
$x_1 = 8, x_2 = 0$: \$48

Therefore, the minimum cost is \$24.

Step 6: Perform sensitivity analysis e.g. What if the cost of raw material is increased by 10%.

Example 2

Ready-Mixes produces both interior and exterior paints from two raw material, M1 and M2. the following table provides the basic data of the problem

	Tons of raw material per ton of		Maximum daily availability (tons)
	Exterior paint	Interior paint	
M1	6	4	24
M2	1	2	6
Profit per ton ($1000)	5	4	

A market survey indicate that's the daily demand for interior paint cannot exceed that for exterior paint by more than 1 ton. Also, the maximum daily demand for interior paint is 2 tons. Determine the optimum (best) product mix of interior and exterior paints that maximizes the total daily profit.

Solution for Example 2:

Step 1 : Define the decision variable,

x_1 = tons produced daily of exterior paint

x_2 = tons produced daily of interior paint

Step 2 : Define the objective function

Max total daily profit = $5x_1 + 4x_2$

Step 3: Define equality & inequality constraint

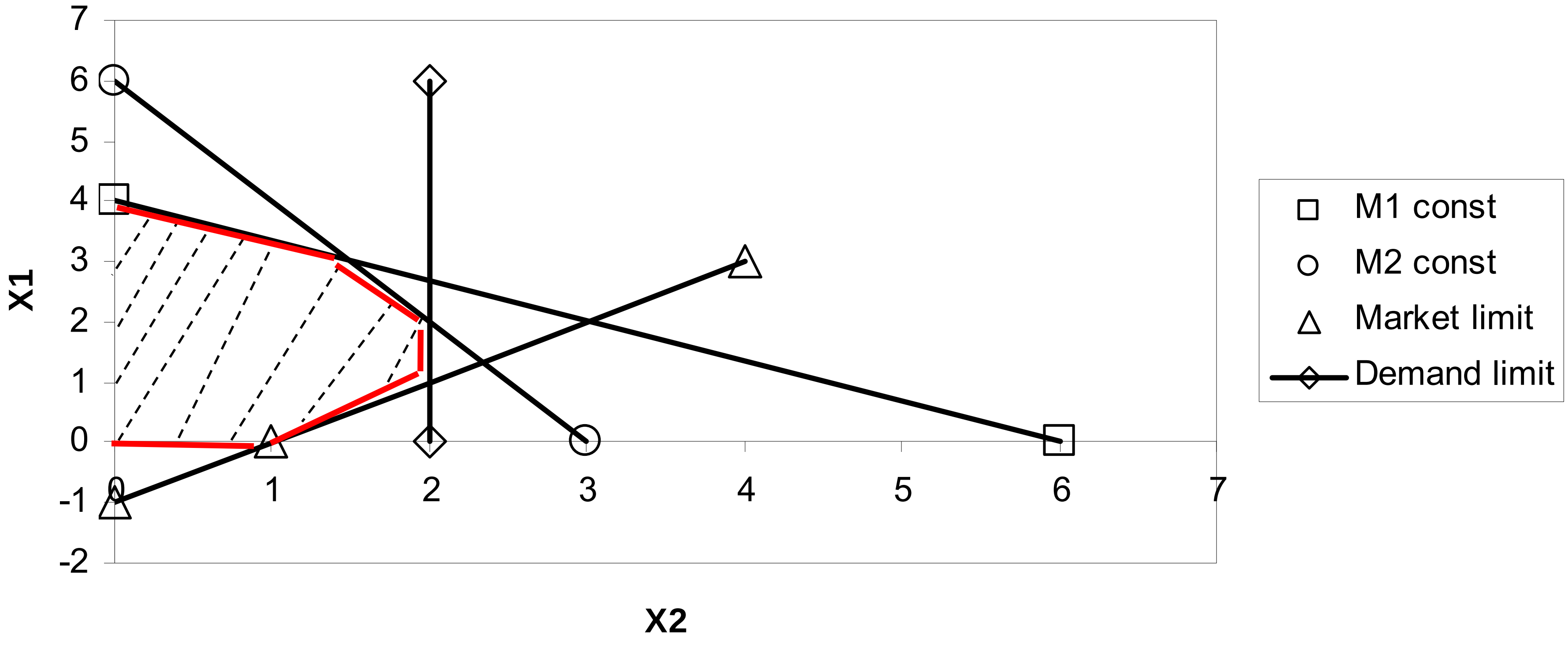
$6x_1 + 4x_2 \leq 24$	(M1 constraint)
$x_1 + 2x_2 \leq 6$	(M2 constraint)
$x_2 - x_1 \leq 1$	(market limit)
$x_2 \leq 2$	(demand limit)

Step 4: Simplification

Not applicable

Step 5: Choose the best suitable optimization technique

The optimization problem is comprised of a linear function and linear constrained, hence linear programming would be the best technique. Use graphical method to find the optimum value. Verify your answer using Excel Solver add-in.



Example 3

During the 2002 Winter Olympics in Salt Lake City, Utah, a local plant X received a rush order for 100 gals of A containing 4.0% vol% alcohol. Although no 4% A was in stock, large quantities of A-4.5 with 4.5% alcohol at a price of \$6.40/gal and A-3.7 with 3.7% alcohol priced at \$5.00/gal were available, as well as water suitable for adding to the blend at no cost. The plant manager wanted to use at least 10 gal of A-4.5. Neglecting any volume change due to mixing, determine the gallons each of A 4.5, A-3.7, and water that should be blended together to produce the desired order at the minimum cost. Use 6 steps approach to solve optimization problem.

Solution to Example 3:

Step 1: Define the decision variables

$V_{4.5}$ = gallons of A-4.5
$V_{3.7}$ = gallons of A-3.7
V_w = gallons of water

Step 2: Define the objective function

Minimize Cost, $ = $6.40V_{4.5} + 5.00\ V_{3.7} + 0.00V_W$ (eq 1)

Step 3: Define the equality & inequality constraints

- Equality constraints
  $0.045V_{4.5} + 0.037\ V_{3.7} + 0.00V_W = 0.04(100) = 4.00$ (eq 2)
  $V_{4.5} + V_{3.7} + V_W = 100$ (eq 3)

- Inequality constraints
  $V_{4.5} \geq 10$
  $V_{3.7} \geq 0$
  $V_W \geq 0$

Step 4 : Simplification.

The problem can be reduced to two decision variables by solving (eq 3) for $V_{3.7}$,

$$V_{3.7} = 100 – V_{4.5} – V_W \qquad \text{(eq 4)}$$

And substituting it into (eqs 1 and 2) to give the following restatement of this problem:

Minimize Cost, $\$ = 1.40V_{4.5} - 5.00V_W +500$
Subject to:

$$0.008V_{4.5} + 0.037\ V_W =0.03 \qquad \text{(eq 5)}$$

$$V_{4.5} \geq 10$$

$$V_{3.7} \geq 0$$

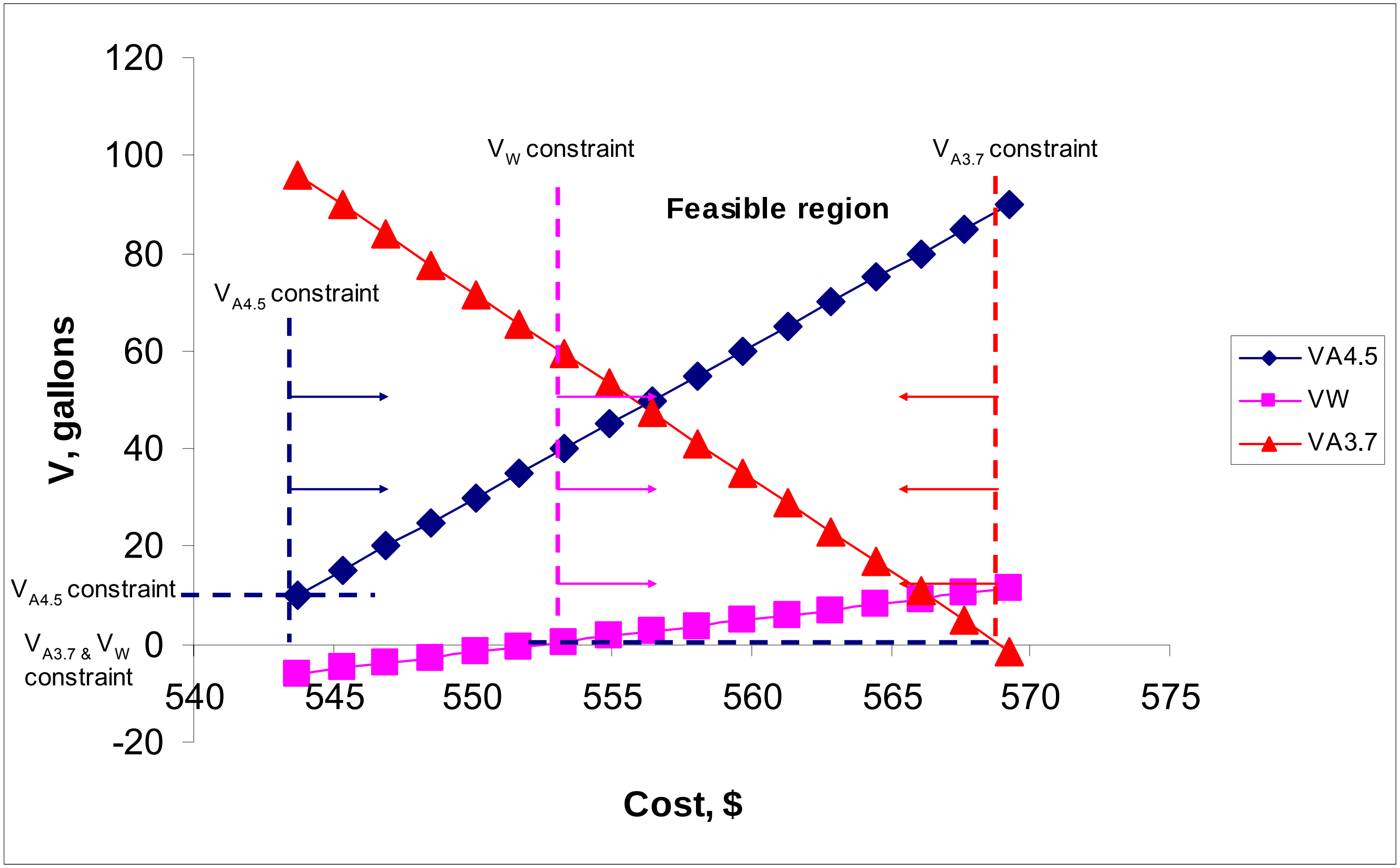
$$V_W \geq 0$$

The optimal volume of A-3.7 need only to be calculated from (eq 4), after the optimal volumes of A-4.5 and water have been determined from (eq 5). Since the objective function, the equality constraints, and the lower and upper bound are all linear, this constitutes an LP problem.

Step 5 : Choose suitable optimization technique

With just two decision variables, the problem can be shown graphically on a plot of V against cost as shown in the following figure.

$V_{4.5}$ (gal)	$V_{3.7}$ (gal)	V_W (gal)	Cost ($)
10	95.95	-5.95	543.73
15	89.86	-4.86	545.32
20	83.78	-3.78	546.92
25	77.70	-2.70	548.51
30	71.62	-1.62	550.11
35	65.54	-0.54	551.70
40	59.46	0.54	553.30
45	53.38	1.62	554.89
50	47.30	2.70	556.49
55	41.22	3.78	558.08
60	35.14	4.86	559.68
65	29.05	5.95	561.27
70	22.97	7.03	562.86
75	16.89	8.11	564.46
80	10.81	9.19	566.05
85	4.73	10.27	567.65
90	-1.35	11.35	569.24



With the constrained given, the optimal solution is when $V_{4.5}$ is 37.5 gal, $V_{3.7}$ is 62.5 and V is 0 with the minimum cost at $552.50.

Example 4

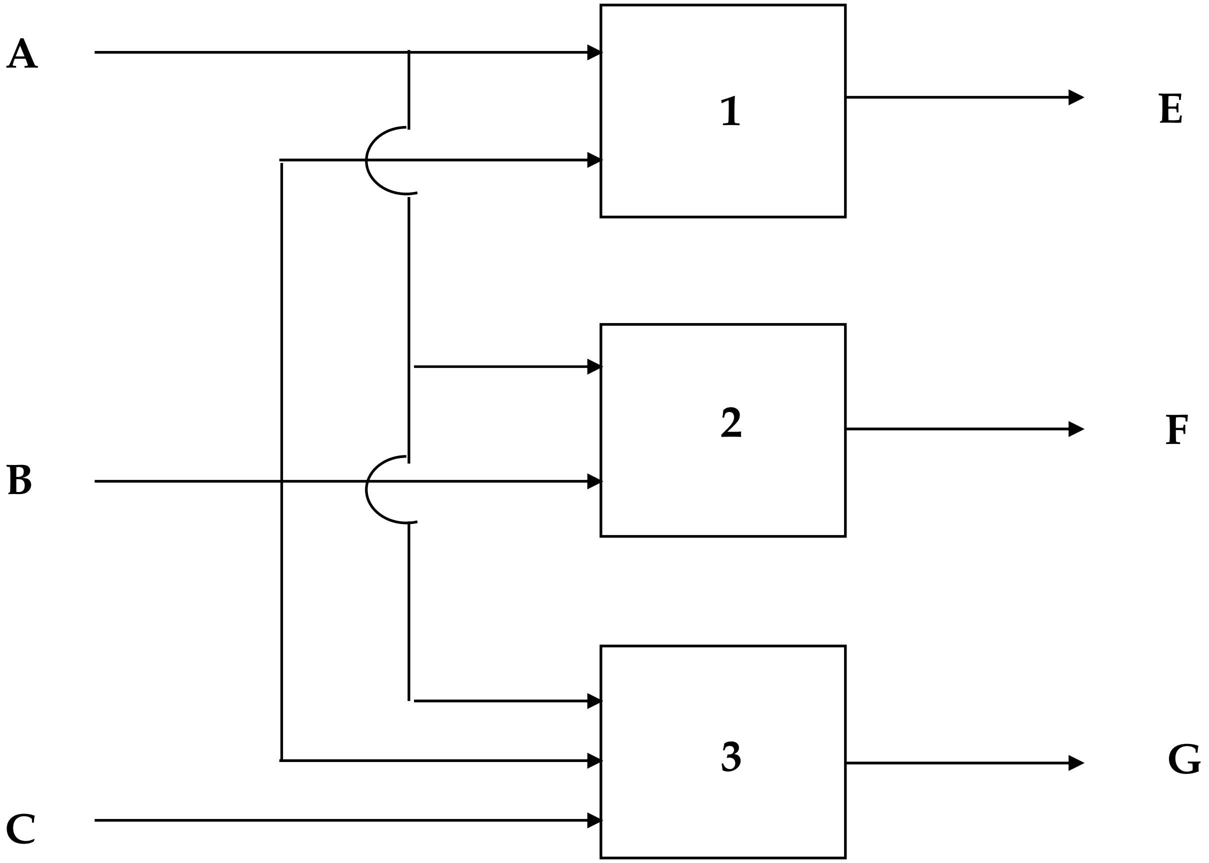
A chemical plant makes three products (E, F, G) and utilizes three raw materials (A, B, C) in limited supply. Each of the three products is produced in a separate process (1, 2, 3); a schematic of the plant is shown in Fig. 2. The available materials A, B and C do not have to be totally consumed. Find the optimum production to maximize the total operating profit per day in $/day.

Raw Material	Maximum available, lb/day	Cost, cent/lb
A	40,000	1.5
B	30,000	2.0
C	25,000	2.5

Process	Product	Reactant requirement (lb) per lb product	Processing cost	Selling price of product
1	E	2/3 A, 1/3 B	1.5 cent/lb E	4.0 cent/lb E
2	F	2/3 A, 1/3 B	0.5 cent/lb F	3.3 cent/lb F
3	G	1/2 A, 1/6 B, 1/3 C	1.0 cent/lb G	3.8 cent/lb G

The reactions involving A, B and C are as follows:

Process 1: A + B → E
Process 2: A + B → F
Process 3: 3A + 2B + C → G



Step 1 : Define the decision variable

$X_A, X_B, X_c, X_E, X_F, X_G$

Step 2 : Define the objective function

Profit = Income – raw material cost – Processing cost

- Income = $0.04\ x_E + 0.033\ x_F + 0.038\ x_G$
- Operating costs in $ per day include:
  - Raw material costs : $0.015x_A + 0.02x_B + 0.025x_C$
  - Processing costs: $0.015x_E + 0.005x_F + 0.01x_G$

$$Min\ f(x) = (0.04x_E + 0.033x_F + 0.038x_G) - (0.015x_A + 0.02x_B + 0.025x_C + 0.015x_E + 0.005x_F + 0.01x_G)$$

Step 3: Define equality & inequality constraint

- Equality constraint, from material balances,

$x_A = 0.667\ x_E + 0.667\ x_F + 0.5\ x_G$

$x_B = 0.333\ x_E + 0.333\ x_F + 0.167\ x_G$

$x_C = 0.333\ x_G$

$x_A + x_B + x_C = x_E + x_F + x_G$

- Inequality constraint

Three are also bounds on the amount of A, B, and C processed:

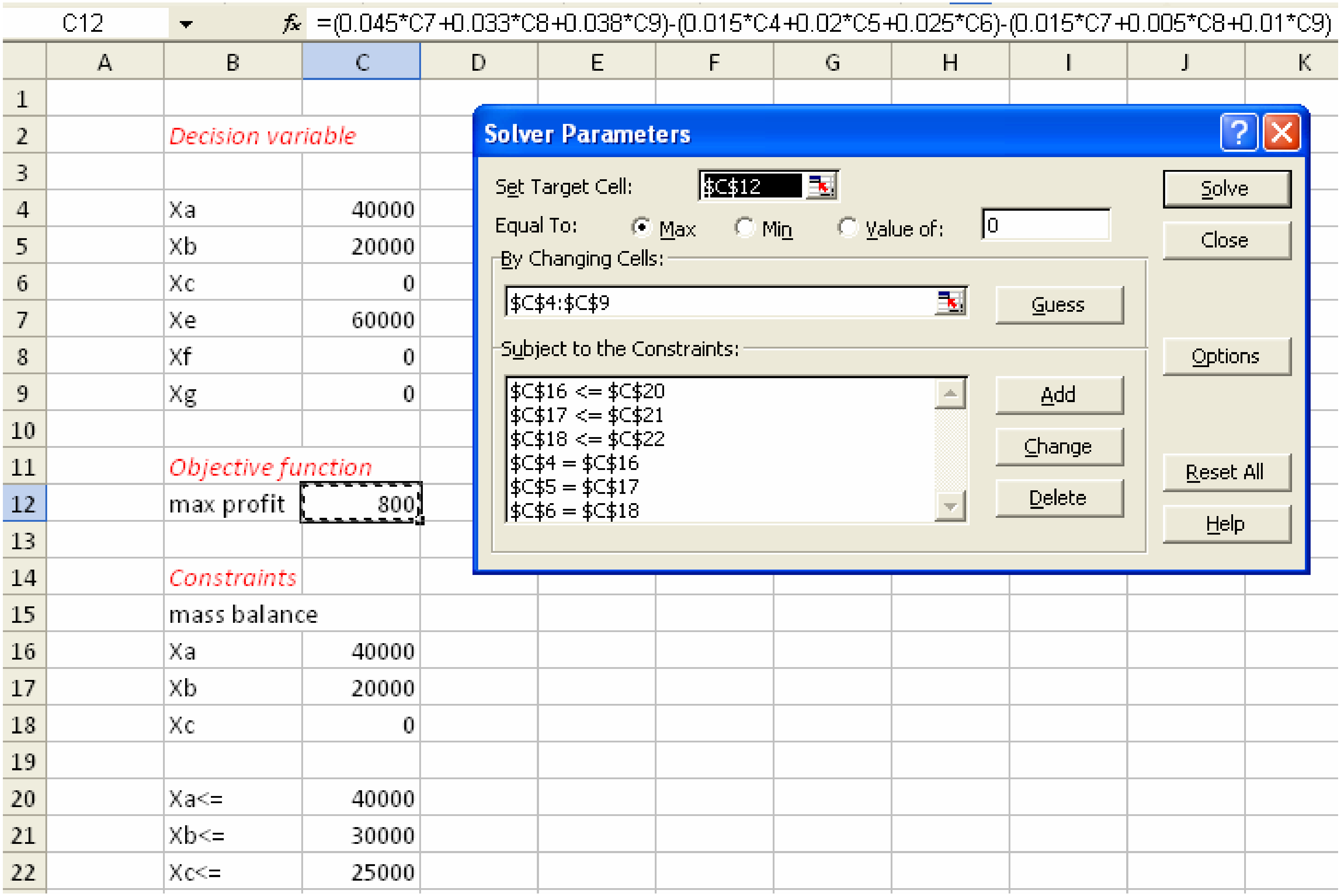
$x_A \leq 40{,}000$
$x_B \leq 30{,}000$
$x_C \leq 25{,}000$

Step 4: Simplification

Not applicable

Step 5: Choose the best suitable optimization technique

The optimization problem is comprised of a linear function and linear constrained, hence linear programming would be the best technique. However, this problem cannot be solve using graphical technique because it consists of more than 2 variables. Solve using computation e.g. Excel Solver.



C12 =(0.045*C7+0.033*C8+0.038*C9)-(0.015*C4+0.02*C5+0.025*C6)-(0.015*C7+0.005*C8+0.01*C9)

	A	B	C
1			
2		*Decision variable*	
3			
4		Xa	40000
5		Xb	20000
6		Xc	0
7		Xe	60000
8		Xf	0
9		Xg	0
10			
11		*Objective function*	
12		max profit	800
13			
14		*Constraints*	
15		mass balance	
16		Xa	40000
17		Xb	20000
18		Xc	0
19			
20		Xa<=	40000
21		Xb<=	30000
22		Xc<=	25000

Solver Parameters

Set Target Cell: $C$12

Equal To: ● Max ○ Min ○ Value of: 0

By Changing Cells: $C$4:$C$9

Subject to the Constraints:

$C$16 <= $C$20
$C$17 <= $C$21
$C$18 <= $C$22
$C$4 = $C$16
$C$5 = $C$17
$C$6 = $C$18

Try this one…

The Nusajaya Fertilizer Company produces two brands of lawn fertilizer - NJ1 and NJ2 at plants in Tanjung Langsat and Tampoi. The plant at Tanjung Langsat has resources available to produce 5,000 kg of fertilizer daily; the plant at Tampoi has enough resources to produce 6000 kg daily. The cost per kg of producing each brand at each plant is as follows:

	Plant	
Product	Tg. Langsat	Tampoi
NJ1	$2	$4
NJ2	$2	$3

The company has a daily budget of $45,000 for both plants combined. Based on past sales, the company knows the maximum demand which is 6000 kg for NJ1 and 7000 kg for NJ2 daily. The selling price is $9/kg for NJ1 and $7/kg for NJ2. The company wants to know how much of fertilizer per brand need to produce at each plant in order to maximize profit.

References

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