OPENCOURSEWARE

SKAA 1213 - Engineering Mechanics

TOPIC 13

Force System Resultants

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Lesson 13 Outline

- Introduction
- Undamped Vibration
- Damped Vibration

Introduction

- ***Vibration*** - Periodic motion of the system of connected bodies displaced from a position of equilibrium point .
- ***Free vibration*** - occurs when the motion is maintained/caused by gravitational or elastic restoring forces.
- ***Forced vibration*** –motion is influenced by an external force applied to the system to vibrate.
- Both of these types of vibration may be either damped or undamped.

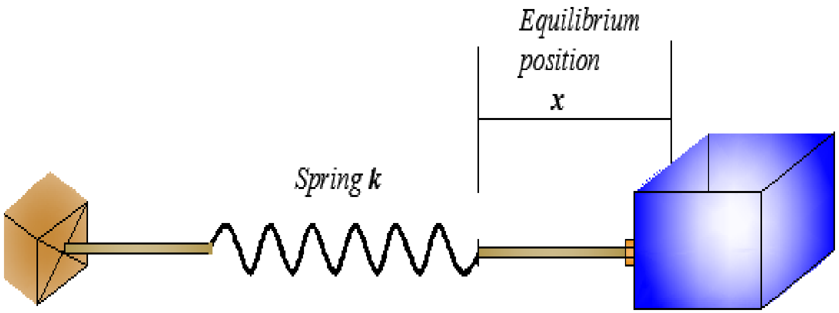
Introduction

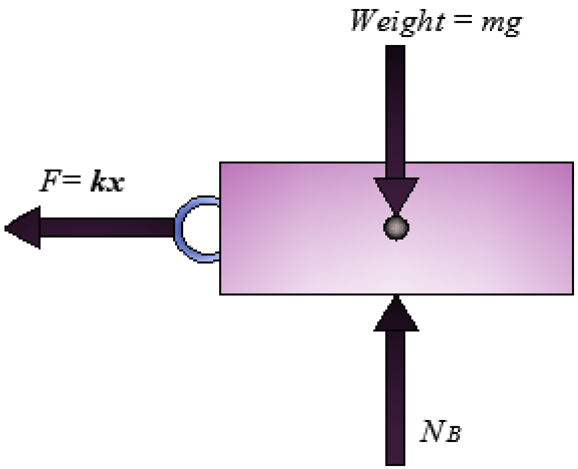
- ***Undamped vibrations*** – Vibration that can continue to vibrate due to the negligence of the frictional factors .

- ***Damped vibrations*** – Actual behavior of vibrations due to the existence of frictional forces from both internal & external sources.

Undamped Free Vibration

- Consider a spring of a stiffness, k which is attached to a block with mass m
- The Elastic restoring force from the spring, $F = kx$ is directed toward the equilibrium position
- Acceleration a is assumed to act in the direction of *positive displacement*





Undamped Free Vibration

Formulation

$$\overset{+}{\rightarrow} \sum F_x = ma_x; \qquad -kx = m\ddot{x}$$

- Rearranging,

$$\ddot{x} + \omega_n^2 x = 0$$

- ω_n is the ***natural frequency*** (rad/s),

$$\omega_n = \sqrt{\frac{k}{m}}$$

Undamped Free Vibration

- In vertical system:

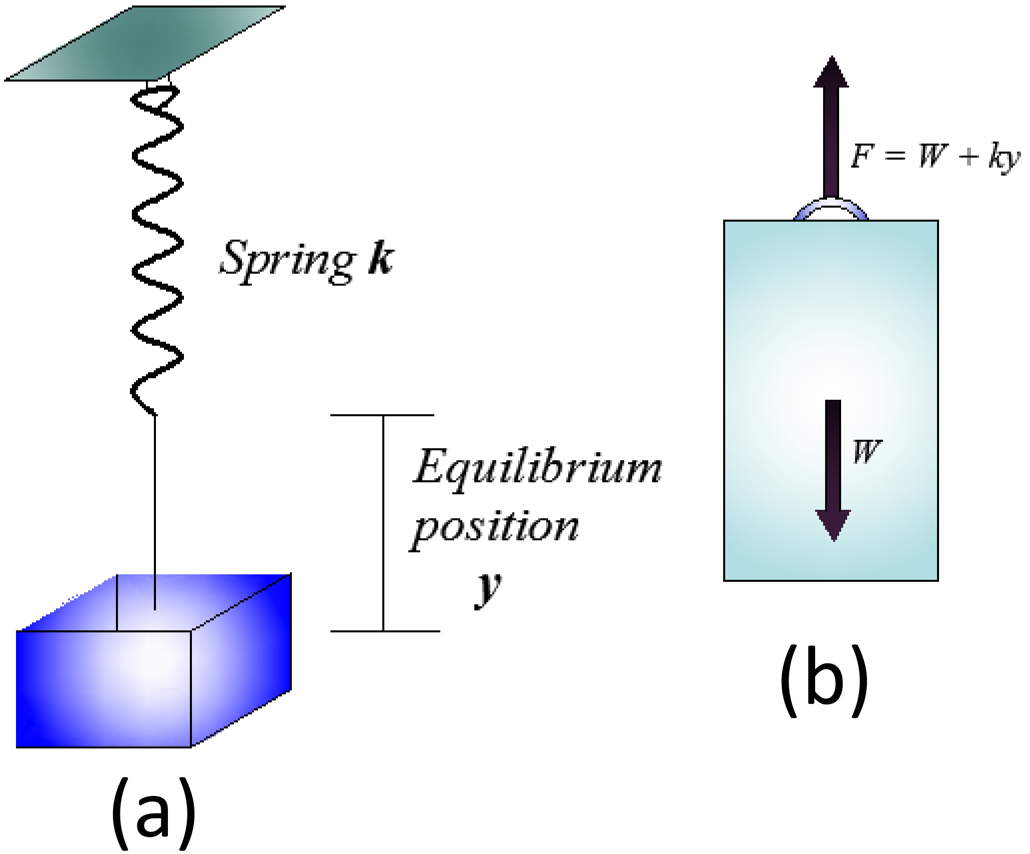
$$\downarrow + \sum F_y = ma_y;$$

$$-W - ky + W = m\ddot{y}$$

$$-ky = m\ddot{y}$$

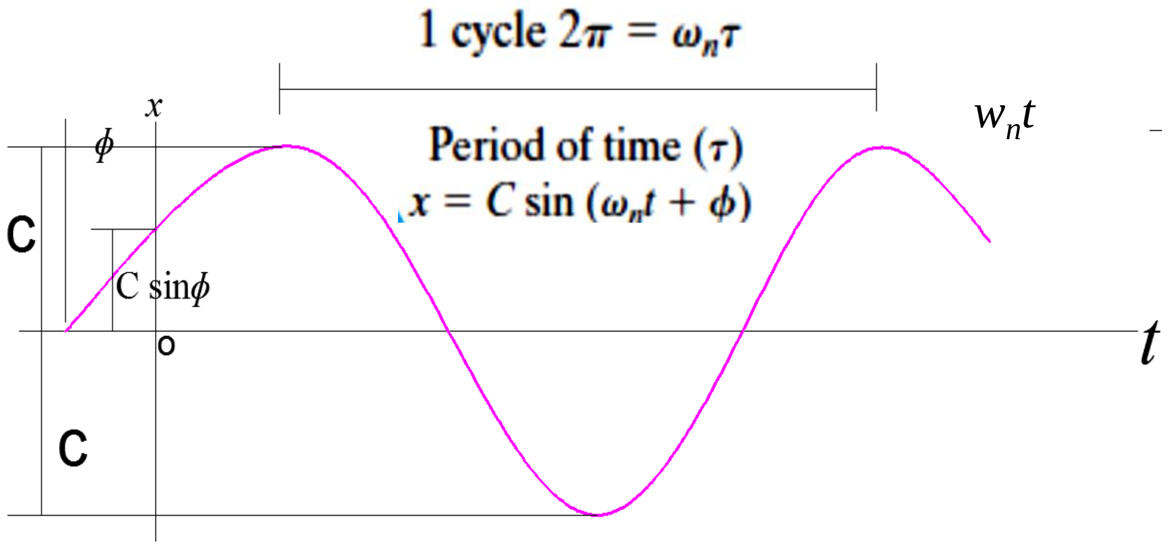
$$\ddot{y} + w_n^2 y = 0$$

$$\omega_n = \sqrt{\frac{k}{m}}$$



Undamped Free Vibration

- The undamped free vibration of the system can be represented by drawing x versus $w_n t$ graph as shown below:



Undamped Free Vibration

- ***Period*** – length of time:

$$\tau = \frac{2\pi}{\omega_n} \qquad \tau = 2\pi\sqrt{\frac{m}{k}}$$

- ***Frequency*** (Hz) - number of cycles completed per unit time

$$f = \frac{1}{\tau} = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Undamped Free Vibration

- In the case of horizontal movement:
- Function "x" is a homogeneous, second-order, linear, differential equation with constant coefficient:

$$x = A\sin\omega_n t + B\cos\omega_n t$$

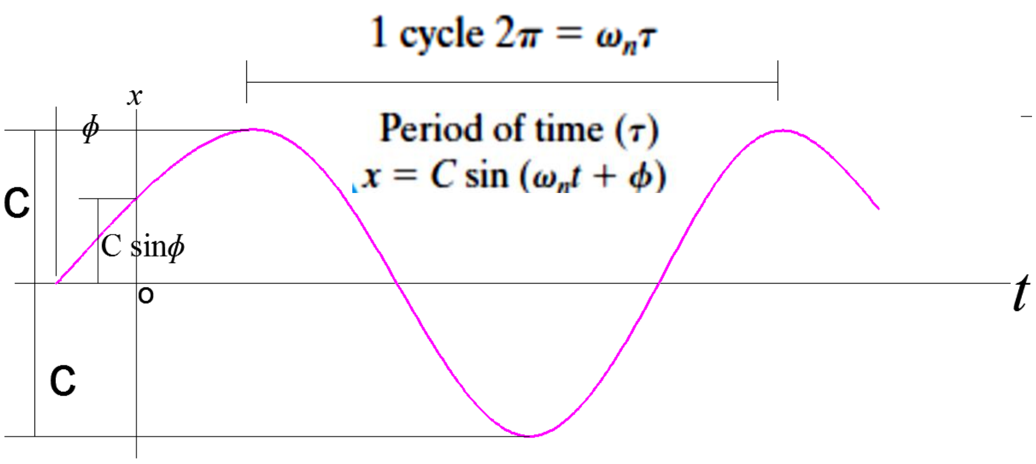
$$v = \frac{dx}{dt} = \dot{x} = A\omega_n\cos(\omega_n t) - B\omega_n\sin(\omega_n t)$$

$$a = \ddot{x} = -A(\omega_n)^2\sin(\omega_n t) - B(\omega_n)^2\cos(\omega_n t)$$

Undamped Free Vibration

- ***Amplitude, C***:

$$C = \sqrt{A^2 + B^2}$$



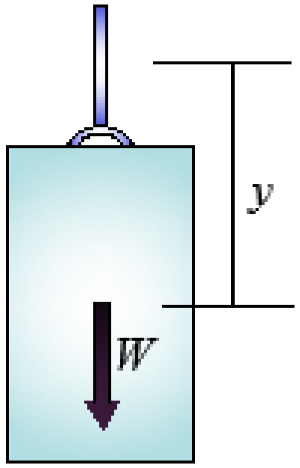
- ***Phase angle Φ***:

$$\phi = \tan^{-1}\frac{B}{A}$$

- A and B are constants which can be obtained from initial condition (t = 0, $x = x_o$)

Example 1

A spring is stretched 175 mm by an 8-kg block. If the block is displaced 100 mm downward from its equilibrium position and given a downward velocity of 1.5 m/s, determine the differential equation which describes the motion. Assume that positive displacement is downward. Also, determine the position of the block when t = 0.22 s.



[Answer : $y = 0.2003 \sin 7.487\, t + 0.1 \cos 7.487\, t$, $y = 0.192$ m]

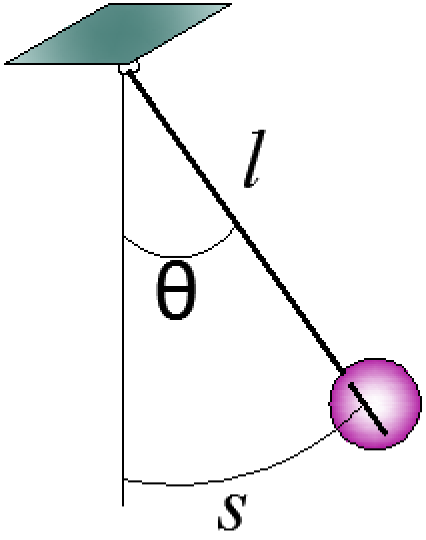
Example 2

When a 2-kg block is suspended from a spring, the spring is stretched a distance of 40 mm. Determine the frequency and the period of vibration for a 0.5-kg block attached to the same spring. *[Answer : f = 4.985 Hz, T = 0.201 s]*

Example 3

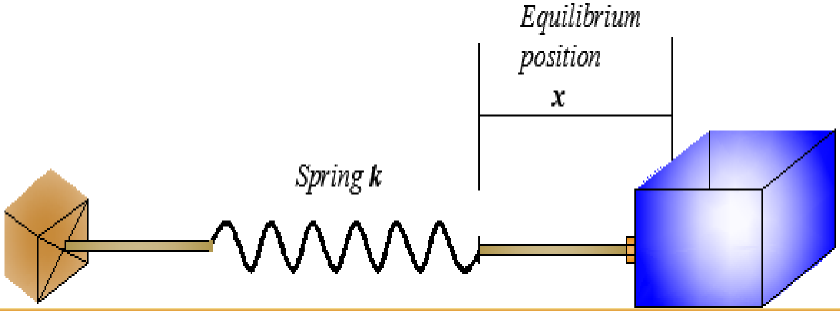
Find the the period of vibration for the simple pendulum shown. The bob has a mass *m* and is attached to a cord of length l.

$$Answer: f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$



Energy Methods

- Simple harmonic motion(SHM) of a body is due only to gravitational and elastic restoring forces acting on the body
- These types of forces are *conservative*
- When the block is displaced an arbitrary amount of x from equilibrium position, the kinetic energy is $T = ½\, mv^2 = ½\, mx^2$ and the potential energy is $V = ½\, kx^2$



Energy Methods

- conservation of energy equation states that ,

$$T + V = \text{constant}$$

$$\frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 = \text{constant}$$

- Differential equation describing the *accelerated motion* of the block can be expressed as

$$m\dot{x}\ddot{x} + kx\dot{x} = 0$$

$$\dot{x}(m\ddot{x} + kx) = 0$$

Energy Methods

- Since the velocity keep changing (not constant)throughout the motion , then

$$\ddot{x} + \omega_n^2 x = 0 \qquad \omega_n = \sqrt{k/m}$$

- If the energy is written for a *system of connected bodies*, the natural frequency or the equation of motion can also be determined by time differentiation

Viscous Damped Free Vibration

- Damping is attributed to the resistance created by the substance, such as water, air, oil, or in which the system vibrates
- This type of force developed under these circumstances is called a *viscous damping force*, $F = c\dot{x}$
- Damping occurs when the piston *P* moves within the enclosed cylinder

Viscous Damped Free Vibration

- Applying the equation of motion yields

$$\rightarrow \sum Fx = ma_x; \qquad -kx - c\dot{x} = m\ddot{x}$$

$$\text{or} \quad m\ddot{x} + c\dot{x} + kx = 0$$

- This linear, second-order, homogeneous, differential equation has solutions of $x = e^{\lambda t}$
- We can obtain two values of λ,

$$\lambda_1 = -\frac{c}{2m} + \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

$$\lambda_2 = -\frac{c}{2m} - \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

Viscous Damped Free Vibration

- The critical damping coefficient c_c is expressed as

$$\left(\frac{c_c}{2m}\right)^2 - \frac{k}{m} = 0 \Rightarrow c_c = 2m\sqrt{\frac{k}{m}} = 2m\omega_n$$

Overdamped System

- When $c < c_c$, the roots λ_1 and λ_2 are both real

$$x = Ae^{\lambda_1 t} + Be^{\lambda_2 t}$$

- Motion corresponding to this solution is *non-vibrating*

Viscous Damped Free Vibration

Critically Damped System

- If $c = c_c$, then $\lambda_1 = \lambda_2 = - c_c/2m = \omega_n$, we have

$$x = (A + B)e^{-\omega_n t}$$

Underdamped System

- Under some Cases where $c < c_c$
- The roots λ_1 and λ_2 are complex numbers and the general solution is

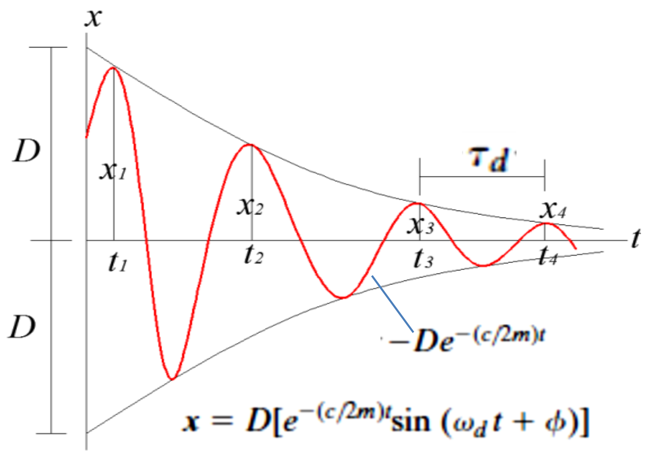
$$x = D\left[e^{-(c/2m)t}\sin(\omega_d t + \phi)\right]$$

Viscous Damped Free Vibration

Underdamped System

- The constant ω_d is called the *damped natural frequency* of the system, and has a value of

$$\omega_d = \sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2} = \omega_n \sqrt{1 - \left(\frac{c}{c_c}\right)^2}$$



- c/c_c is called *damping factor*
- Period of damped vibration is

$$\tau_d = \frac{2\pi}{\omega_d}$$

The End of Lesson 13