SKAA 1213 - Engineering Mechanics

5
Equilibrium of a Rigid Body

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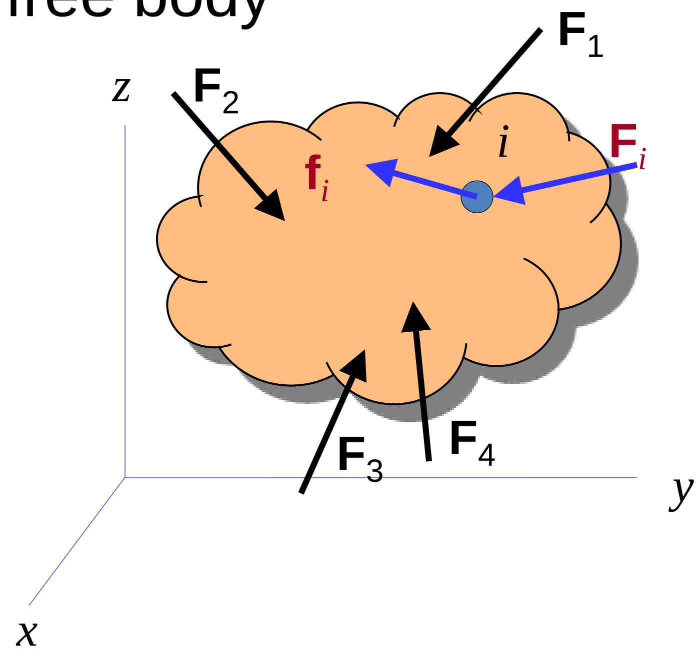
Condition for Rigid Body Equilibrium

Consider a rigid body in equilibrium , a free body of a particle *i.*



$\mathbf{F}_i$ is the **external force** due to gravitational, contact forces between the particle and the surrounding bodies.

$\mathbf{F}_i$ is the resultant **internal force** caused by interaction within adjacent particles.

According to Newton's 1st law; $\mathbf{F}_i + \mathbf{f}_i = 0$, that is $\Sigma\mathbf{F}_i + \Sigma\mathbf{f}_i = 0$

$\Sigma\mathbf{f}_i = 0$ since the internal forces will be cancelled due to the collide between particles within the body which have equal magnitude but opposite collinear pairs (Newton 3rd law).

$$\Sigma\mathbf{F}_i = 0 \quad \text{or} \quad \Sigma\mathbf{F} = 0$$

From the **moments** of forces acting on the particle about point O.

$$\mathbf{r}_i \times (\mathbf{F}_i + \mathbf{f}_i) = \mathbf{r}_i \times \mathbf{F}_i + \mathbf{r}_i \times \mathbf{f}_i = 0$$

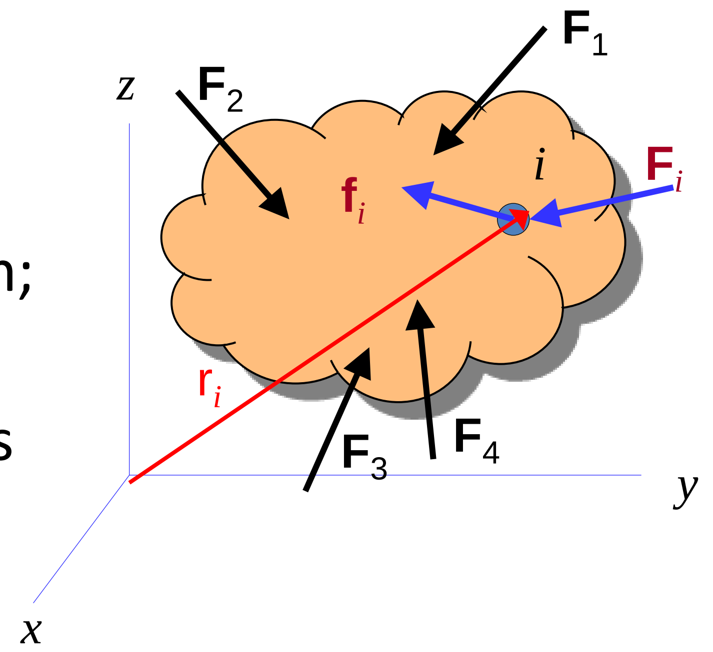
Applying to other particles and adding them;

$$\Sigma \mathbf{r}_i \times \mathbf{F}_i + \Sigma \mathbf{r}_i \times \mathbf{f}_i = 0$$

The *second term* is 0 because internal forces occur in equal and opposite collinear pairs

Therefore, resultant moment about O,

$$\Sigma \mathbf{M}_o = \Sigma \mathbf{r}_i \times \mathbf{F}_i = 0$$



Conclusion : Two equations of equilibrium for rigid body are

$$\Sigma \mathbf{F} = 0$$
$$\Sigma \mathbf{M}_o = 0$$

Free Body Diagrams (FBD)

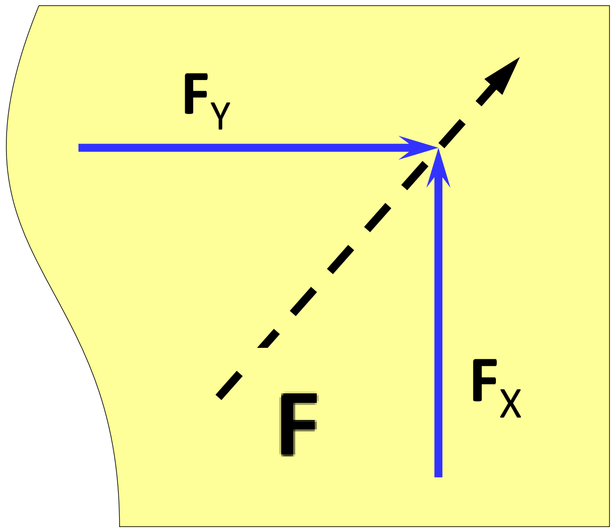
A FBD is a way of sketching a body which represents it's being isolated from the surroundings, and show all the forces and couple moments that are acting on the body.

When the body is being isolated from the surroundings , types of reaction must be applied correctly. There are few types of reactions that can occur at supports and point of supports between bodies.

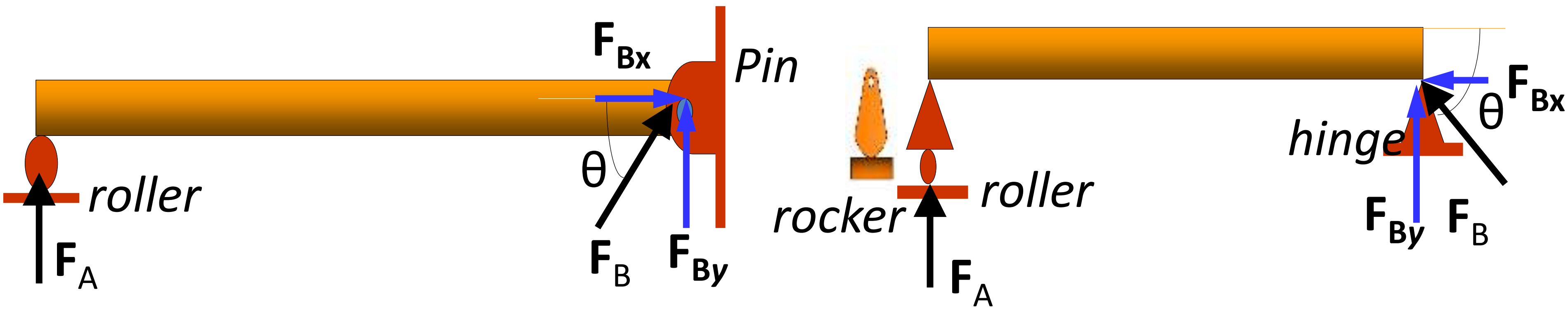
Equilibrium in Two Dimensions

Support Reactions in 2D

Resolution of force to rectangular components for analytical purposes



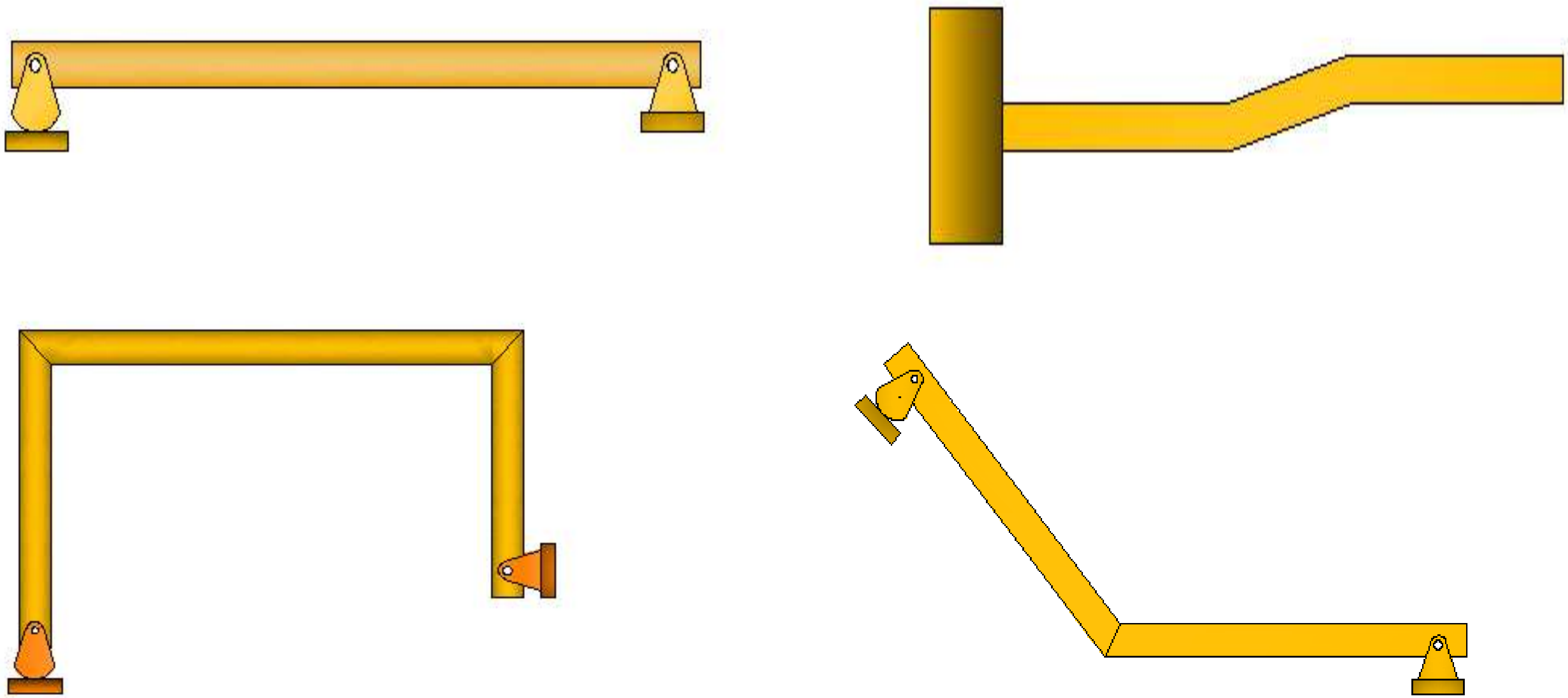
1) If translation is prevented by a body in a given ,then a force is developed on the body in that direction.



2) If rotation is prevented, a couple moment is acting on the body.

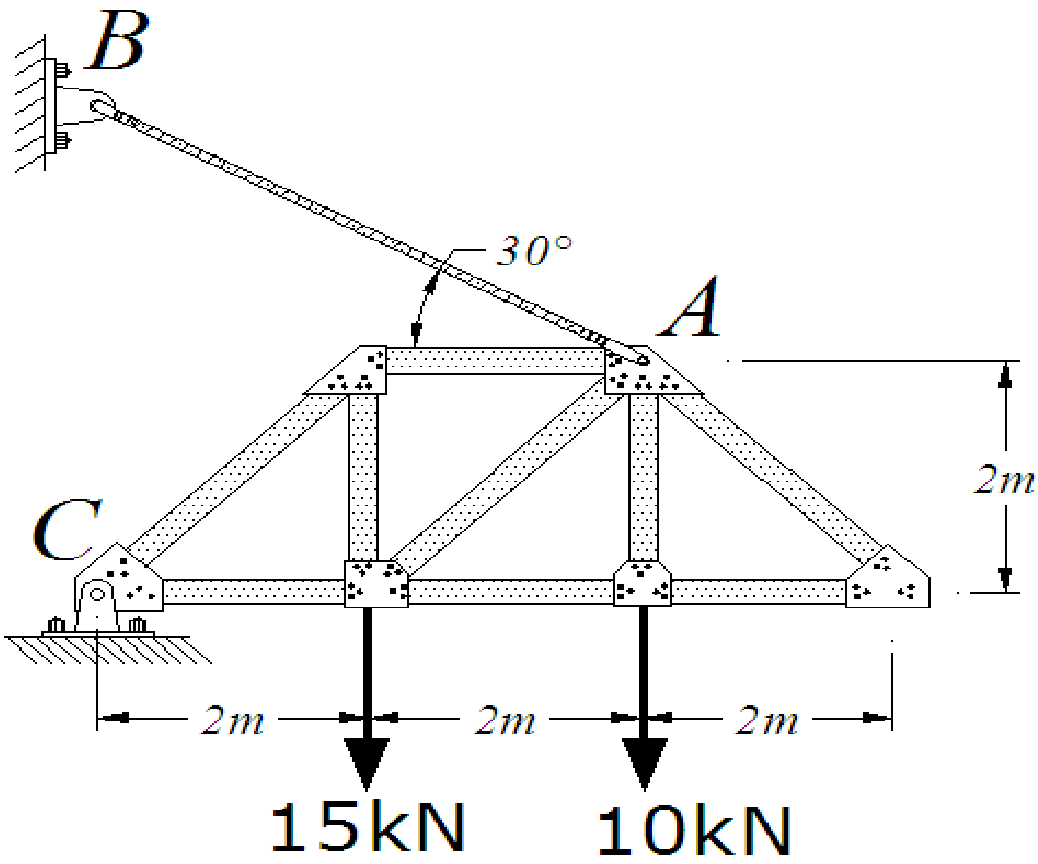
Example 1

Draw the reaction components of the supports of the following structure.



Example 2

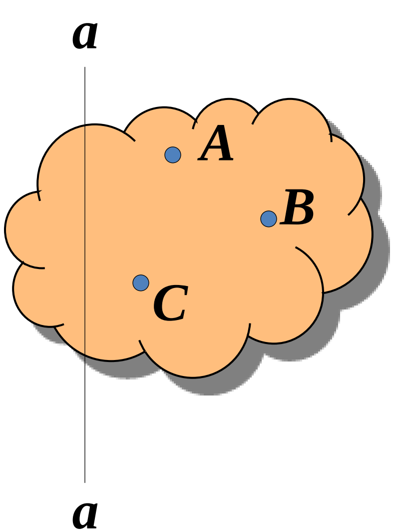
Draw the free body diagram of the truss that is supported by the cable AB and pin C.



Equations of Equilibrium

$$\Sigma F_x = 0 \;, \Sigma F_y = 0, \Sigma M_o = 0$$

Under certain situation, some alternative sets of three independent equations can solve problem in equilibrium.



A line passing through points A and B must NOT be perpendicular to a- axis

$\Sigma F_a = 0$
$\Sigma M_A = 0$
$\Sigma M_B = 0$

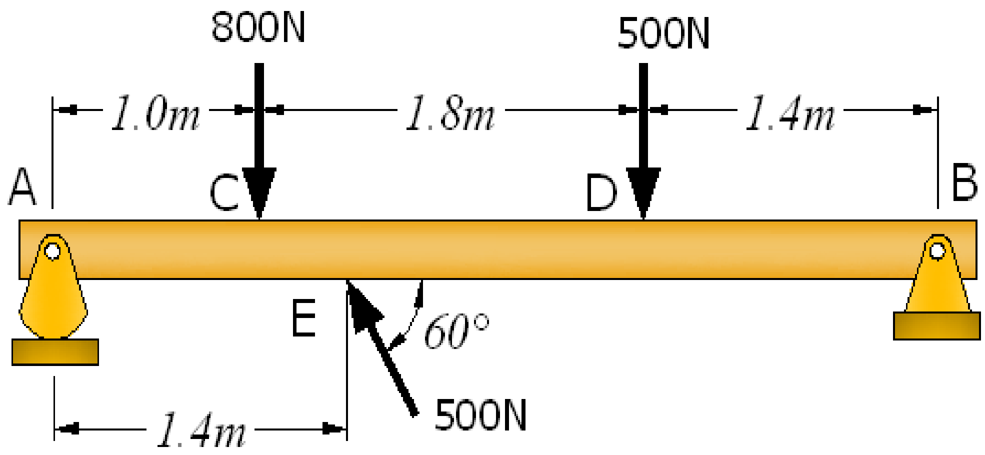
Points A, B and C must NOT lie on a straight line

$\Sigma M_A = 0$
$\Sigma M_B = 0$
$\Sigma M_C = 0$

Example 3

Determine the reactions at supports A and B.

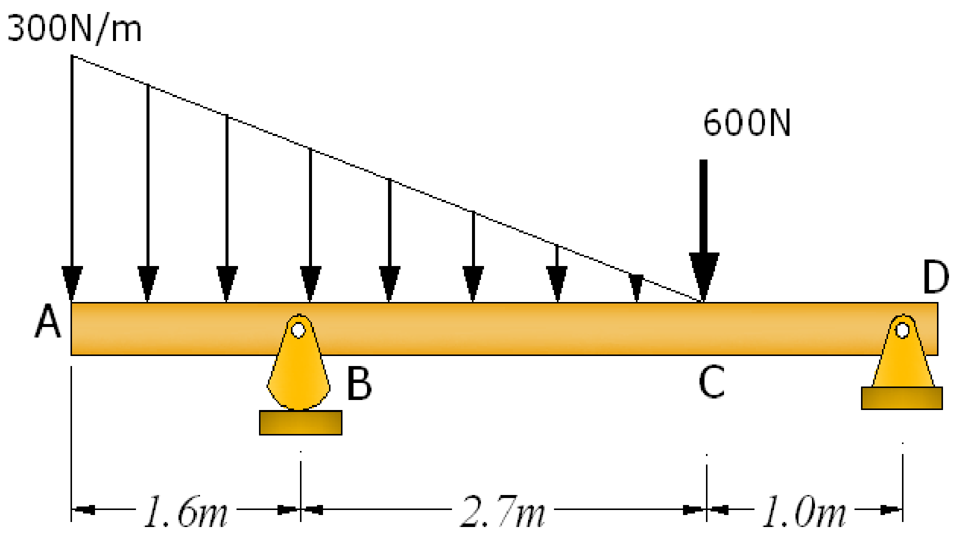
[Answer : B_x = 250 N , B_y = 379.47 N , A_y = 487.52 N]



Example 4

Determine the reactions at supports B and D.

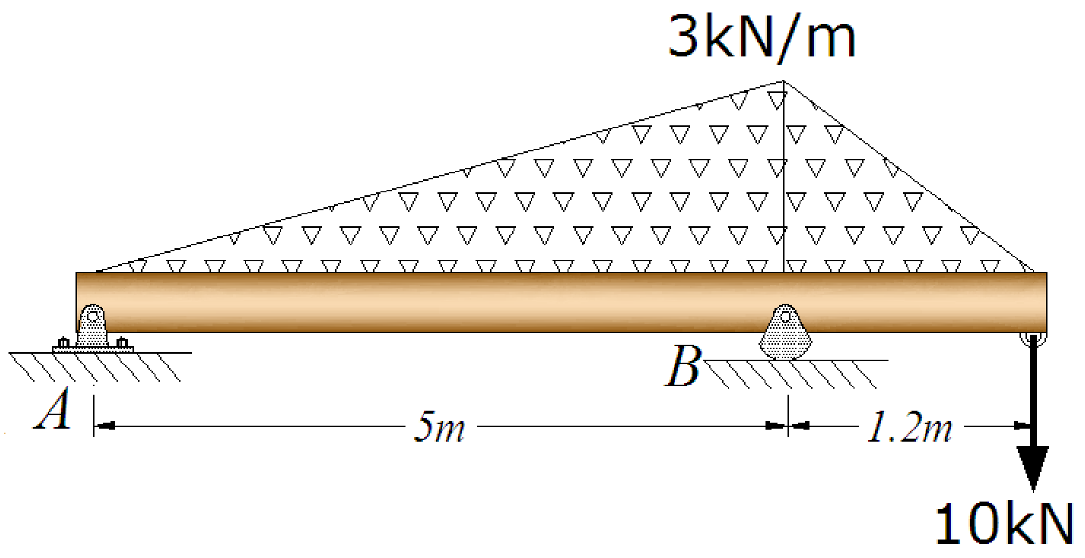
[Answer : D_y = 408 N , B_y = 837 N]



Example 5

Determine the reactions at A and B.

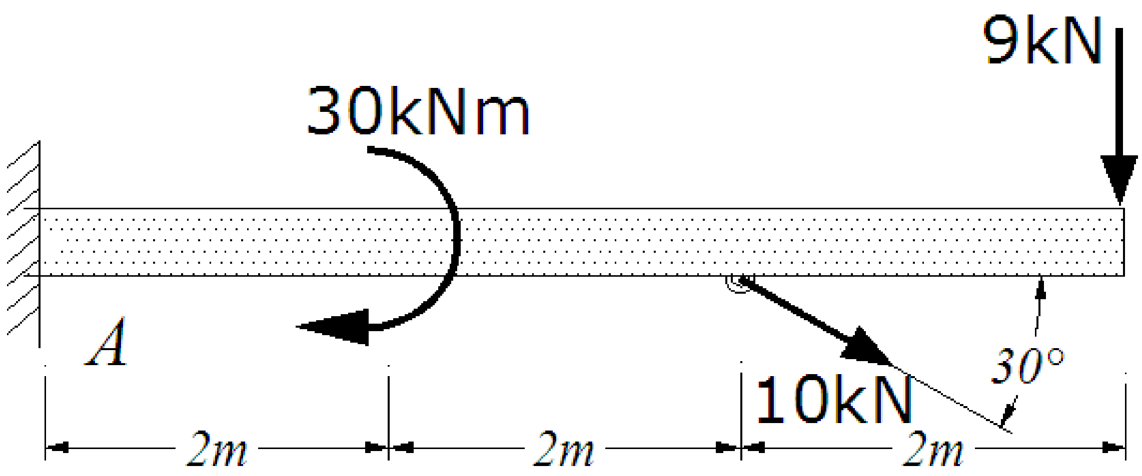
[Answer : $A_x = 0$,$B_y = 19.3N$, $A_y = 0N$ }



Example 6

Determine the reactions at field support A .

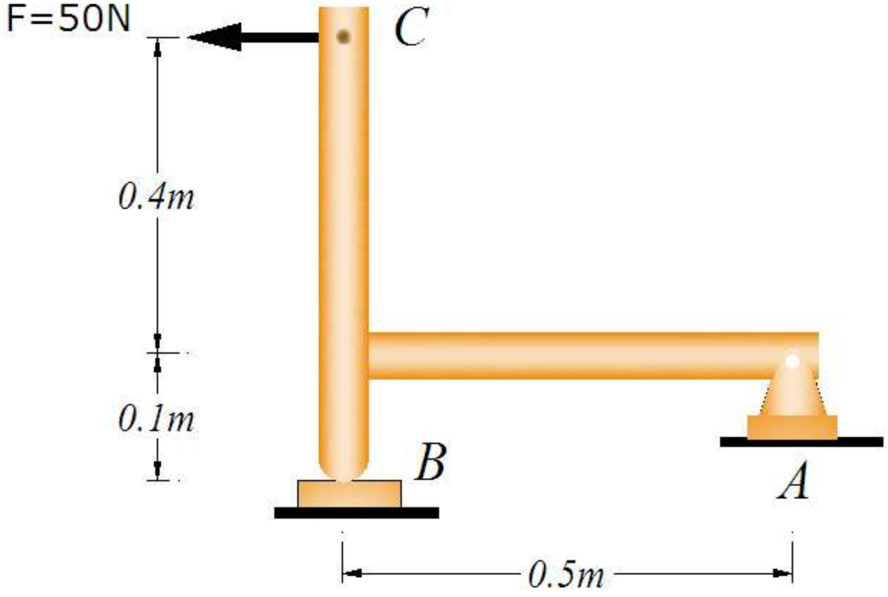
[Answer : A_x = 8.7kN , A_y = 14kN , M_A = 104kNm]



Example 7

Determine the vertical and horizontal components of reaction. Neglect the weight of the structure.

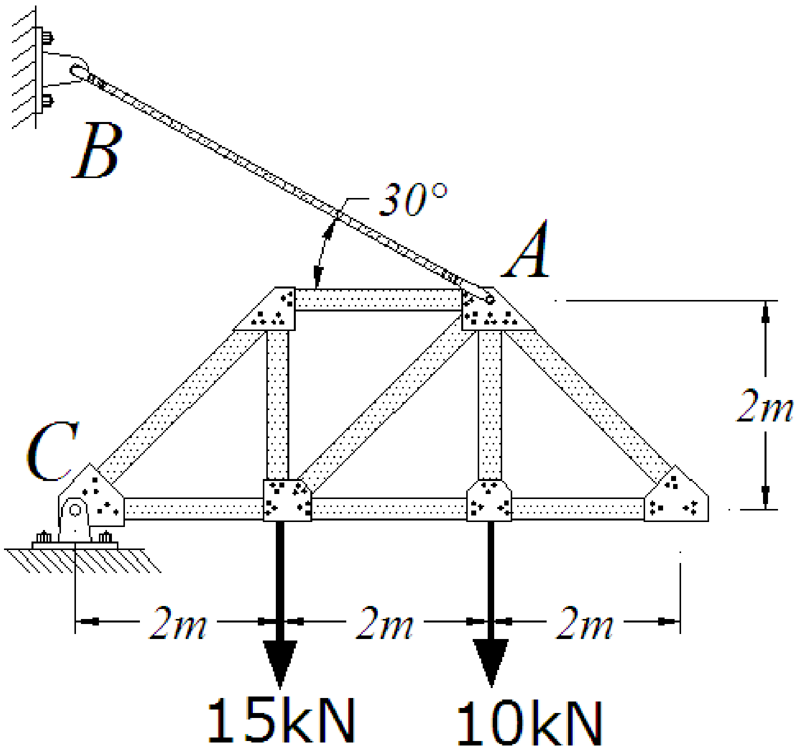
[Answer : A_x = 50 N , A_y = -40 N , B_y = 40 N]



Example 8

Determine the reactions at B and C.

[Answer : C_x = 16.2N , C_y = 15.7N , T = 18.7N]



Equilibrium in Three Dimensions(3D)

$$\Sigma \mathbf{F} = 0 \, , \, \Sigma \mathbf{M}_o = 0$$

In Cartesian vector form ; it can be written as

$$\Sigma F = \Sigma F_x i + \Sigma F_y j + \Sigma F_z k$$

$$\Sigma M_o = \Sigma M_x i + \Sigma M_y j + \Sigma M_z k$$

Condition to satisfy the equation above :

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma F_z = 0$$

These 6 scalar equilibrium equations may be used to solve 6 unknowns shown in the free-body diagram.

$$\Sigma M_x = 0$$
$$\Sigma M_y = 0$$
$$\Sigma M_z = 0$$

Example 9

Determine the x, y, z components of reaction at the fixed wall A if $\mathbf{F}_1$={50**i** + 40**j** - 25**k**}N and $\mathbf{F}_2$={40**i** - 35**j** - 20**k**}N.

[Answer : A_x= -90N , A_y = -5NA_z = 45N , M_x = 75Ncm,M_y = -60Ncm, M_z = 255Ncm]