Prestressed Concrete Design

(SAB 4323)

Elastic Analysis of Section for Flexure

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Stages of Loading

The analysis of prestressed members can be different for the different stages of loading. The stages of loading are as follows:

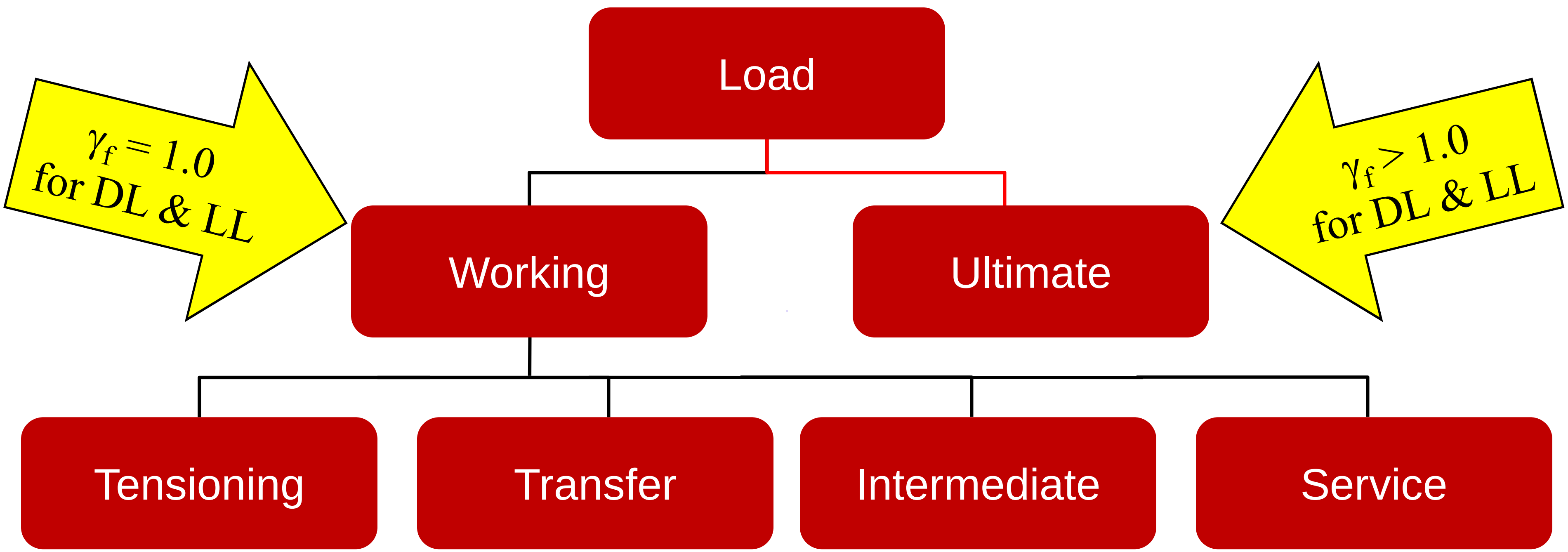
- **Initial** : It can be subdivided into two stages:
  - a) During tensioning of steel
  - b) At transfer of prestress to concrete.
- **Intermediate** : This includes the loads during handling, transportation and erection of the prestressed members
- **Final** : It can be subdivided into two stages:
  - a) At service, during operation.
  - b) At ultimate, during extreme events.

UTM

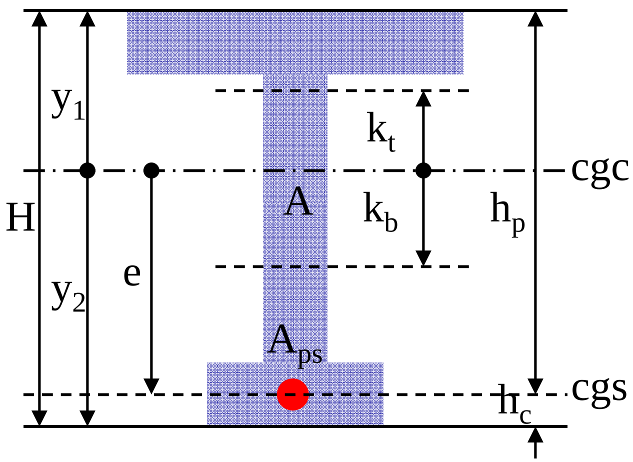
Stages of Loading

- Normally the most critical loading conditions are:
  - Initial Loading (Transfer)
  - Final Loading (Service)
- For certain cases, some intermediate loading, such as special conditions during handling, transportation and erection of precast elements **may** become critical

Loadings to be Considered



Notation & Sign Convention



A – area of cross section

A_{ps} – area of prestressing steel

I – moment of inertia of section

cgc – centre of gravity of concrete

cgs – centre of gravity of steel

e – eccentricity of the prestressing steel

h_c – concrete cover to cgs

y_1 – distance of extreme top fibres from cgc

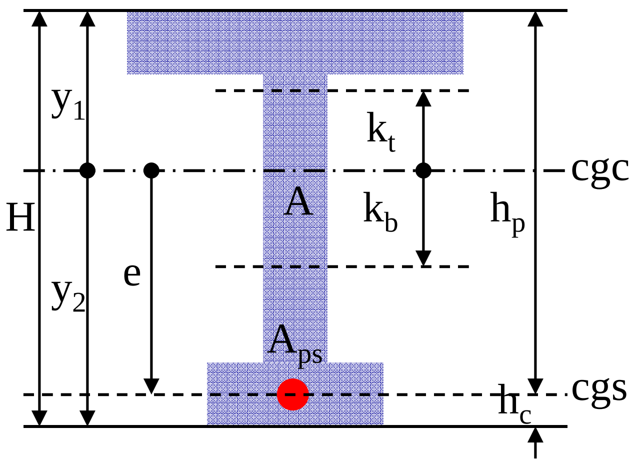
y_2 – distance of extreme bottom fibres from cgc

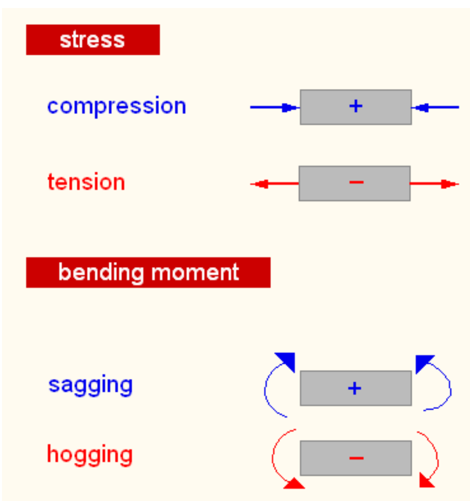
z_1 – section modulus of top fibre = I/y_1

z_2 – section modulus of bottom fibre = I/y_2

k_t – distance from cgc to upper limit of central kern

k_b – distance from cgc to lower limit of central kern





f_{1t} – extreme top fibre stress at transfer

f_{2t} – extreme bottom fibre stress at transfer

f_{1s} – extreme top fibre stress at service

f_{2s} – extreme bottom fibre stress at service

f_{ct} – allowable compressive stress at transfer

f_{tt} – allowable tensile stress at transfer

f_{cs} – allowable compressive stress at service

f_{ts} – allowable tensile stress at service

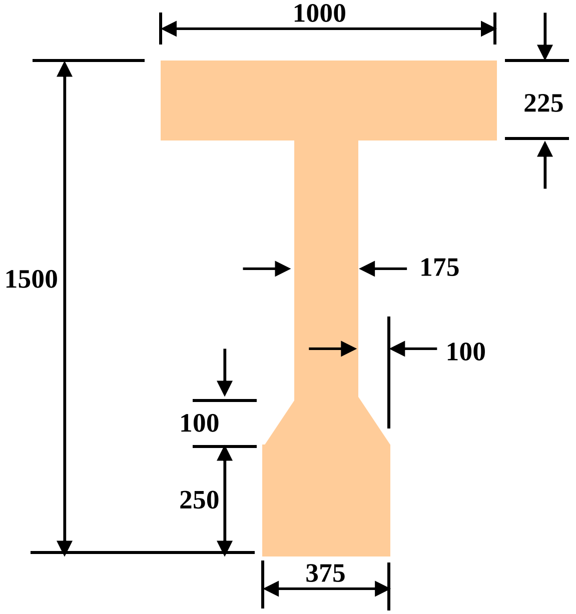
P_i – inertial prestress force

α – coefficient of short term losses

β – coefficient of long term losses

Example 2-1

- Determine the geometric property of the section shown below

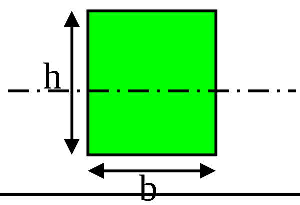
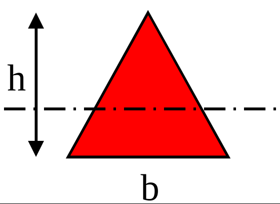
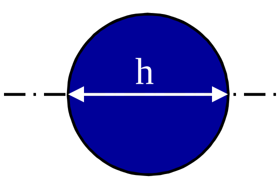


All dimensions are in mm

Geometric Properties

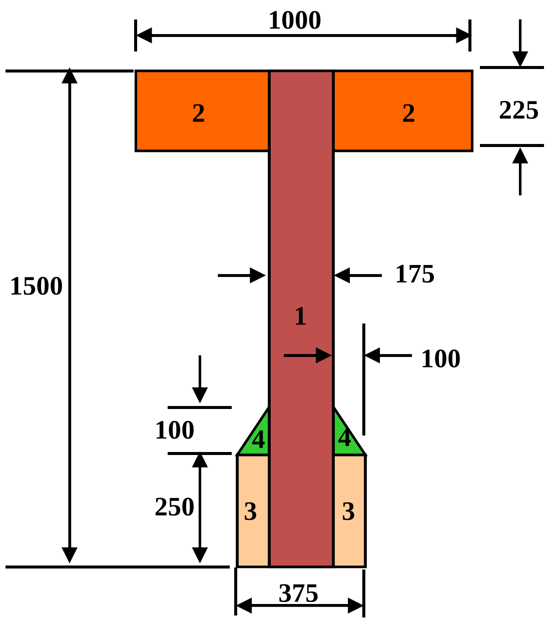
- Area of Section, A
- y_1 & y_2
- I
- z_1 & z_2
- k_t & k_b

Geometric Properties of Basic Shapes

Shape	y_1 & y_2	Area	2^{nd} Moment of Area
	y_1=h/2 y_2=h/2	A = b x h	I = b x h^3 / 12
	y_1=2h/3 y_2=h/3	A = b x h / 2	I = b x h^3 / 36
	y_1=h/2 y_2=h/2	A = π x h^2 / 4	I = = π x h^4 / 64

Solution

- Divide the section into the basic geometrical shapes as shown



1. Calculate area of section, A

A1: 175 x 1500 = 262,500

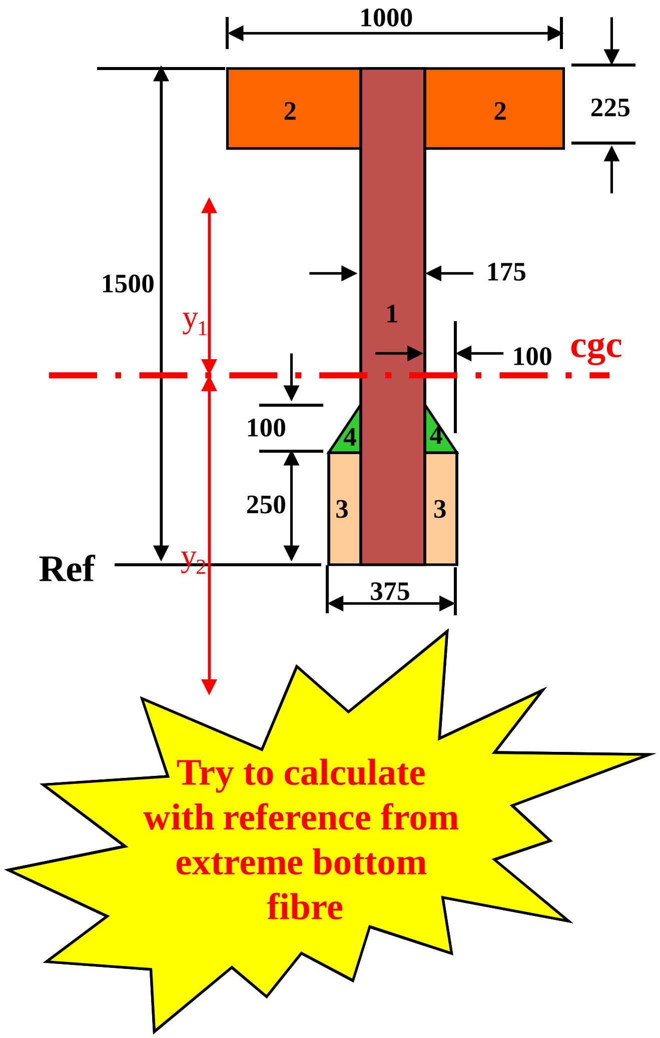
A2: (1000-175) x 225 = 185,625

A3: (375-175) x 250 = 50,000

A4: 2 x 0.5 x 100 x 100 = 10,000

Total, A = 508,125 mm^2

Solution



2. Calculate the distance of the centroid from bottom fibre, y_2

$y_2 = \sum A_i y_i / A$

A_1y_1: 262,500 x 1500/2 = 196,875 x 10^3

A_2y_2: 185,625 x (1500-225/2) = 257,554 x 10^3

A_3y_3: 50,000 x 250/2 = 6,250 x 10^3

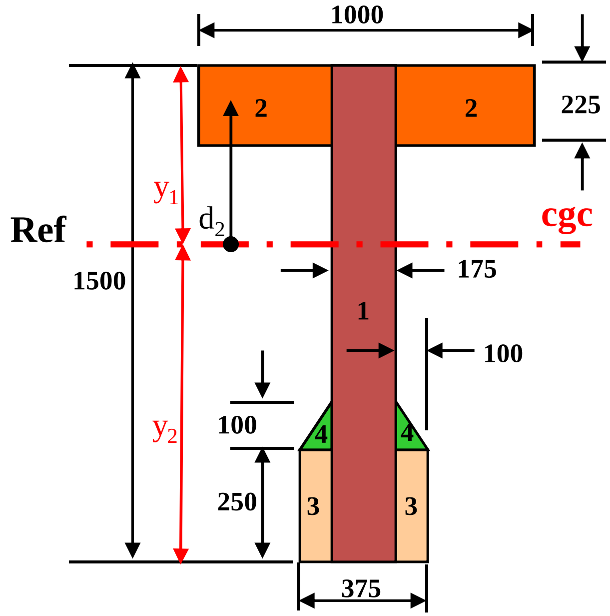
A_4y_4: 10,000 x (250+100/3) = 2,833 x 10^3

$\sum A_i y_i$ = 463,513 x 10^3

Therefore, y_2 = 463,513 x 10^3/ 508,125 = 912 mm

y_1 = 1500 – 912 = 588 mm

Solution



3. Calculate I

$I = \sum (I_i + A_i d_i^2)$

1: $175 \times 1500^3/12 + 262{,}500 \times (1500/2 - 588)^2$

$= 56.10 \times 10^9$

2: $(1000-175) \times 225^3/12 + 185{,}625 \times (588 - 225/2)^2$

$= 42.75 \times 10^9$

3: $(375-175) \times 250^3/12 + 50{,}000 \times (1500-588 - 250/2)^2$

$= 31.23 \times 10^9$

4: $2 \times 100 \times 100^3/36 + 10{,}000 \times (1500-588 - 250 - 100/3)^2$

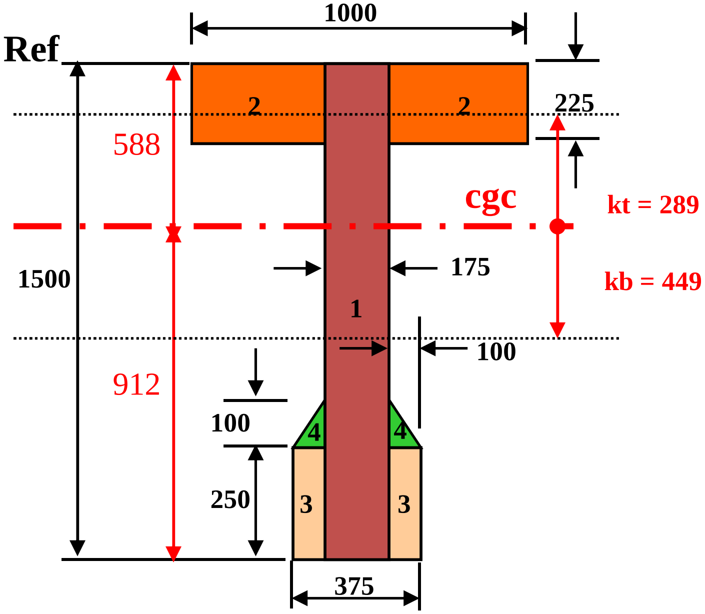
$= 3.92 \times 10^9$

Therefore, $I = (56.10 + 42.75 + 31.23 + 3.92) \times 10^9$

$= 134 \times 10^9 \text{ mm}^4$

Try to calculate with reference from extreme bottom fibre

Solution



4. Section moduli

$z_1 = I/y_1 = 228 \times 10^6 \text{ mm}^3$

$z_2 = I/y_2 = 147 \times 10^6 \text{ mm}^3$

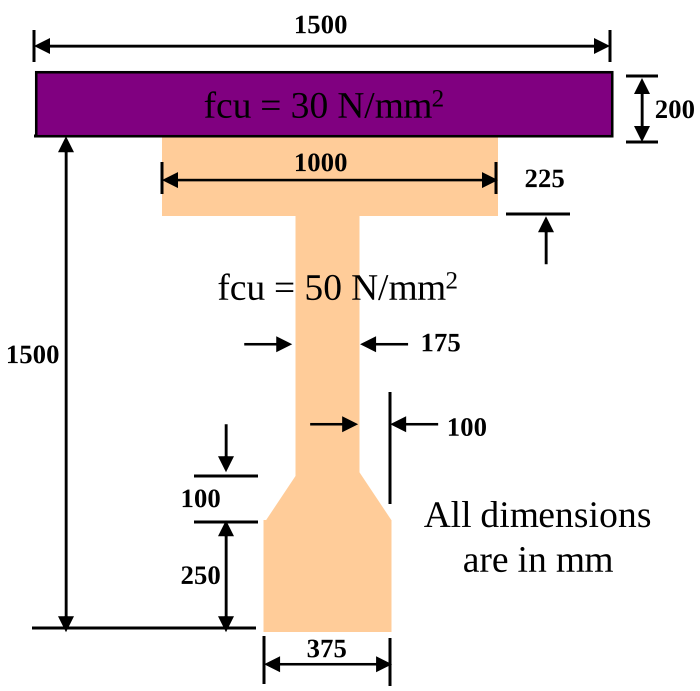
5. Kern distances

$k_t = z_2/A = 289 \text{ mm}$

$k_b = z_1/A = 449 \text{ mm}$

Example 2-2

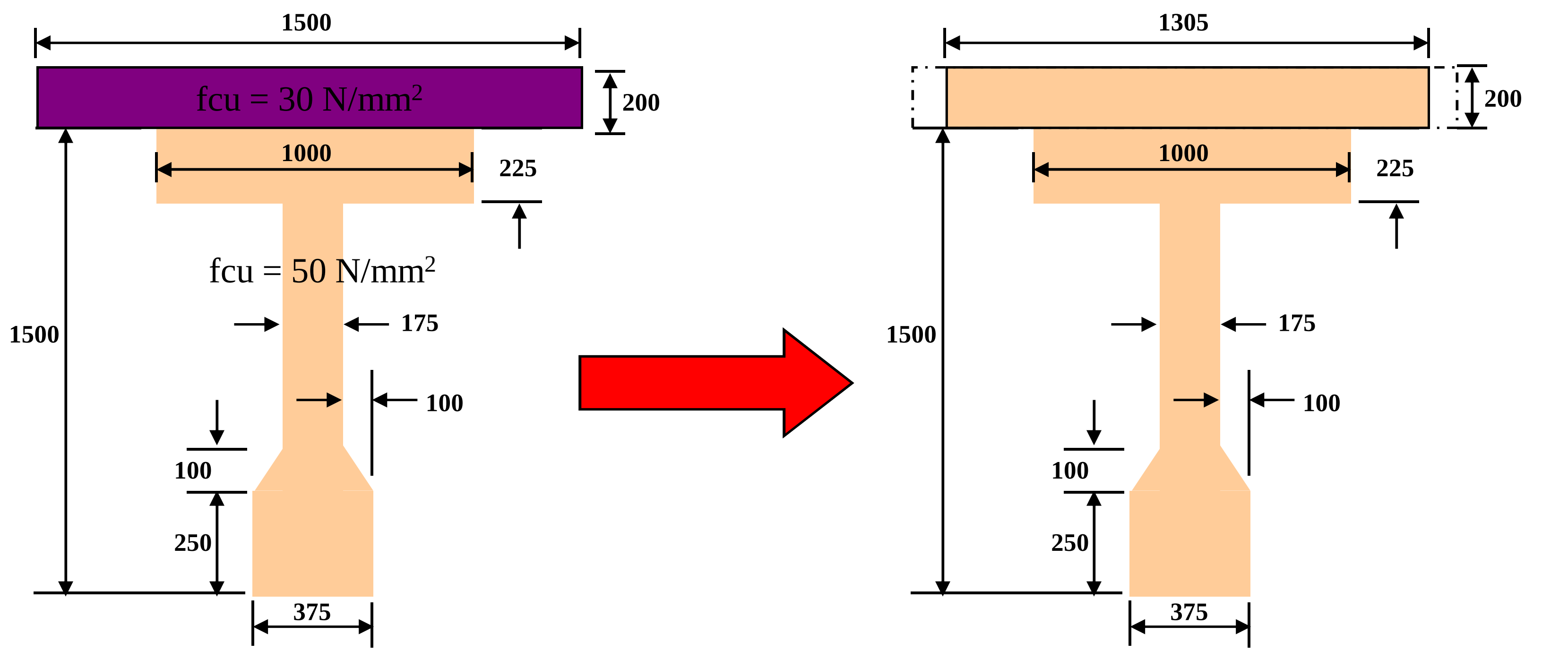
- Determine the geometric property of the composite section shown below



- calculate modular ratio, m
- $m = E_{in\text{-}situ}/E_{precast}$
- $fcu=50N/mm^2$, $E=30\ kN/mm^2$
- $fcu=30N/mm^2$, $E=26\ kN/mm^2$
- E from Table 7.2 BS 8110 Part2
- Modify cast insitu length by m
- m = 26/30 = 0.87
- new bf = 0.87*1500=1305mm

Solution

- Transformation of section

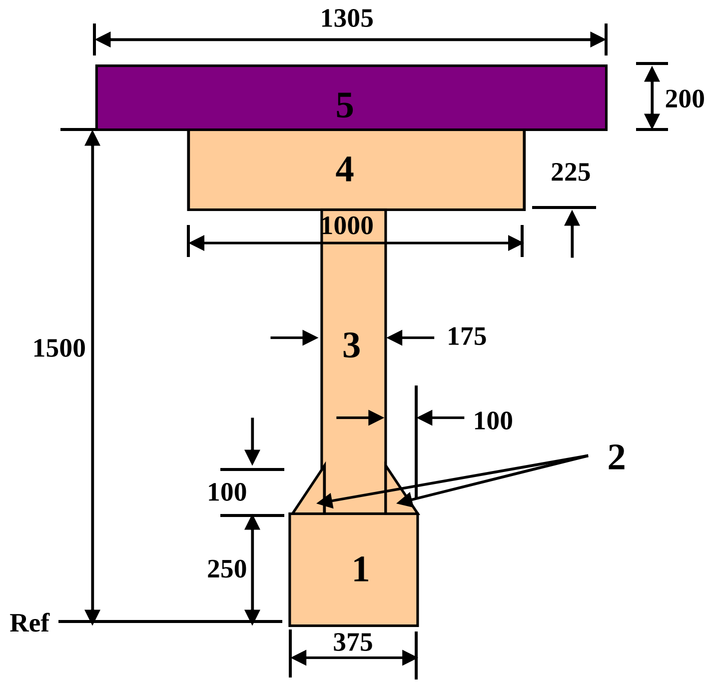


Solution

- Calculate the section properties from scratch
- Calculate the section properties based on the known precast section's properties

Solution

Calculate the section properties from scratch



Solution

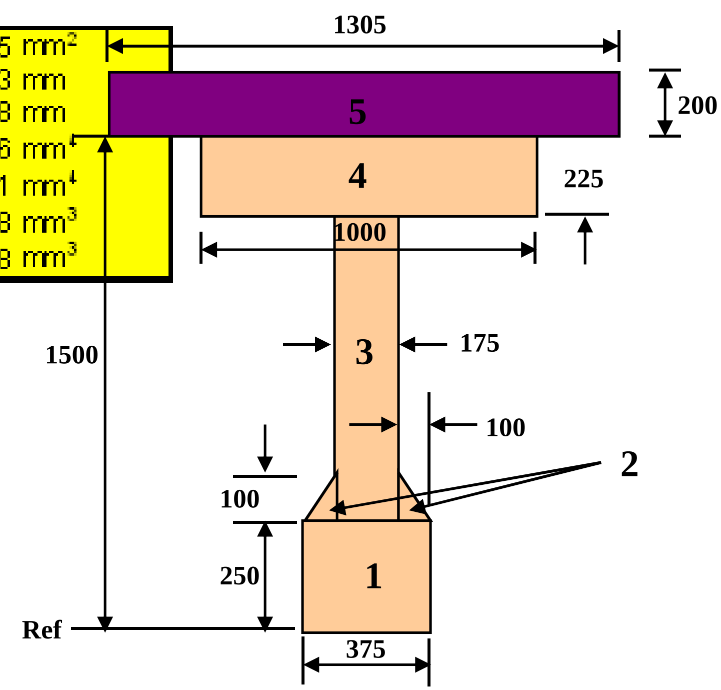
Sectional Properties Calculations (with reference from soffit of section)

H = 1700 mm

No	Type	b	h	ref_l	A	y	yi	Ayi	I	di	Adi^2
1	R	375	250	0	93750	125	125	11718750	488281250	1020.604	97653134873.87
2	T1	200	100	250	10000	33.33333	283.3333333	2833333.33	5555555.556	862.2711	7435114746.62
3	R	175	1025	250	179375	512.5	762.5	136773438	15704654948	383.1044	26326692575.70
4	R	1000	225	1275	225000	112.5	1387.5	312187500	949218750	-241.8956	13165528100.62
5	R	1305	200	1500	261000	100	1600	417600000	870000000	-454.3956	53890057986.17
					769125			881113021	1.802E+10		198470528282.98

Types	Shapes	Input
R		b,h
T1		b,h
T2		b,h
C		b=h
	Blank	

A =	769125 mm^2
y1 =	554.3955523 mm
y2 =	1145.604448 mm
I =	216488238786 mm^4
=	2.1649E+11 mm^4
z1 =	3.905E+08 mm^3
z2 =	1.890E+08 mm^3



Calculate the section properties from scratch

Solution

Calculate the section properties based on the known precast section's properties

Properties of Precast Section:

A = 508,125 mm^2

Ixx = 134 x 10^9 mm^4

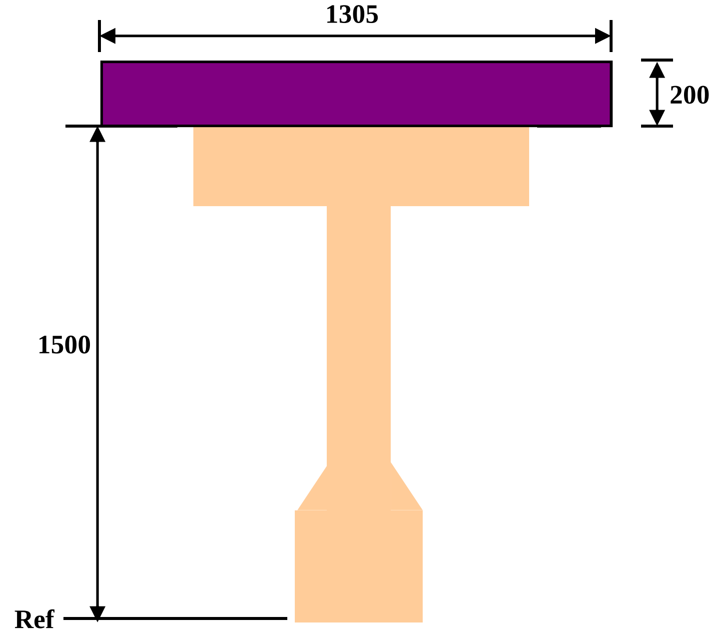
y1 = 587.8 mm, y2 = 912.2 mm

Properties of Insitu Slab:

A = 1305 x 200 = 261,000 mm^2

Ixx = 1305 x 200^3/12 = 0.87 x 10^9 mm4

y1 = y2 = 100 mm



Solution

From Ref point,

yc2 = (508125*912.2 + 261000*1600)/(508125+26100)

= 1145.6 mm

yc1 = 1700 – 1145.6 = 554.4 mm

Icxx = $0.87 \times 10^9 + (26100)(554.4-100)^2 +$

$134 \times 10^9 + (508125)(1145.6-912.2)^2$

$= 0.87 \times 10^9 + 53.89 \times 10^9 + 134 \times 10^9 + 27.68 \times 10^9$

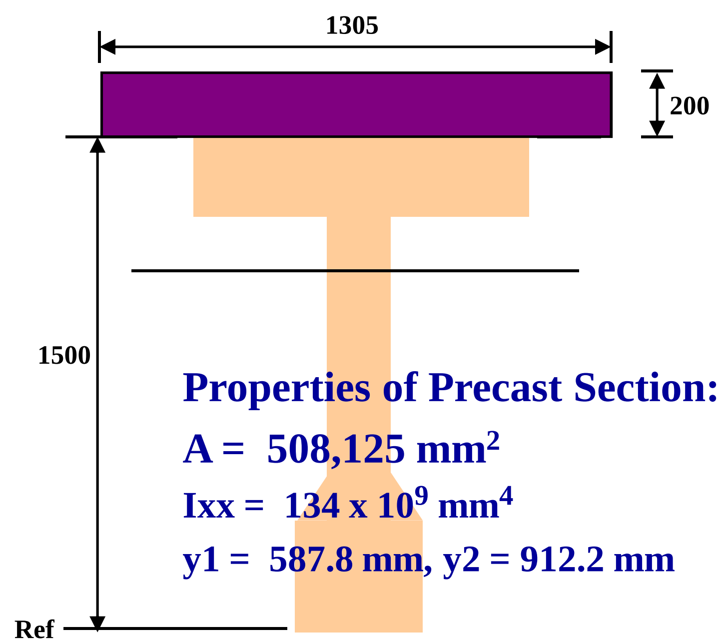
$= 216.44 \times 10^9$ mm^4

Properties of Insitu Slab:

A = 261,000 mm^2

Ixx = 0.87×10^9 mm4

y1 = y2 = 100 mm



Properties of Precast Section:

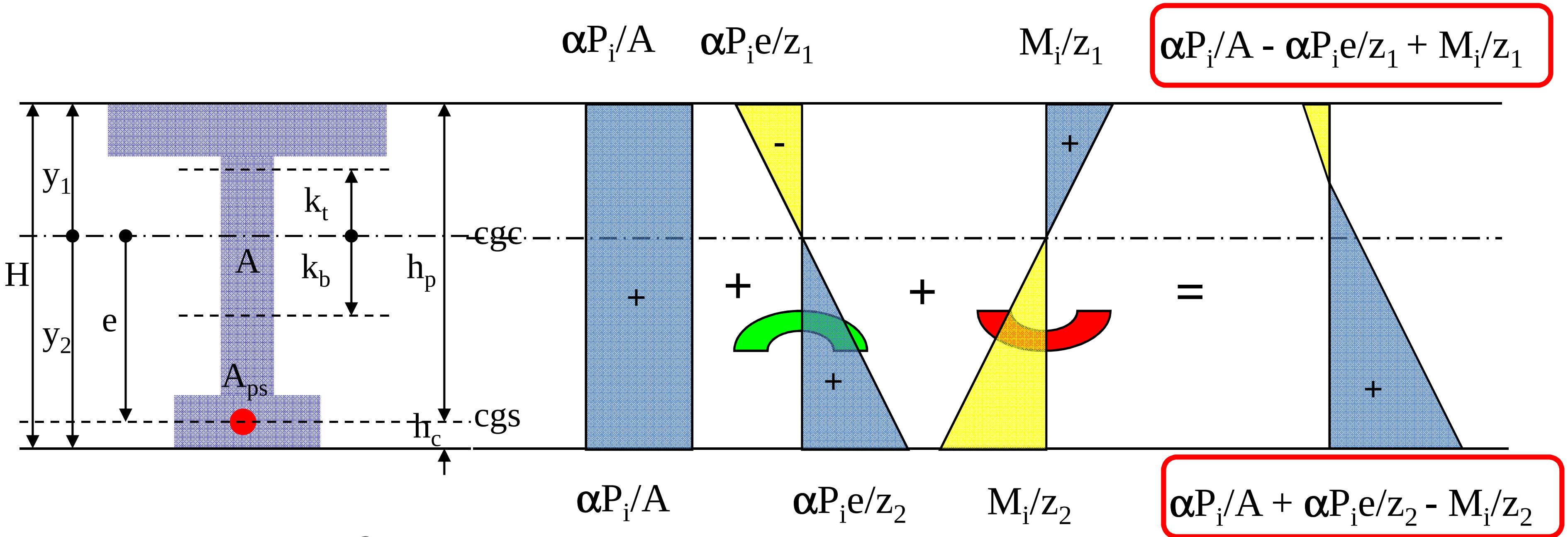
A = 508,125 mm^2

Ixx = 134×10^9 mm^4

y1 = 587.8 mm, y2 = 912.2 mm

Stresses in Concrete At Transfer

- Consider at mid span of a simply supported beam



$M_i = W_{sw} \times L^2/8$

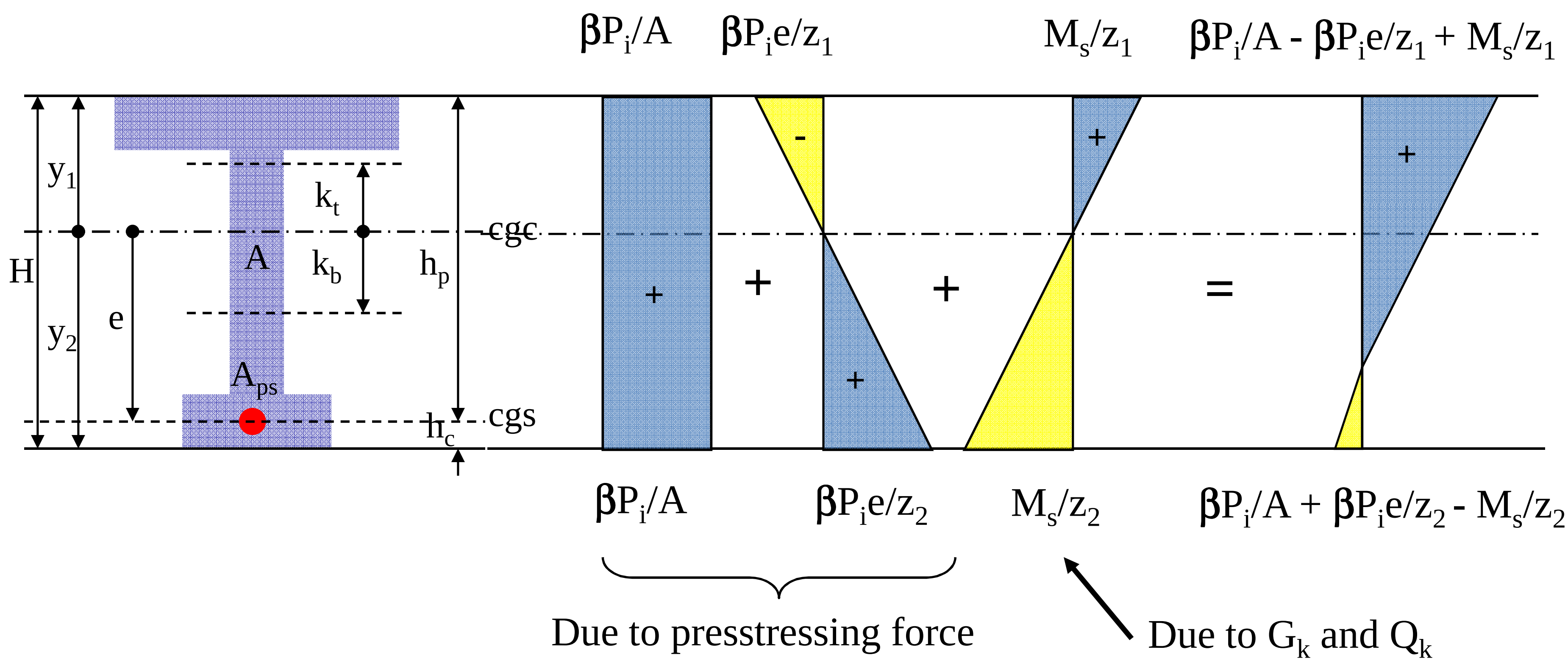
L = beam's span

Due to presstressing force

Due to self weight, W_{sw}

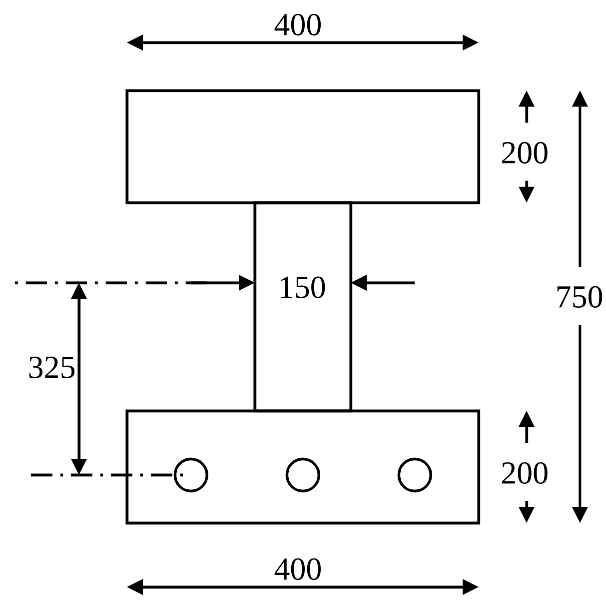
Stresses in Concrete At Service

- Consider at mid span of a simply supported beam



Example 2-3

A simply supported pretensioned concrete beam has dimensions as shown and spans 15m. It has an initial prestress force of 1100kN applied to it and it carries a uniformly distributed imposed load of 12 kN/m. Determine the extreme fibre stresses at midspan (i) under the self weight of the beam, if the short-term losses are 10% and eccentricity is 325mm below the beam centroid; (ii) under service load, when the prestress force has been reduced by a further 10%



Solution

The beam x-section properties (try to calculate) are as follow:

$A = 2.13 \times 10^5\ mm^2$ & $z_1 = z_2 = 35.12 \times 10^6\ mm^3$

$W_{sw} = 5.1$ kN/m (= $A \times \gamma_{con} = 0.213 \times 24 = 5.1$)

Total service load = 5.1 + 12 = 17.1 kN/m

Span, L = 15 m

Moment at midspan = $WL^2/8$

Moment at transfer, $M_i = 5.1 \times 15^2/8 = 143.4$ kNm

Moment at service, $M_s = 17.1 \times 15^2/8 = 480.9$ kNm

$\alpha = 0.9$, $\beta = 0.8$

$\alpha P_i = 0.9 \times 1100 = 990$ kN, $\beta P_i = 0.8 \times 1100 = 880$ kN

Solution

i. **At Transfer** $\alpha P_i/A - \alpha P_i e/z_1 + M_i/z_1$

f_{1t} = $(990\times10^3/2.13\times10^5) - (990\times10^3 \times 325/35.12\times10^6) + (143.4\times10^6/35.12\times10^6)$

= 4.65 – 9.16 + 4.08

= - 0.43 N/mm^2 $\alpha P_i/A + \alpha P_i e/z_2 - M_i/z_2$

f_{2t} = $(990\times10^3/2.13\times10^5) + (990\times10^3 \times 325/35.12\times10^6) - (143.4\times10^6/35.12\times10^6)$

= 4.65 + 9.16 - 4.08

= 9.73 N/mm^2

ii. **At Service** $\beta P_i/A - \beta P_i e/z_1 + M_s/z_1$

f_{1s} = $(880\times10^3/2.13\times10^5) - (880\times10^3 \times 325/35.12\times10^6) + (480.9\times10^6/35.12\times10^6)$

= 4.13 – 8.14 + 13.69

= 9.68 N/mm^2 $\beta P_i/A + \beta P_i e/z_2 - M_s/z_2$

f_{2s} = $(880\times10^3/2.13\times10^5) + (880\times10^3 \times 325/35.12\times10^6) - (480.9\times10^6/35.12\times10^6)$

= 4.13 + 8.14 – 13.69

= -1.42 N/mm^2

Example 2-4

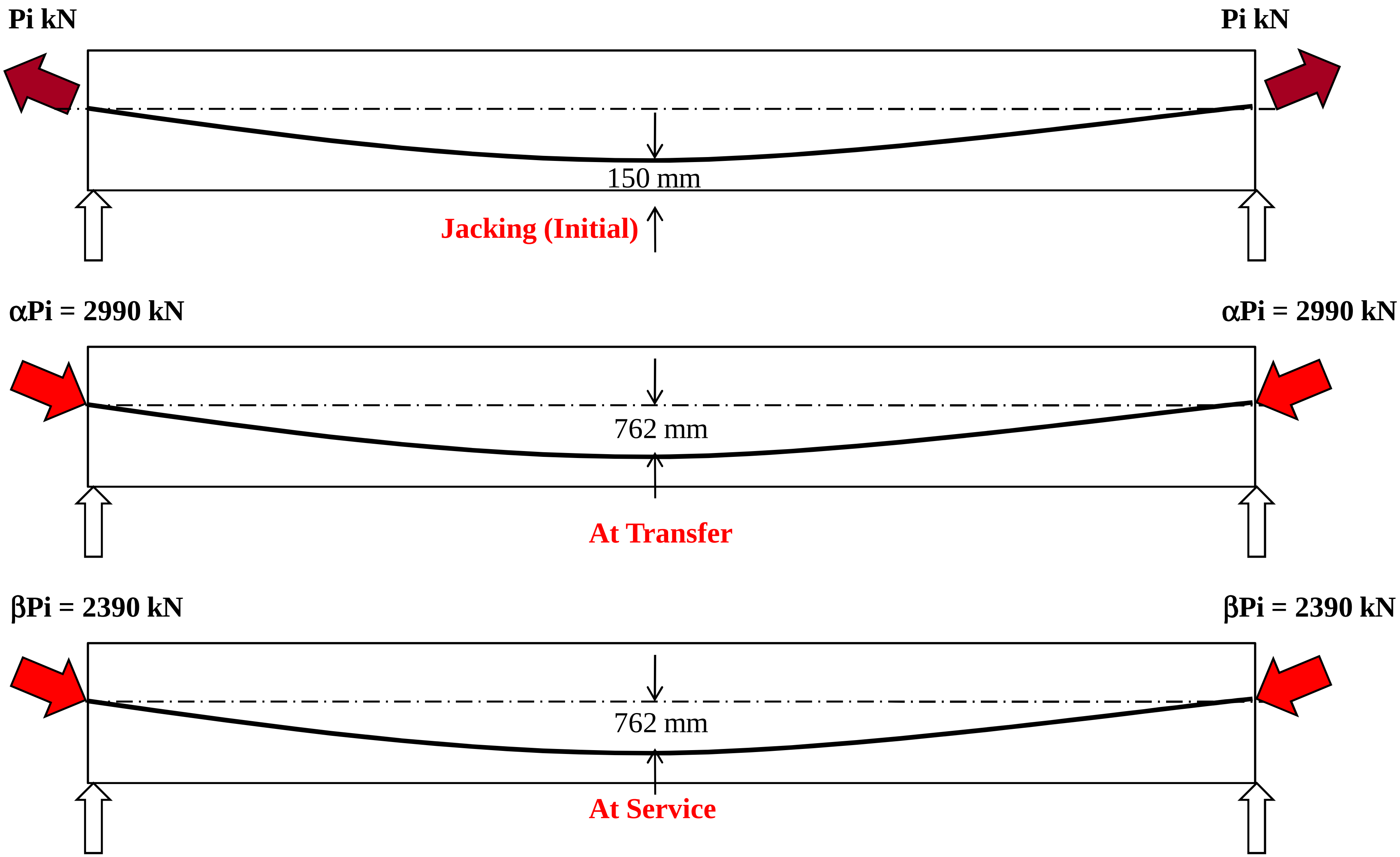
A post-tensioned concrete beam (designed as class 2), simply supported over a span of 30m carries a uniformly distributed characteristic imposed dead load of 4 kN/m, in addition to its own self weight, and characteristic live load of 9 kN/m. The cable is laid out as a parabola with a concrete cover to c.g.s of 150mm at mid span and zero eccentricity at each end. The prestressing force immediately after transfer is 2990 kN, which reduces, after all losses, to an effective value of 2390 kN. The following information is given:

f_{ci} = 30 N/mm^2; f_{cu} = 50 N/mm^2; use the cross section of example 1a.

Determine the extreme fibre stresses at mid span for the critical loading conditions.

Also, check that these stresses are within the permissible values.

Example 2-4


Pi kN
Pi kN
150 mm
Jacking (Initial)
αPi = 2990 kN
αPi = 2990 kN
762 mm
At Transfer
βPi = 2390 kN
βPi = 2390 kN
762 mm
At Service

Solution

The beam x-section properties (refer to e.g. 1-1) are as follow:

$A = 5.08 \times 10^5\ mm^2$; $z_1 = 228 \times 10^6\ mm^3$; $z_2 = 147 \times 10^6\ mm^3$

$W_{sw} = 0.508 \times 24 = 12.19$ kN/m

Total service load = 12.19 + 4 + 9 = 25.19 kN/m

Span, L = 30 m

Moment at midspan = $WL^2/8$

Moment at transfer, $M_i = 12.19 \times 30^2/8 = 1372$ kNm

Moment at service, $M_s = 25.19 \times 30^2/8 = 2834$ kNm

$y_2 = 912$ mm; Therefore, $e = 912 - 150 = 762$ mm

$\alpha P_i = 2990$ kN, $\beta P_i = 2390$ kN

Solution

i. **At Transfer**

f_{1t} = (2990x10^3/5.08x10^5) – (2990x10^3 x 762 /228x10^6) + (1372x10^6 /228x10^6)

= 5.89 – 9.99 + 6.02

= 1.92 N/mm^2

f_{2t} = (2990x10^3/5.08x10^5) + (2990x10^3 x 762 /147x10^6) - (1372x10^6 /147x10^6)

= 5.89 + 15.50 – 6.02

= 12.06 N/mm^2

ii. **At Service**

f_{1s} = (2390x10^3/5.08x10^5) – (2390x10^3 x 762/228x10^6) + (2834x10^6 /228x10^6)

= 4.71 – 7.99 + 12..43

= 9.15 N/mm^2

f_{2s} = (2390x10^3/5.08x10^5) + (2390x10^3 x 762 /147x10^6) - (2834x10^6 /147x10^6)

= 4.71 + 12.39 – 19.28

= -2.18 N/mm^2

Stress Limits

(i) At Transfer

$f_{tt} = -0.36(f_{ci})^{1/2}$ N/mm^2

(Class 2, post-tensioned)

$= -0.36 \times 30^{1/2} = -1.97$ N/mm^2

$f_{ct} = 0.5 f_{ci}$ N/mm^2 (flexural members)

$= 0.5 \times 30 = 15.00$ N/mm^2

$f_{1t} = 1.92$ N/mm$^2 > f_{tt} = -1.97$ N/mm^2

$f_{2t} = 12.06$ N/mm$^2 < f_{ct} = 15$ N/mm^2

(ii) At Service

$f_{ts} = -0.36(f_{cu})^{1/2}$ N/mm^2

(Class 2, post-tensioned)

$= -0.36 \times 50^{1/2} = -2.54$ N/mm^2

$f_{cs} = 0.33 f_{cu}$ N/mm^2 (flexural members)

$= 0.33 \times 50 = 16.5$ N/mm^2

$f_{1s} = 9.15$ N/mm$^2 < f_{cs} = 16.5$ N/mm^2

$f_{2s} = -2.18$ N/mm$^2 > f_{ts} = -2.54$ N/mm^2

Therefore, stresses at transfer and service are within the allowable limits

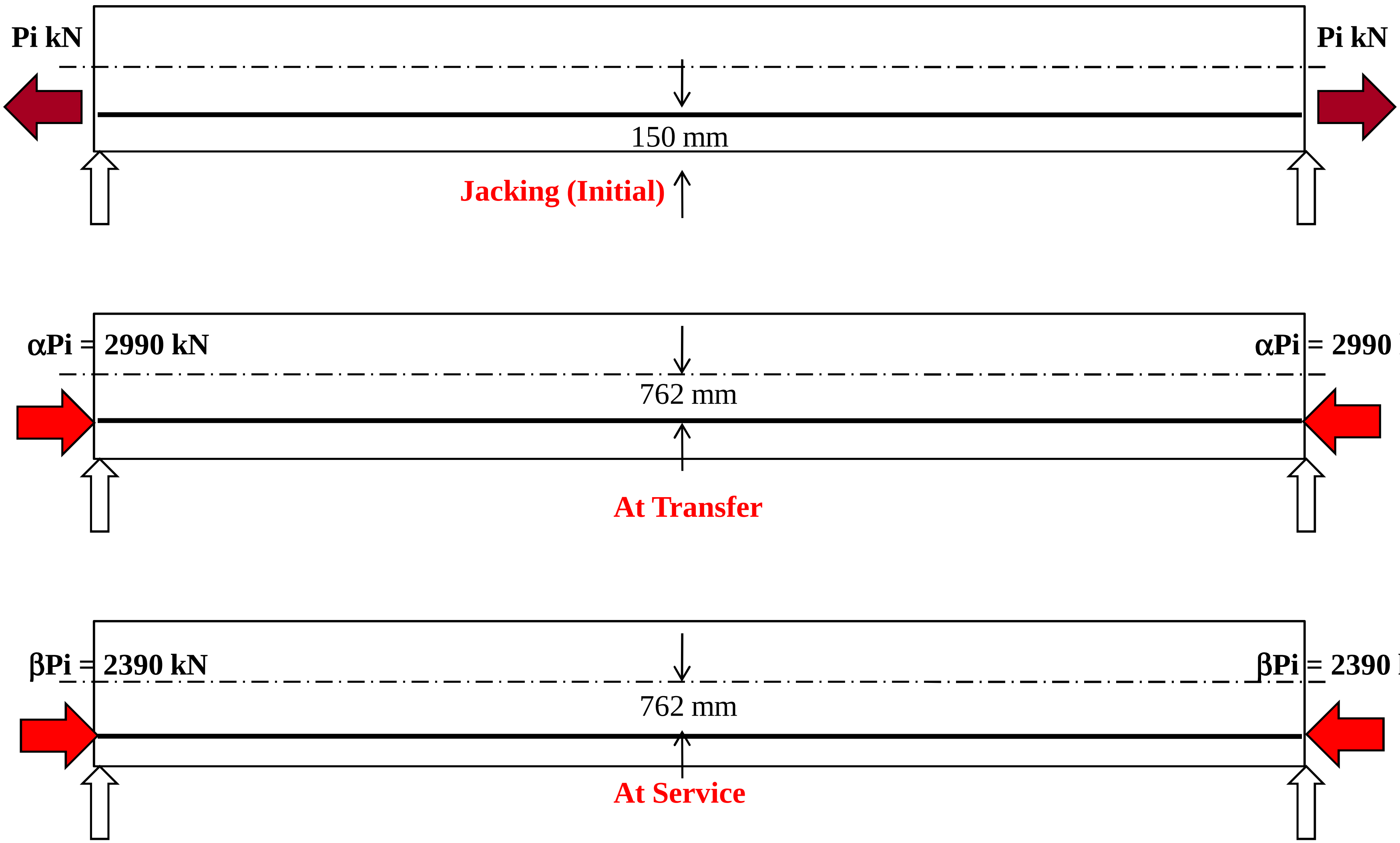
Example 2-4

Using the same beam geometry and loadings in Example 2-3, determine the extreme fibre stresses at support, quarter span and mid span. Assume the followings:

- Pre-tensioned class 2
- e = 762 mm (constant along the span)

Also, check that these stresses are within the permissible values.

Example 2-4


Pi kN
Pi kN
150 mm
Jacking (Initial)
αPi = 2990 kN
αPi = 2990 kN
762 mm
At Transfer
βPi = 2390 kN
βPi = 2390 kN
762 mm
At Service

Solution 2-4

Solution for Example 3a

Structural Cassification : Class 2 Pre-tensioned

Concrete

fcu =	50	N/mm^2
fci =	30	N/mm^2

Prestressing Steel

αPi =	2990	kN
βPi =	2390	kN
cover =	150	mm
e =	762	mm

Loads & Moments

Self-weight =	12.192	kN/m
Gk =	4	kN/m
Qk =	9	kN/m
Span (L) =	30	m
Wi =	12.19	kN/m
Ws =	25.19	kN/m

Mx = 0.5w*x(L - x)

Stress Limits

At Transfer

ftt =	-2.46	N/mm^2
fct =	15.00	N/mm^2

At Service

fts =	-3.18	N/mm^2
fcs =	16.50	N/mm^2

Section Properties

A =	508000	mm^2
I =	8.425E+09	mm^4
y_1 =		mm
y_2 =	912	mm
Z_1 =	2.280E+08	mm^3
Z_2 =	1.470E+08	mm^3

Solution

Stress Calculations

	x = 0	x = L/4	x = L/2	
x	0.00	7.50	15.00	m
Mi	0.00	1028.70	1371.60	kNm
Ms	0.00	2125.58	2834.10	kNm
αPi/A	5.89	5.89	5.89	N/mm^2
βPi/A	4.70	4.70	4.70	N/mm^2
αPie/z1	9.99	9.99	9.99	N/mm^2
αPie/z2	15.50	15.50	15.50	N/mm^2
βPie/z1	7.99	7.99	7.99	N/mm^2
βPie/z2	12.39	12.39	12.39	N/mm^2
Mi/z1	0.00	4.51	6.02	N/mm^2
Mi/z2	0.00	7.00	9.33	N/mm^2
Ms/z1	0.00	9.32	12.43	N/mm^2
Ms/z2	0.00	14.46	19.28	N/mm^2
f1t	**-4.11**	**0.40**	**1.91**	**N/mm^2**
f2t	**21.39**	**14.39**	**12.05**	**N/mm^2**
f1s	**-3.28**	**6.04**	**9.15**	**N/mm^2**
f2s	**17.09**	**2.63**	**-2.19**	**N/mm^2**

Check Stresses

	x = 0	x = L/4	x = L/2	
x	0.00	7.50	15.00	m
f1t	-4.11	0.40	1.91	N/mm^2
ftt	-2.46	-2.46	-2.46	N/mm^2
Status	not ok	ok	ok	
f2t	21.39	14.39	12.05	N/mm^2
fct	15.00	15.00	15.00	N/mm^2
Status	not ok	ok	ok	
f1s	-3.28	6.04	9.15	N/mm^2
fcs	16.50	16.50	16.50	N/mm^2
Status	Not ok	ok	ok	
f2s	17.09	2.63	-2.19	N/mm^2
fts	-3.18	-3.18	-3.18	N/mm^2
Status	Not ok	ok	ok	

Example 2-4

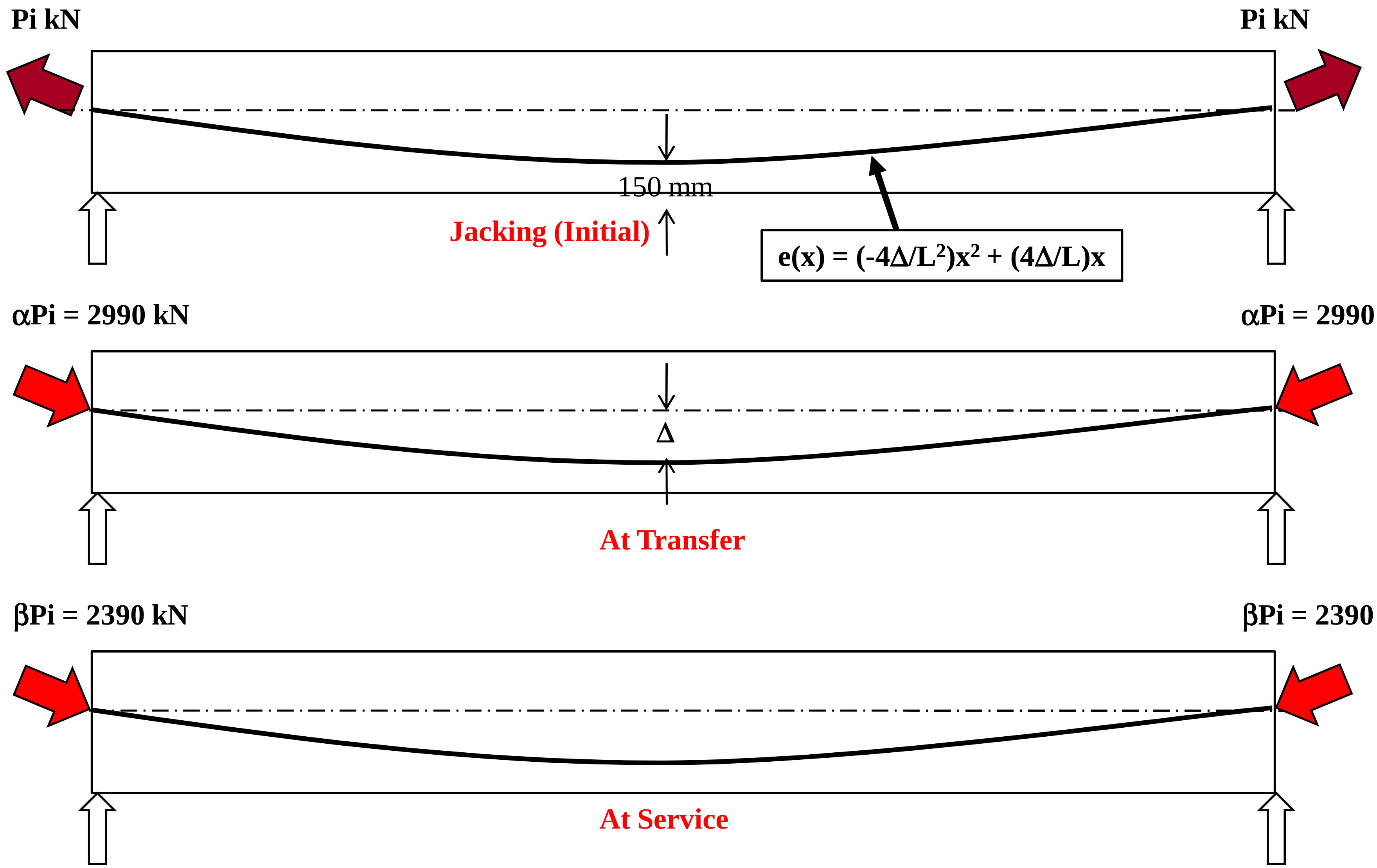
Using the same beam and loadings in Example 2-3, determine the extreme fibre stresses at support and quarter span. Taking eccentricity of tendon along the span at distance x (in m) from left support as:

$$e(x) = (-4\Delta/L^2)x^2 + (4\Delta/L)x$$

Where L = 30 m and Δ = 762 mm

Also, check that these stresses are within the permissible values.

Example 2-4


Pi kN
Pi kN
150 mm
Jacking (Initial)
e(x) = (-4Δ/L²)x² + (4Δ/L)x
αPi = 2990 kN
αPi = 2990 kN
Δ
At Transfer
βPi = 2390 kN
βPi = 2390 kN
At Service

Solution for Example 3b

Structural Cassification : Class 2 Post-tensioned

Concrete

fcu =	50	N/mm^2
fci =	30	N/mm^2

Prestressing Steel

αPi =	2990	kN
βPi =	2390	kN
cover =	150	mm
e_{max} =	762	mm

Loads & Moments

Self-weight =	12.192	kN/m
Gk =	4	kN/m
Qk =	9	kN/m
Span (L) =	30	m
Wi =	12.19	kN/m
Ws =	25.19	kN/m

Mx = 0.5w*x(L - x)

Stress Limits

At Transfer

ftt =	-1.97
fct =	15.00

At Service

fts =	-2.55
fcs =	16.50

Section Properties

A =	508000
I =	8.425E+09
y_1 =	
y_2 =	912
Z_1 =	2.280E+08
Z_2 =	1.470E+08

Solution

Stress Calculations

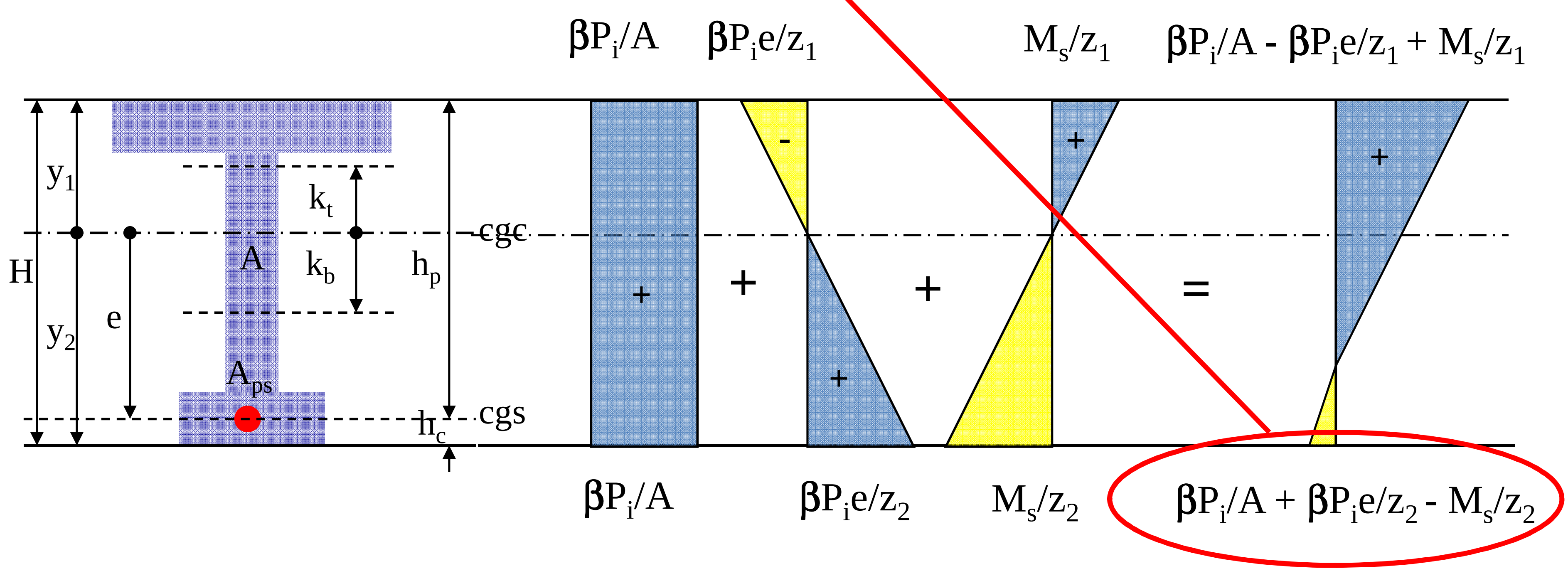
	x = 0	x = L/4	x = L/2	
x	0.00	7.50	15.00	m
e	0.00	571.50	762.00	mm
Mi	0.00	1028.70	1371.60	kNm
Ms	0.00	2125.58	2834.10	kNm
αPi/A	5.89	5.89	5.89	N/mm^2
βPi/A	4.70	4.70	4.70	N/mm^2
αPie/z1	0.00	7.49	9.99	N/mm^2
αPie/z2	0.00	11.62	15.50	N/mm^2
βPie/z1	0.00	5.99	7.99	N/mm^2
βPie/z2	0.00	9.29	12.39	N/mm^2
Mi/z1	0.00	4.51	6.02	N/mm^2
Mi/z2	0.00	7.00	9.33	N/mm^2
Ms/z1	0.00	9.32	12.43	N/mm^2
Ms/z2	0.00	14.46	19.28	N/mm^2
f1t	5.89	2.90	1.91	N/mm^2
f2t	5.89	10.51	12.05	N/mm^2
f1s	4.70	8.04	9.15	N/mm^2
f2s	4.70	-0.46	-2.19	N/mm^2

Check Stresses

	x = 0	x = L/4	x = L/2	
x	0.00	7.50	15.00	m
f1t	5.89	2.90	1.91	N/mm^2
ftt	-1.97	-1.97	-1.97	N/mm^2
Status	ok	ok	ok	
f2t	5.89	10.51	12.05	N/mm^2
fct	15.00	15.00	15.00	N/mm^2
Status	ok	ok	ok	
f1s	4.70	8.04	9.15	N/mm^2
fcs	16.50	16.50	16.50	N/mm^2
Status	ok	ok	ok	
f2s	4.70	-0.46	-2.19	N/mm^2
fts	-2.55	-2.55	-2.55	N/mm^2
Status	ok	ok	ok	

Cracking Moment

- Crack will start to develop when the tensile stress at the bottom fibre equals to the flexural tensile stress of the concrete (f_{tu} – modulus of rupture ≈ $0.59(f_{cu})^{0.5}$)
- Ms ➔ Mcr = $\beta Pi(e - k_t) + f_{tu}z_2$



Try to solve this…

Write down the stresses at the extreme fibres (f_{1t}, f_{2t}, f_{1s} & f_{2s}) at transfer & service

in terms of α, β, P_i, e, M_i and M_s at

section x-x.