

LECTURE 3

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Institute of Geospatial and Science Technology (INSTeG)

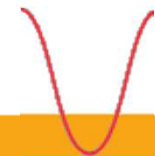
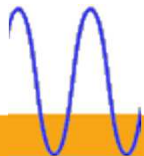
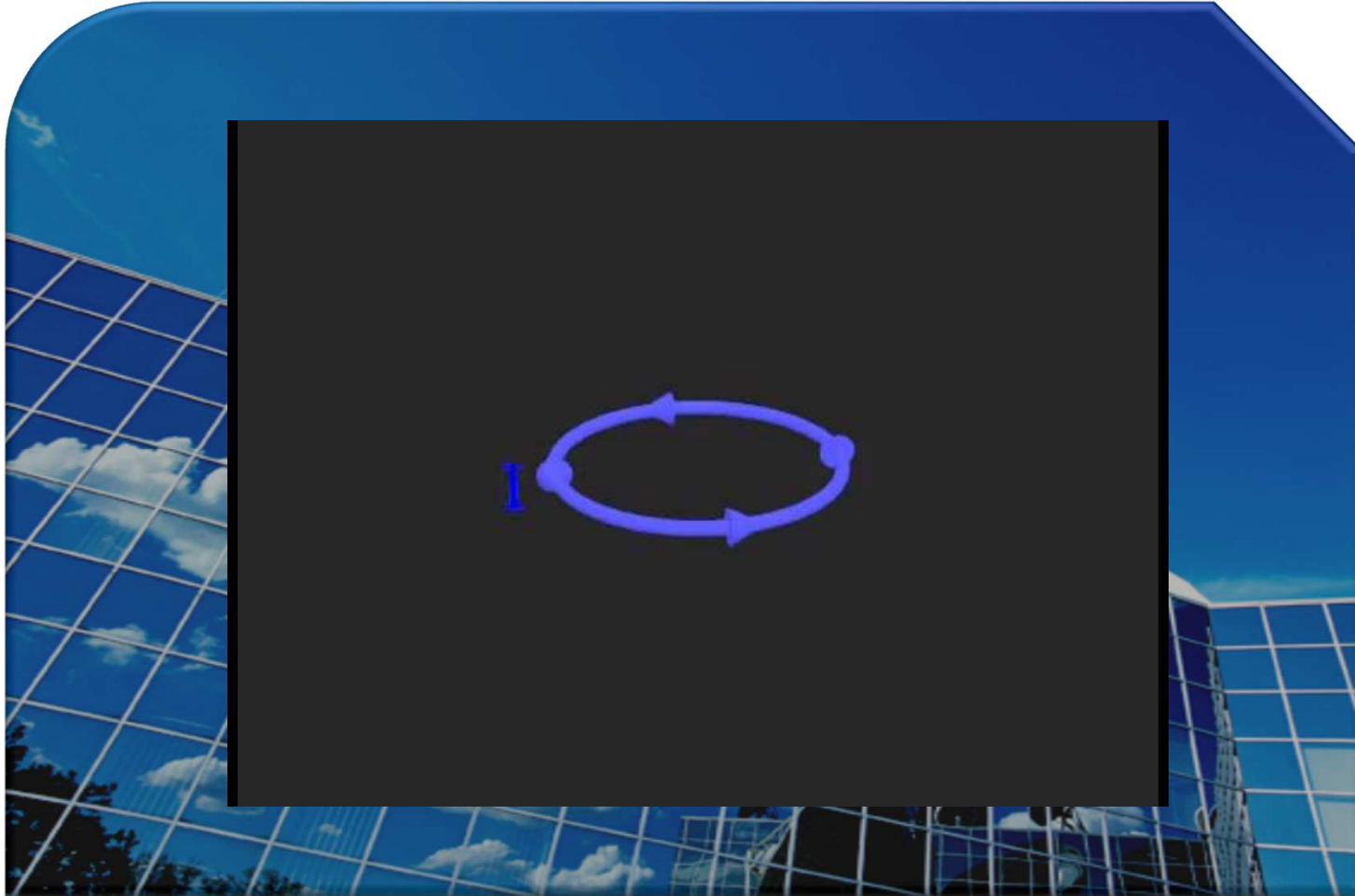
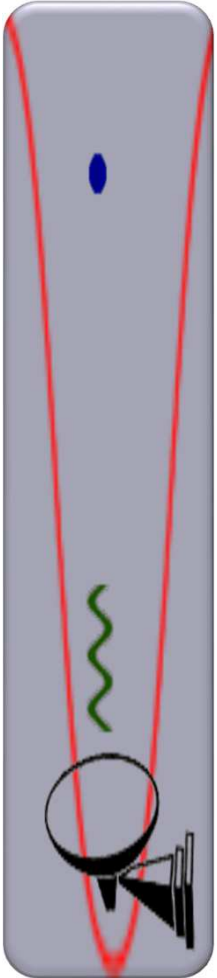
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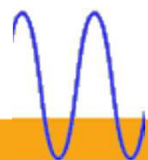
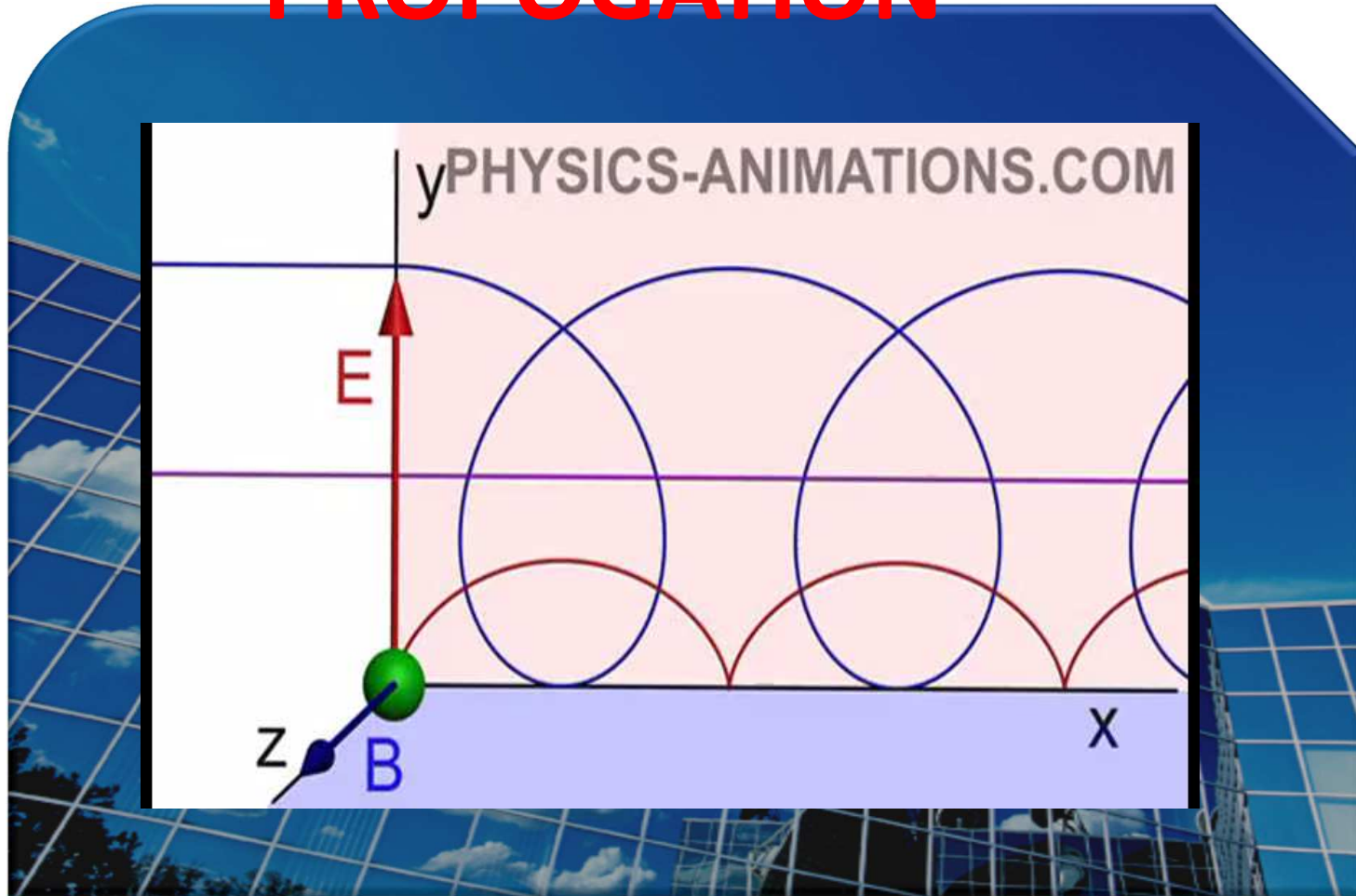
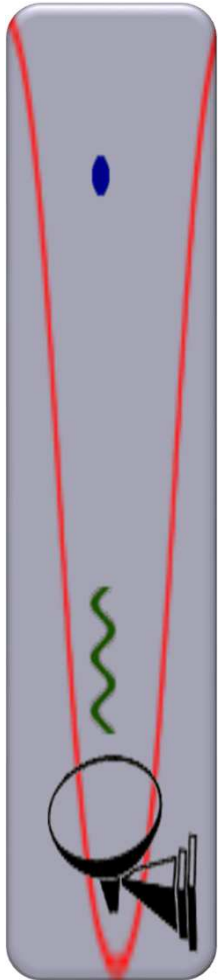
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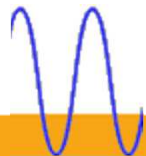
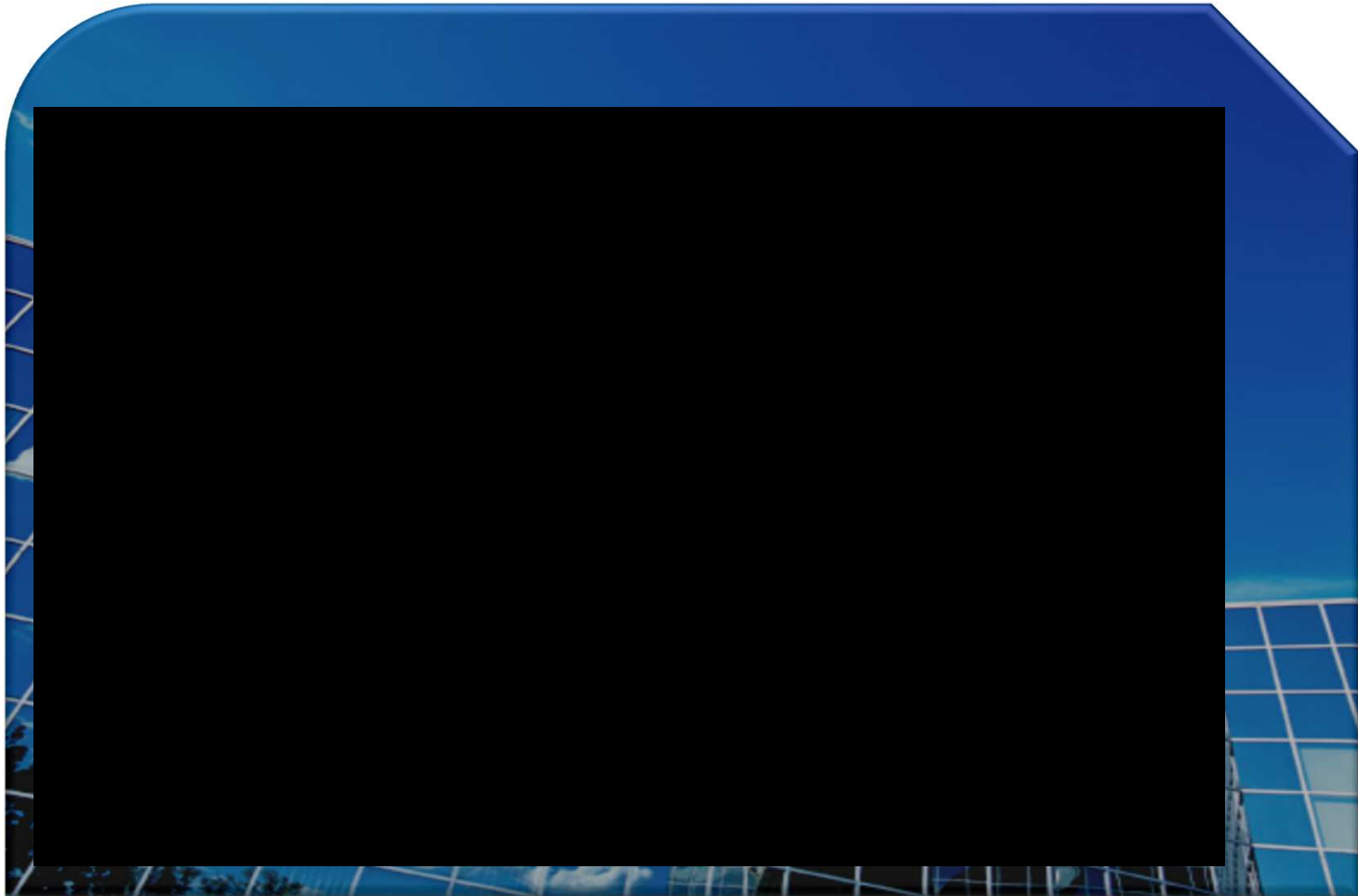
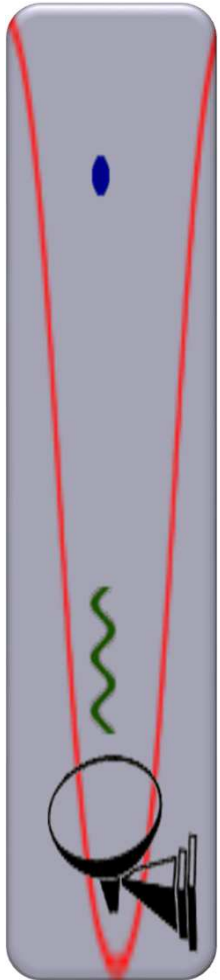
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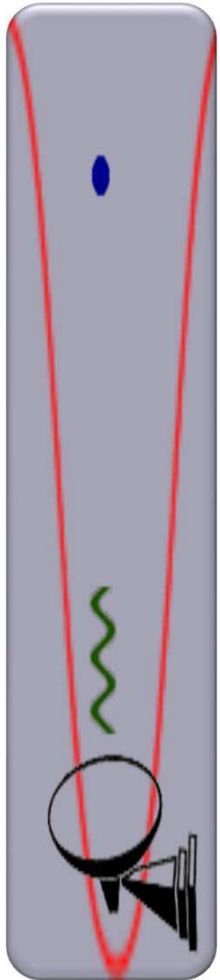
RIGHT HAND RULE



ELECTROMAGNETIC WAVE PROPOGATION







There are three descriptions of light.

- Light behaves as a geometric line (ray)
- Light behaves as a wave (second most complex and 99% valid)
- Light behaves as a quantum mechanical particle

We propagate E (field):

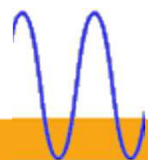
$$\nabla^2 \vec{E}(t, \vec{r}) - \frac{1}{c^2} \frac{d^2 \vec{E}(t, \vec{r})}{dt^2} = 0 \quad \text{Wave Equation}$$

One solution for E (field) is the plane wave:

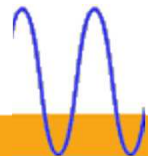
$$\vec{E}(t, z) = \vec{E}_0 e^{j(\omega t - k z)} \Rightarrow \hat{y} E_0 e^{j(\omega t - k z)}$$

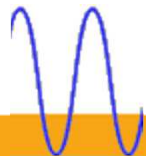
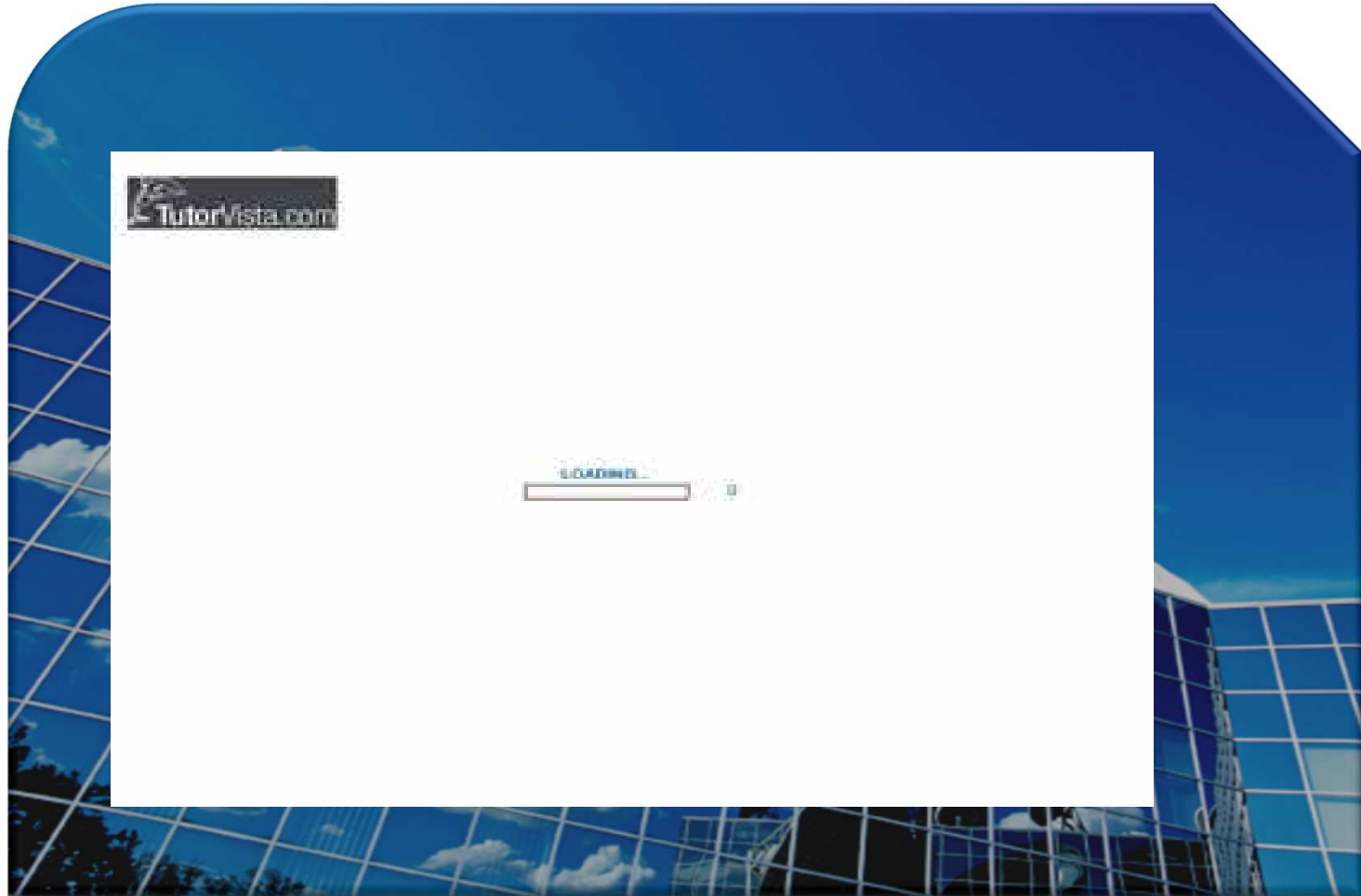
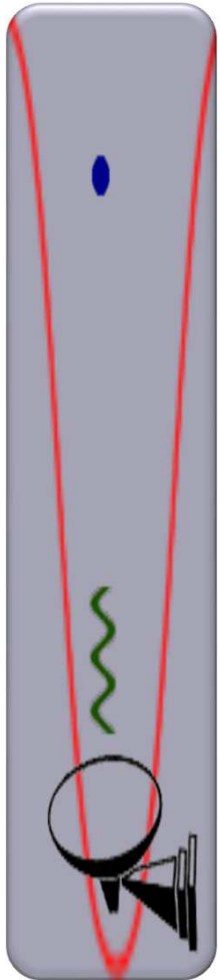
$\omega = \frac{2\pi c}{\lambda_0}$ $c = \frac{\omega}{k} = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$
 $\lambda = \frac{2\pi}{k}$

Relate space and time through phase $e^{j\Phi} \Rightarrow \Phi = \omega t - k z$

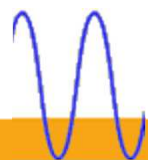
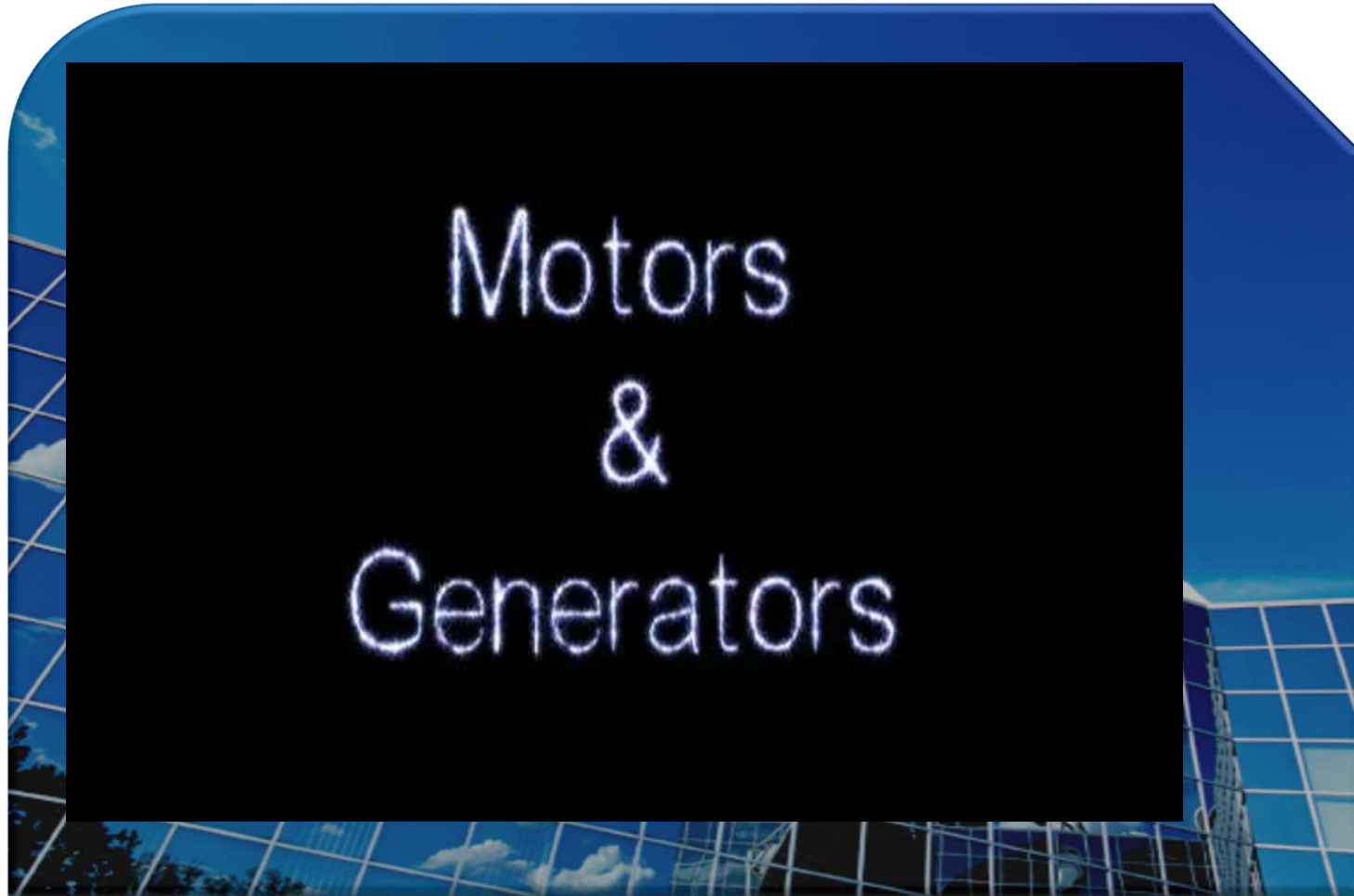
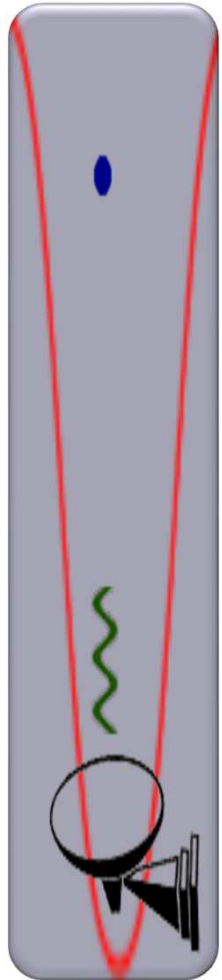


Faraday

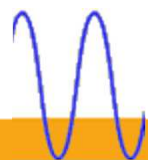
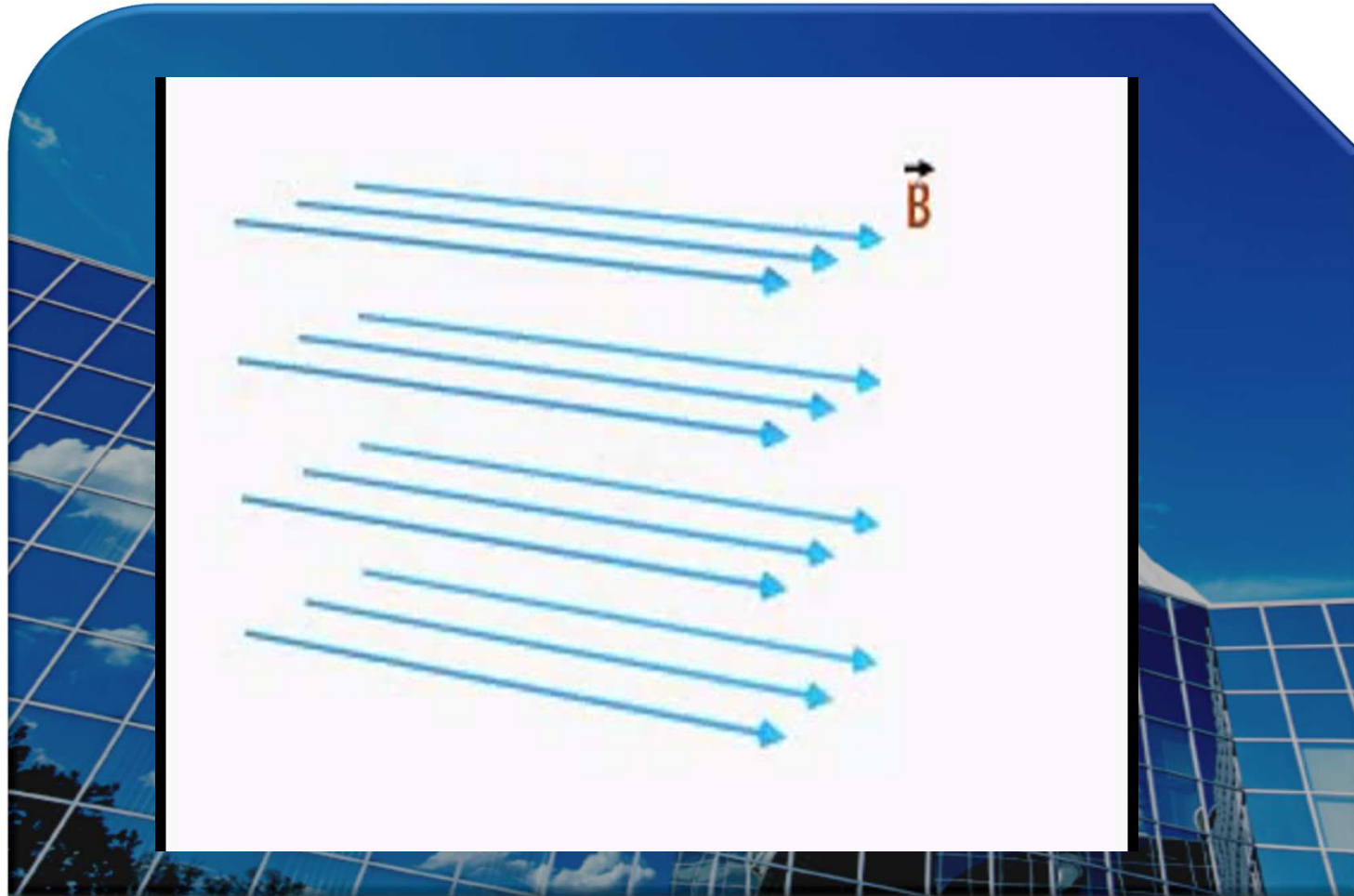
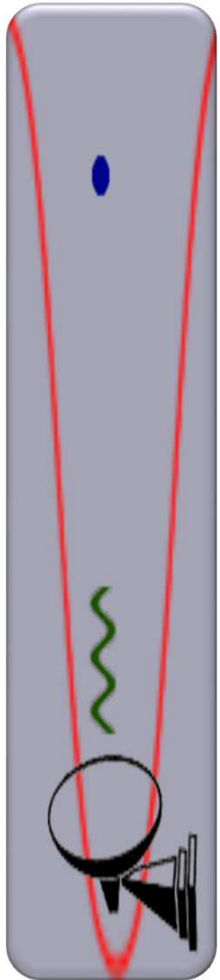




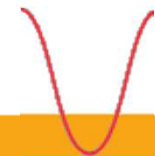
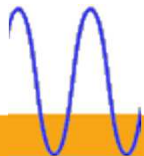
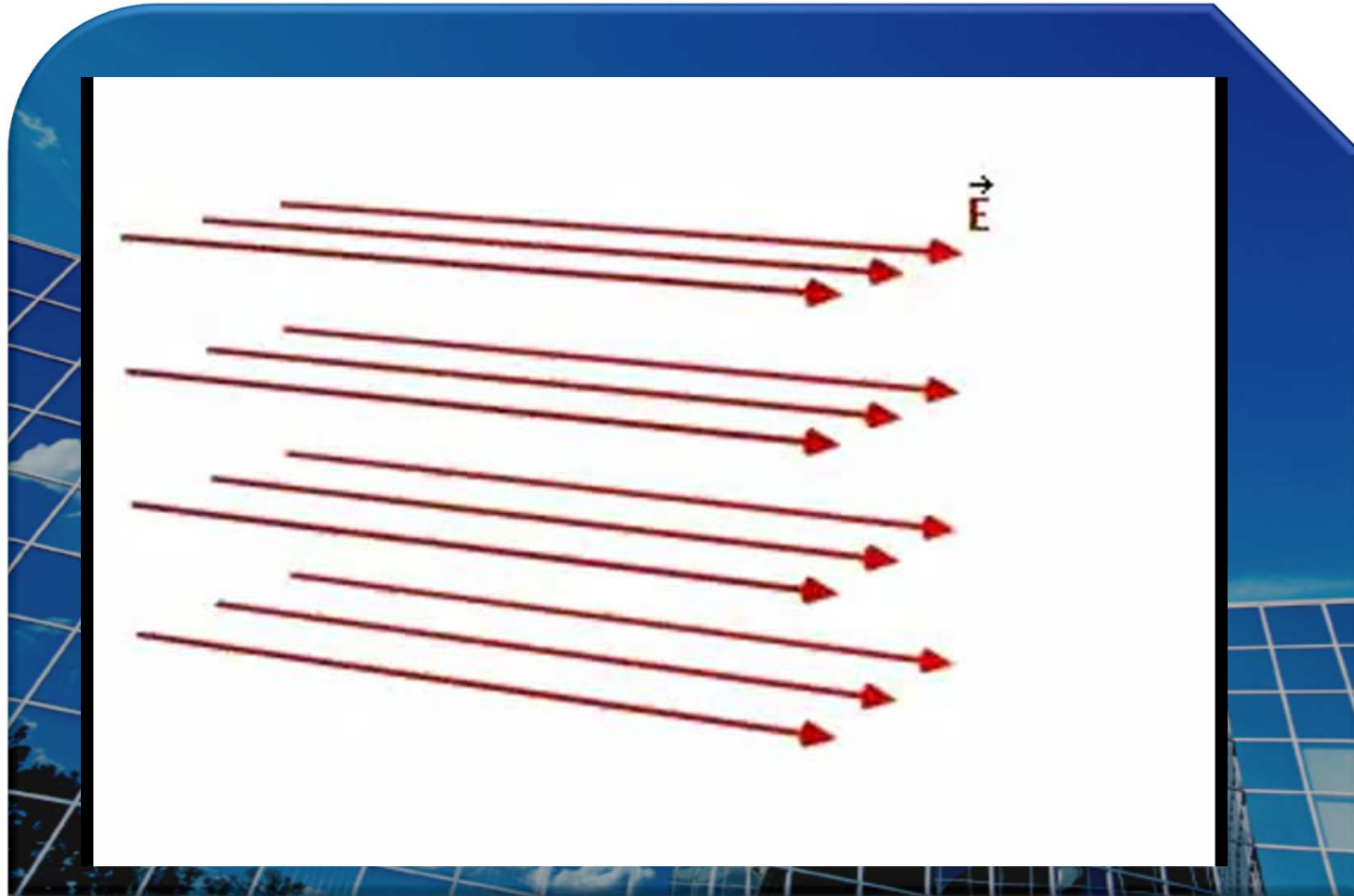
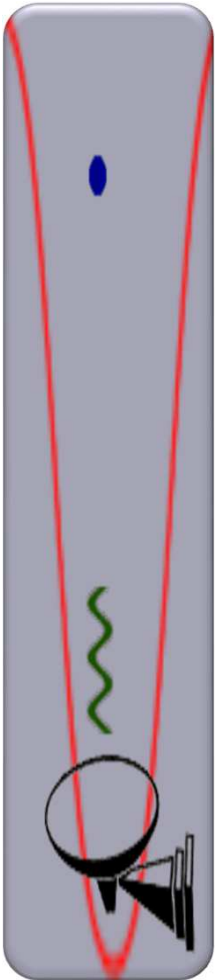
SIMPLE EXAMPLE OF HOW EM WORKS



MAGNETIC FLUX



ELECTRIC FLUX



MAXWELL'S EQUATIONS



are four differential equations summarizing nature of electricity and magnetism: (formulated by James Clerk Maxwell around 1860):

$$\vec{\nabla} \cdot \vec{D} = \rho$$

Gauss's Law

$$\vec{\nabla} \cdot \vec{B} = 0$$

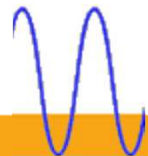
Gauss's Law for Magnetism

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Faraday's Law

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Ampere's Law



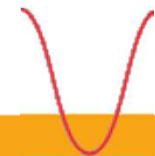
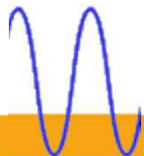
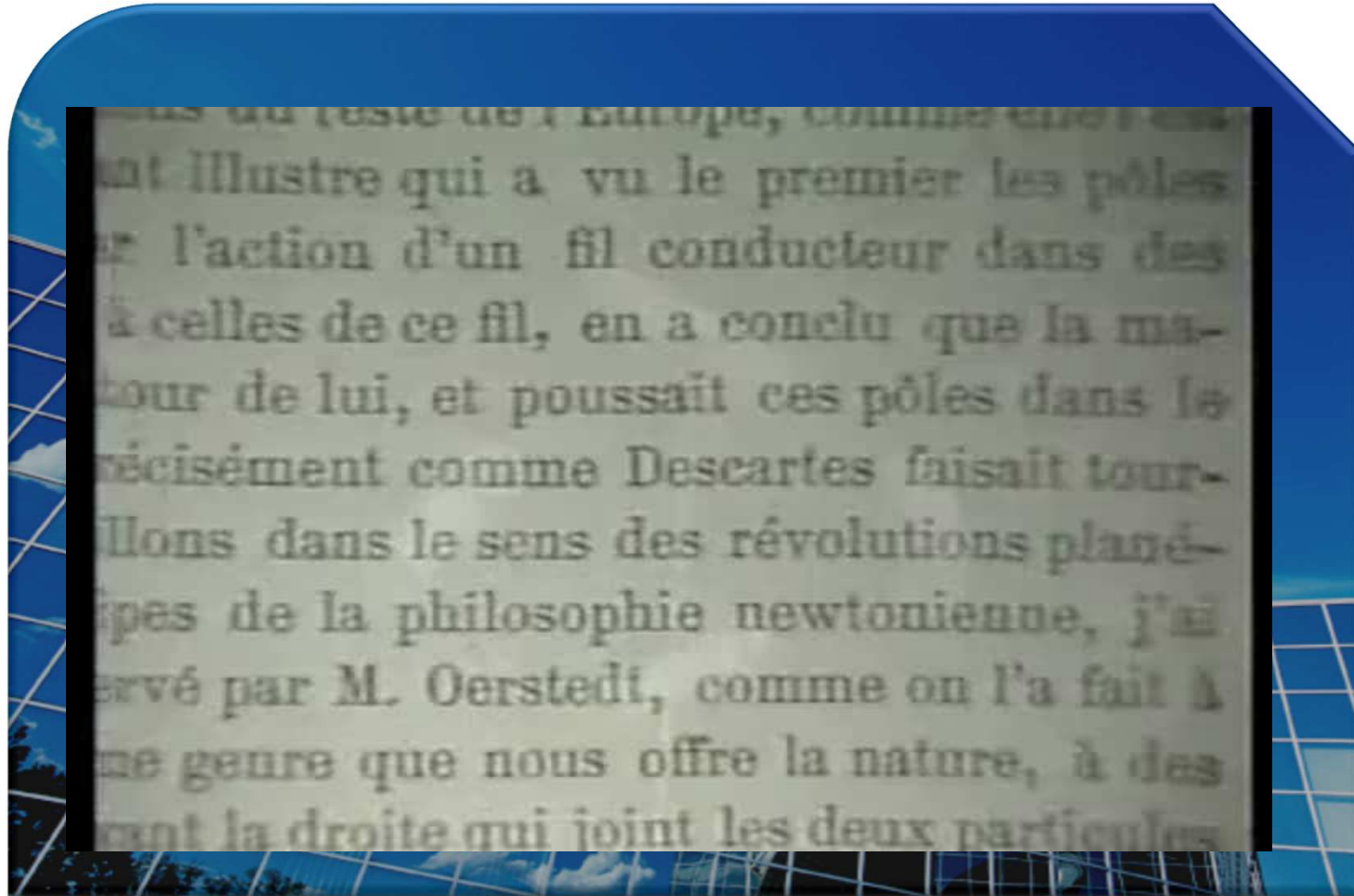
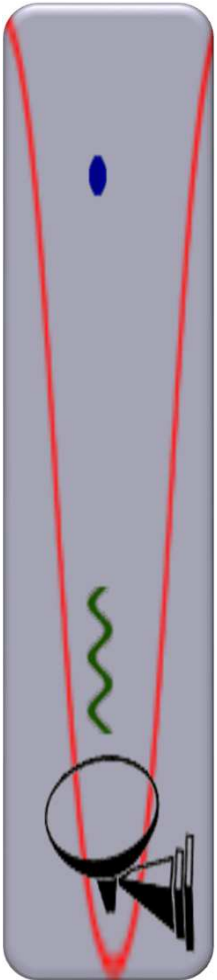
MAXWELL'S EQUATIONS



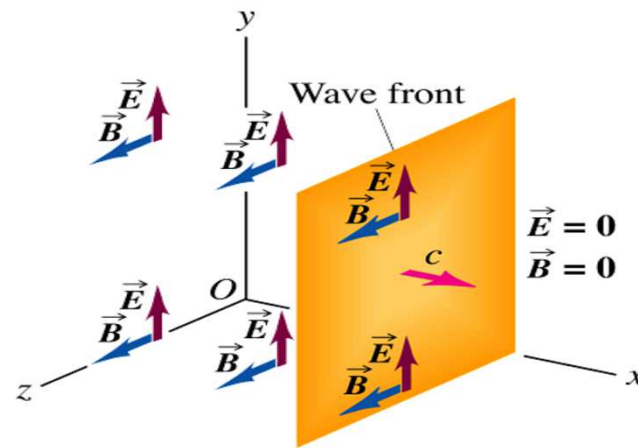
- (1) Electric charges generate electric fields.
- (2) Magnetic field lines are closed loops; there are no magnetic monopoles.
- (3) Currents and changing electric fields produce magnetic fields.
- (4) Changing magnetic fields produce electric fields.

From Maxwell's equations one can derive another equation which has the form of a “wave equation”.

ELECTROMAGNETIC

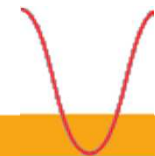
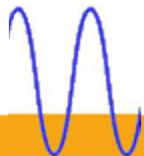


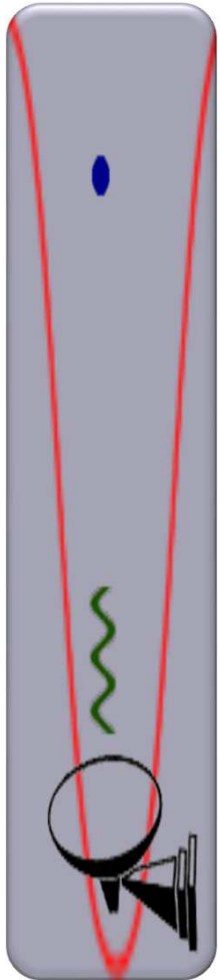
Faraday's law to a plane wave



An electromagnetic wave front.
The plane representing the wave front (yellow) moves to the right with speed c .

The \vec{E} and \vec{B} fields are uniform over the region behind the wave front but are zero everywhere in front of it.

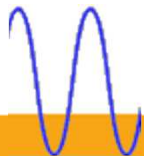




$$\int \mathbf{E} \cdot d\mathbf{l} = -d\Phi_B / dt$$

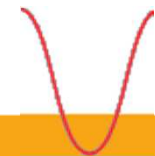
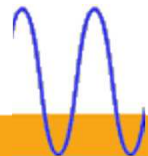
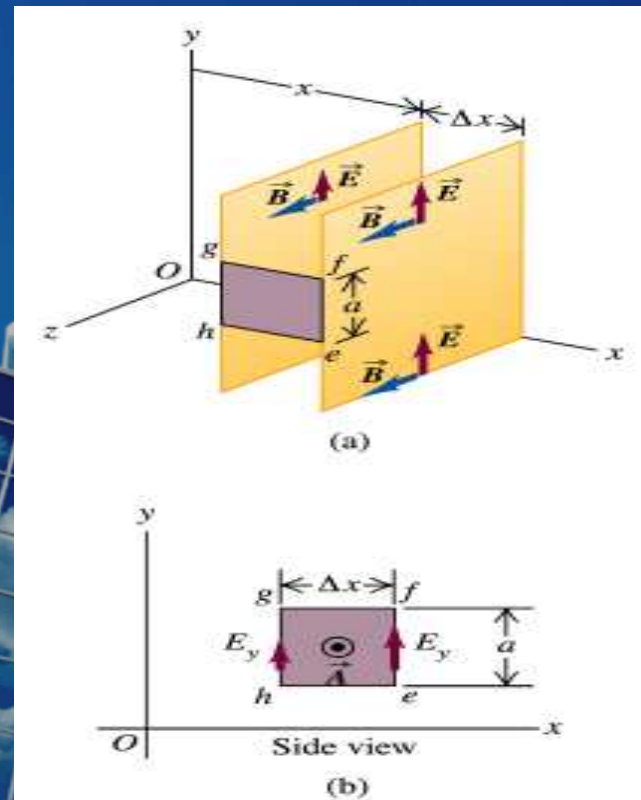
1. $\int \mathbf{E} \cdot d\mathbf{l} = -Ea \quad (\cos 90^\circ = 0)$
2. In time dt the wave front moves to the right a distance $c dt$. The **magnetic flux** through the rectangle in the xy -plane increases by an amount $d\Phi_B$ equal to the flux through the **shaded rectangle** in the xy -plane with area $ac dt$, that is,
 $d\Phi_B = Bac dt$. So
 $-d\Phi_B / dt = -Bac$ and

$$\mathbf{E} = Bc$$



Faraday's law to a plane wave

Faraday's Law applied to a rectangle with height a and width Δx parallel to the xy -plane.



Ampere's law to a plane wave

$$\int \mathbf{B} \cdot d\mathbf{l} = \mu_0 \epsilon_0 d\Phi_E / dt$$

$$1. \int \mathbf{B} \cdot d\mathbf{l} = Ba \quad (\cos 90^\circ = 0)$$

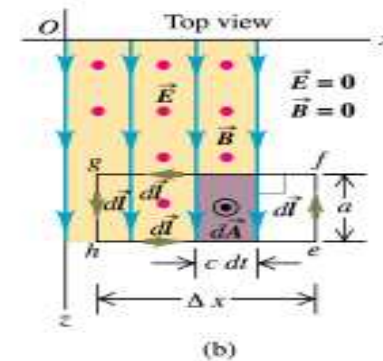
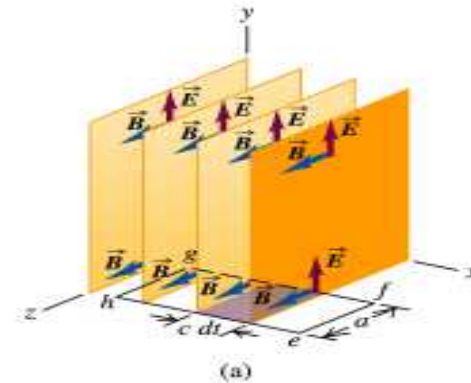
2. In time dt the wave front moves to the right a distance $c dt$. The **electric flux** through the rectangle in the xz -plane increases by an amount $d\Phi_E$ equal to E times the area $ac dt$ of the **shaded rectangle**, that is, $d\Phi_E = Eac dt$. Thus $d\Phi_E / dt = Eac$.

$$Ba = \mu_0 \epsilon_0 Eac \rightarrow \mathbf{B} = \mu_0 \epsilon_0 \mathbf{E}c$$

and from $\mathbf{E} = \mathbf{B}c$ and $\mathbf{B} = \mu_0 \epsilon_0 \mathbf{E}c$

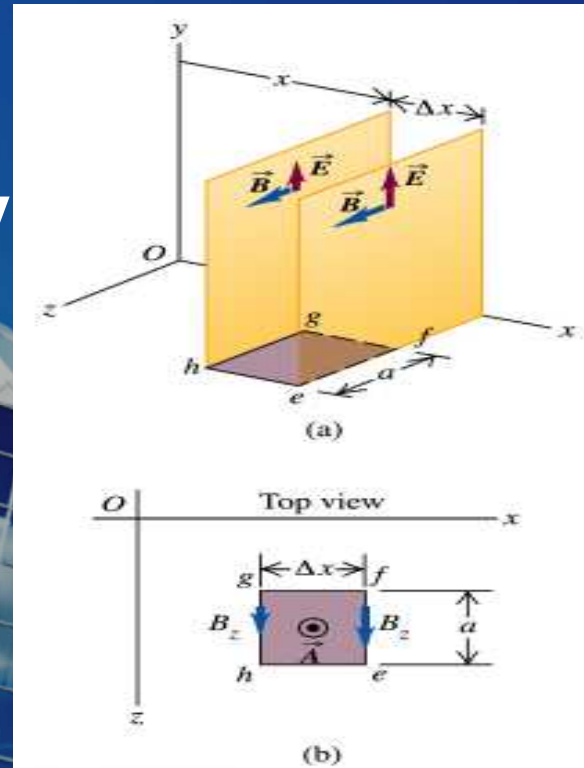
We must have $c = 1 / (\mu_0 \epsilon_0)^{1/2}$

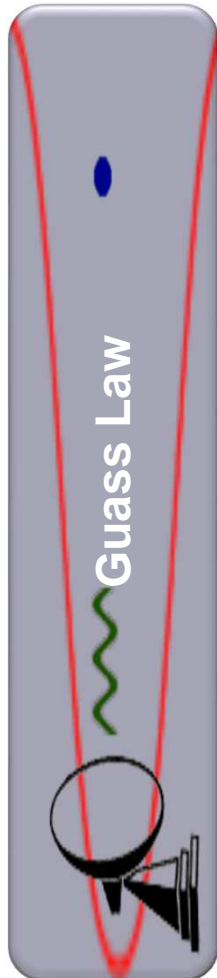
$$c = 3.00 (10)^8 \text{ m/sec}$$



Ampere's law to a plane wave

Ampere's Law
applied to a rectangle with
height a and width
 Δx parallel to the
 xz -plane.





Gauss's law (electrical):

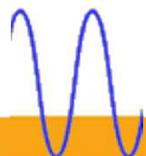
- The total electric flux through any closed surface equals the net charge inside that surface divided by ϵ_0
- This relates an electric field to the charge distribution that creates it

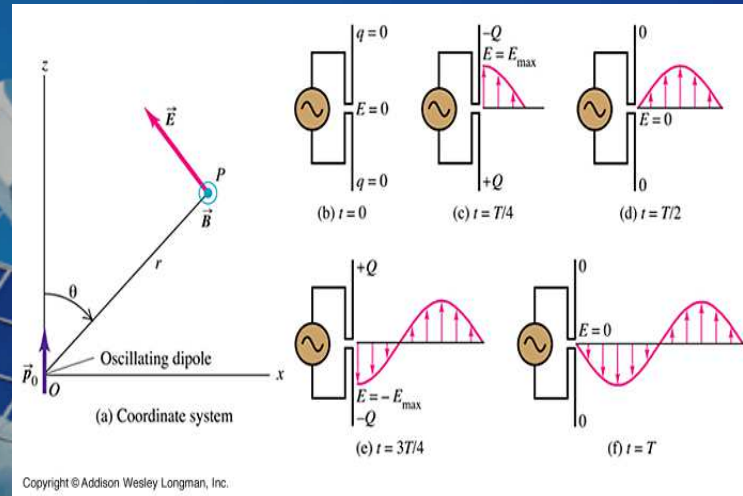
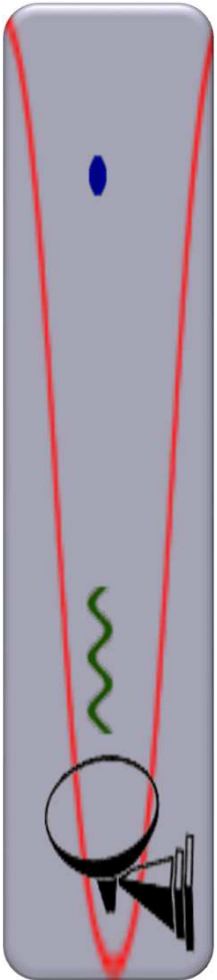
$$\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon_0}$$

Gauss's law (magnetism):

- The total magnetic flux through any closed surface is zero
- This says the number of field lines that enter a closed volume must equal the number that leave that volume
- This implies the magnetic field lines cannot begin or end at any point
- Isolated magnetic monopoles

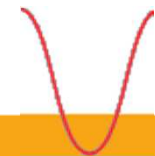
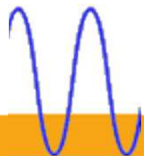
$$\oint_S \mathbf{B} \cdot d\mathbf{A} = 0$$

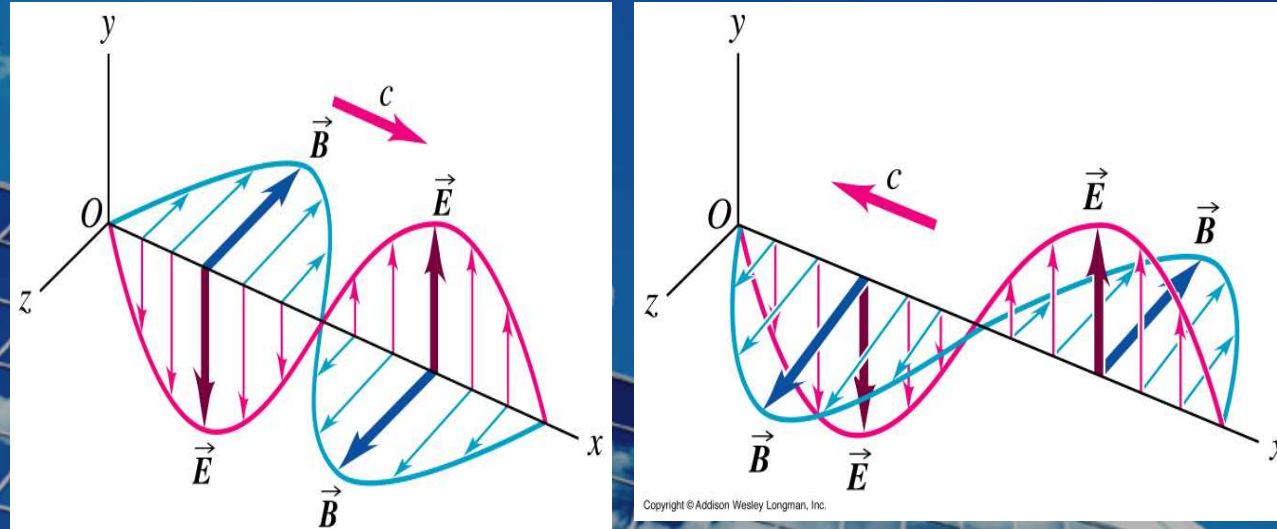
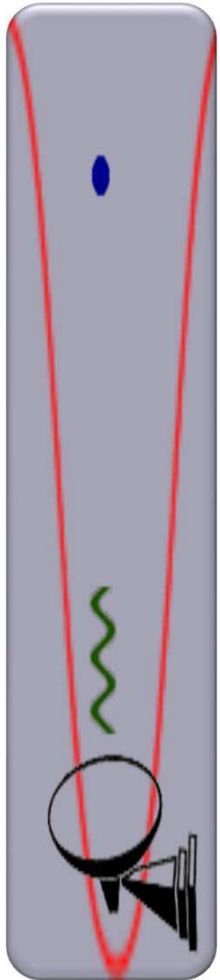




•One cycle in the production of an electro-magnetic wave by an oscillating electric dipole antenna.

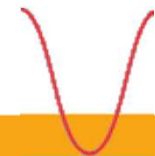
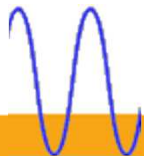
•The red arrows represent the E field. (B not shown.)





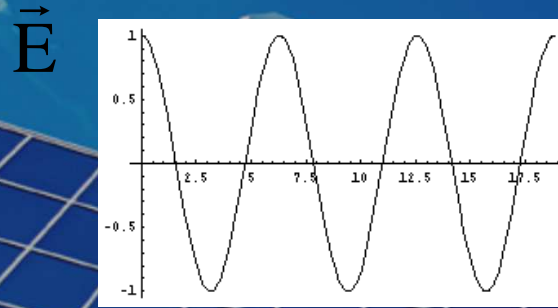
•Representation of the electric and magnetic fields in a propagating wave. One wavelength is shown at time $t = 0$.

•Propagation direction is $E \times B$.

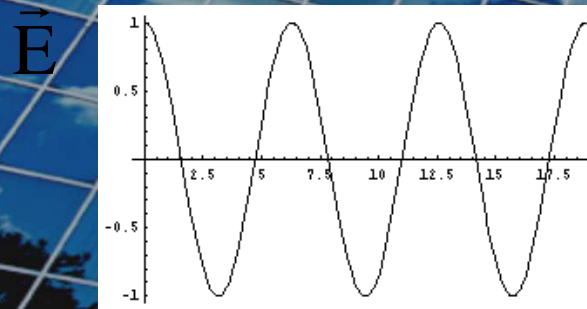


Harmonic Plane Waves

At $t = 0$



At $x = 0$



$\lambda =$ spatial period or wavelength

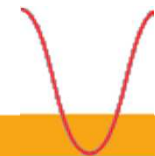
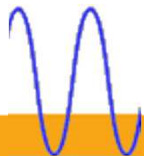


phase velocity

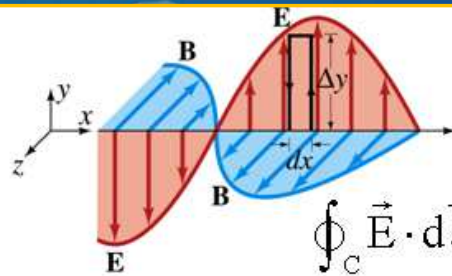
$$v = \frac{\lambda}{T} = f\lambda = \frac{2\pi}{T} \frac{\lambda}{2\pi} = \frac{\omega}{k}$$



$T =$ temporal period



Applying Faraday to radiation



$$\oint_C \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

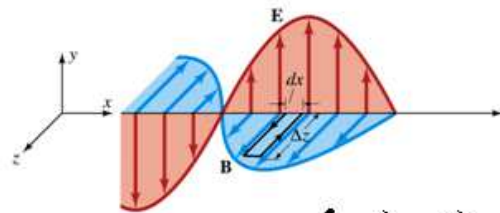
$$\oint_C \vec{E} \cdot d\vec{\ell} = (E + dE)\Delta y - E\Delta y = dE\Delta y$$

$$\frac{d\Phi_B}{dt} = \frac{dB}{dt} dx\Delta y$$

$$dE\Delta y = -\frac{dB}{dt} dx\Delta y$$

$$\frac{dE}{dx} = -\frac{dB}{dt}$$

Applying Ampere to radiation



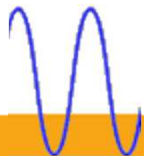
$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = B\Delta z - (B + dB)\Delta z = -dB\Delta z$$

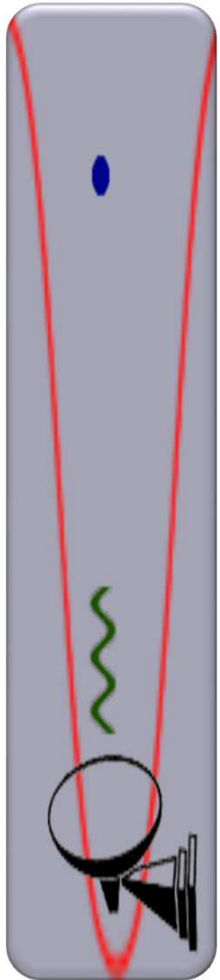
$$\frac{d\Phi_E}{dt} = \frac{dE}{dt} dx \Delta z$$

$$-dB\Delta z = \mu_0 \epsilon_0 \frac{dE}{dt} dx \Delta z$$

$$\frac{dB}{dx} = -\mu_0 \epsilon_0 \frac{dE}{dt}$$



Fields are functions of both position (x) and time (t)



$$\frac{dB}{dx} = -\frac{dE}{dt}$$

Partial derivatives
are appropriate

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

$$\frac{dB}{dx} = -\mu_0 \epsilon_0 \frac{dE}{dt}$$

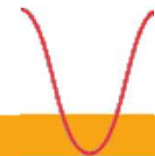
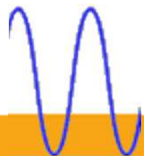
$$\frac{\partial B}{\partial x} = -\mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

$$\frac{\partial^2 E}{\partial x^2} = -\frac{\partial}{\partial x} \frac{\partial B}{\partial t}$$

$$\frac{\partial}{\partial t} \frac{\partial B}{\partial x} = -\mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

This is a wave equation!



The Trial Solution

The simplest solution to the partial differential equations is a sinusoidal wave:

$$E = E_{\max} \cos(kx - \omega t)$$

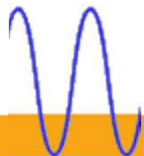
$$B = B_{\max} \cos(kx - \omega t)$$

The angular wave number is $k = 2\pi/\lambda$

λ is the wavelength

The angular frequency is $\omega = 2\pi f$

f is the wave frequency



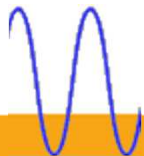
$$E = E_y = E_o \sin(kx - \omega t)$$

$$\frac{\partial^2 E}{\partial x^2} = \mu_o \epsilon_o \frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 E}{\partial x^2} = -k^2 E_o \sin(kx - \omega t) \quad \frac{\partial^2 E}{\partial t^2} = -\omega^2 E_o \sin(kx - \omega t)$$

$$-k^2 E_o \sin(kx - \omega t) = -\mu_o \epsilon_o \omega^2 E_o \sin(kx - \omega t)$$

$$\frac{\omega^2}{k^2} = \frac{1}{\mu_o \epsilon_o}$$

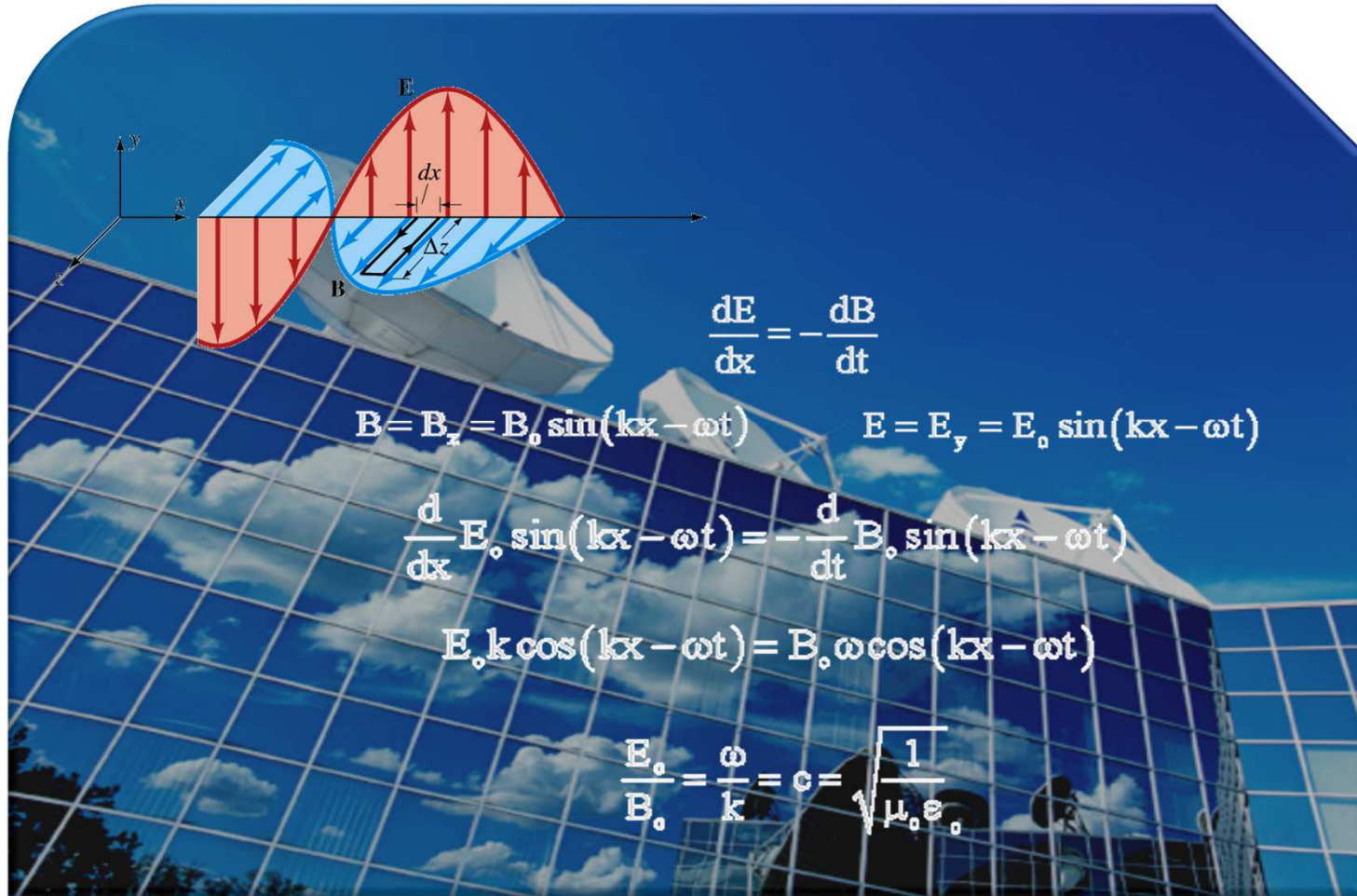


The speed of light

$$v = \frac{\lambda}{T} = f\lambda = \frac{2\pi}{T} \frac{\lambda}{2\pi} = \frac{\omega}{k}$$

$$v = c = \frac{\omega}{k} = \sqrt{\frac{1}{\mu_0 \epsilon_0}}$$

Another look



$$\frac{dE}{dx} = -\frac{dB}{dt}$$

$$B = B_z = B_0 \sin(kx - \omega t) \quad E = E_y = E_0 \sin(kx - \omega t)$$

$$\frac{d}{dx} E_0 \sin(kx - \omega t) = -\frac{d}{dt} B_0 \sin(kx - \omega t)$$

$$E_0 k \cos(kx - \omega t) = B_0 \omega \cos(kx - \omega t)$$

$$\frac{E_0}{B_0} = \frac{\omega}{k} = c = \sqrt{\frac{1}{\mu_0 \epsilon_0}}$$

