OPENCOURSEWARE

Prestressed Concrete Design (SAB 4323)

Loss of Prestress Force

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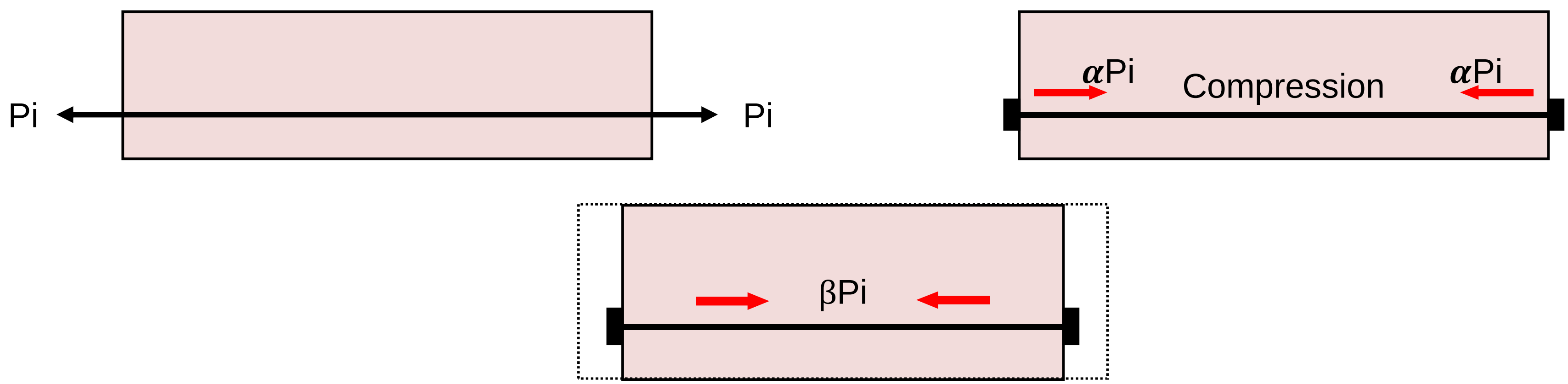
Introduction

- The force applied to the tendon is measured by the pressure gauge mounted on the hydraulic jack and is known as the jacking force, P_i.
- However, this force cannot entirely be transmitted to the concrete because some losses of prestress occur during the process of stretching and anchoring the tendons.

Introduction

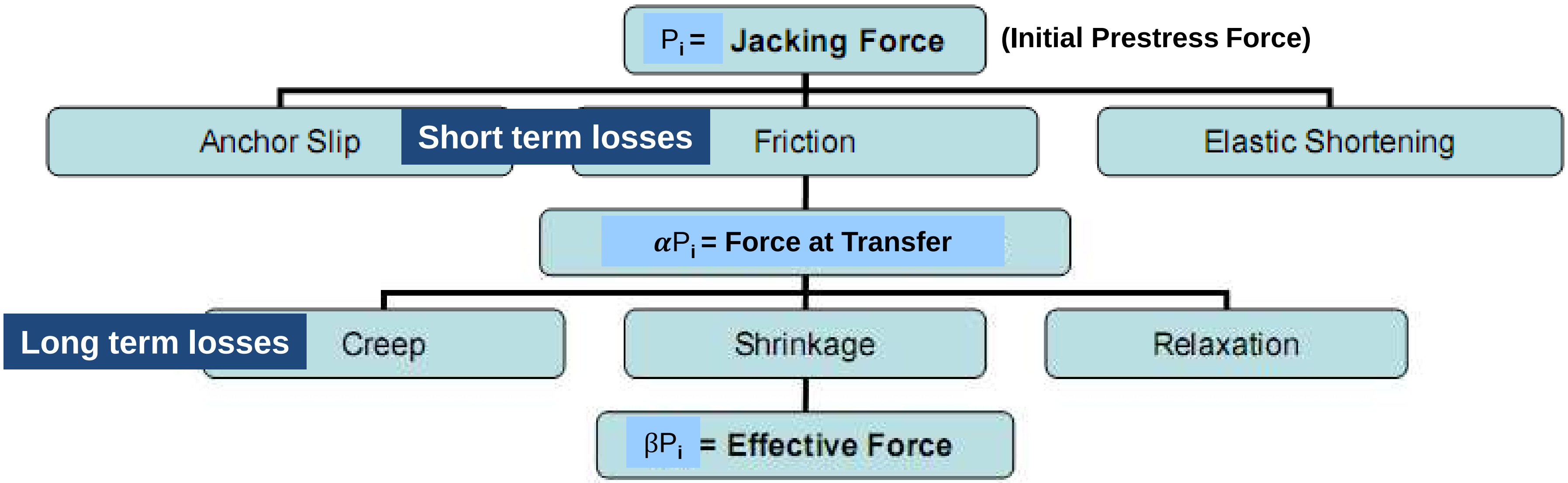
- The fraction of the jacking force that is eventually transferred to the concrete after releasing the temporary anchor or withdrawal of the hydraulic jack is the αP_i.
- This force also keeps on decreasing with time due to time-dependent response of constituent materials; steel and concrete and reduced to a final value known as effective prestressing force, βP_i.



Introduction

- These short and long-term losses are summarised as follows:
- Short-term losses:
  - Elastic shortening
  - Anchorage draw-in
  - Friction
- Long-term losses:
  - Concrete shrinkage
  - Concrete creep
  - Steel relaxation

Prestress Force Levels



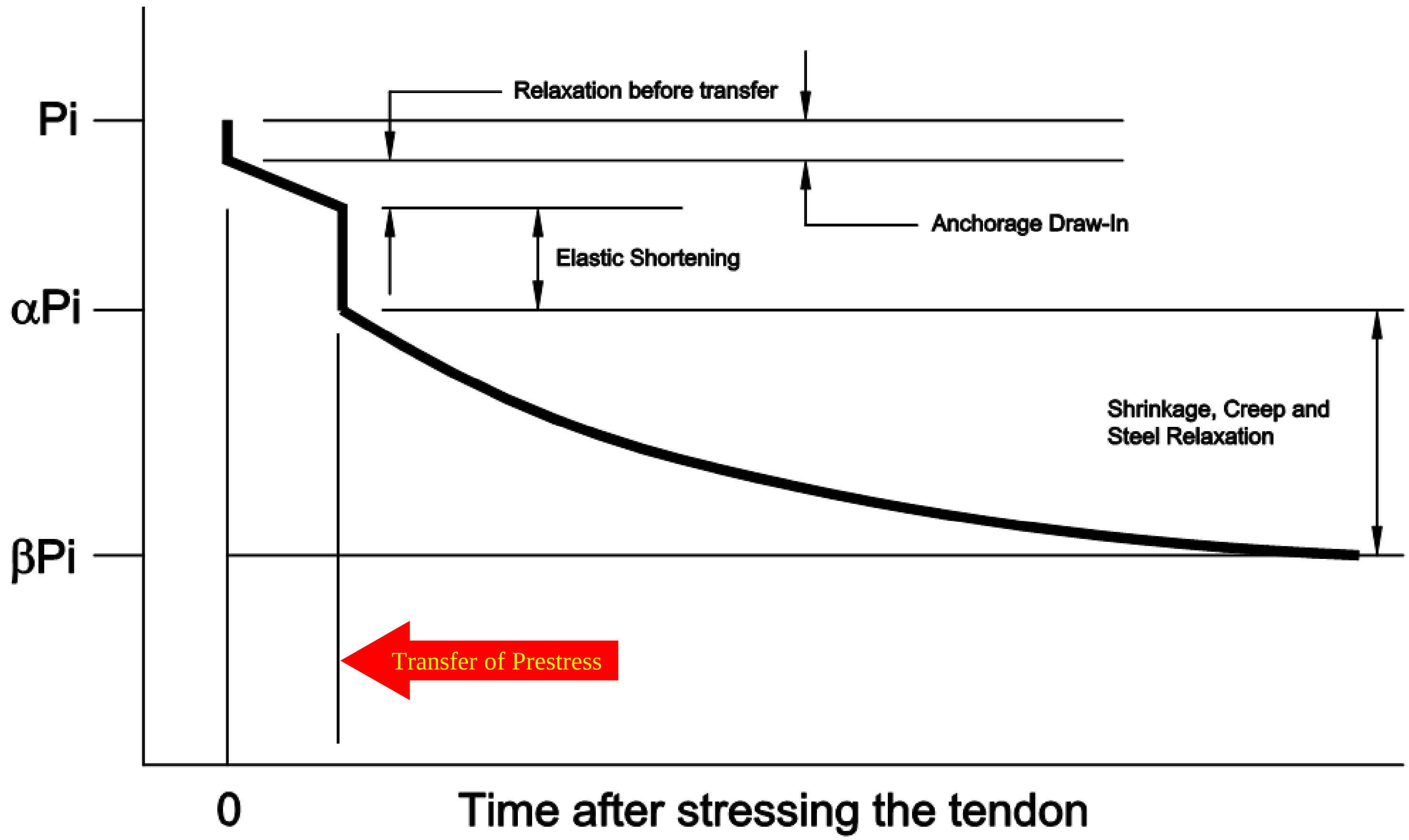
Losses in Pre-Tensioning

- During the process of anchoring, the stressed tendon tends to slip before the full grip is established, thus losing some of its imposed strain or in other words, induced stress. This is known as loss due to **anchorage draw-in**.
- From the time the tendons are anchored until transfer of prestressing force to the concrete, the tendons are held between the two abutments at a constant length. The stretched tendon during this time interval will lose some of its induced stress due to the phenomenon known as **relaxation of steel**.
- As soon as the tendons are cut, the stretched tendons tend to go back to their original state, but are prevented from doing so by the interfacial bond developed between the concrete and the tendons.

Losses in Pre-Tensioning

- The concrete will therefore be subjected to a compressive force, which results in an instantaneous shortening of the member. Since the tendons are bonded to the concrete, they will lose an equal amount of deformation, meaning a reduction of induced stress. This is known as loss due to **elastic deformation**.
- Subsequent to the transfer of prestress, concrete keeps on shrinking due to the loss of free water and continues shortening under sustained stress, thus resulting in a loss of tension in the embedded tendon. These are known as **loss due to shrinkage** and **loss due to creep** respectively.
- Also, loss due to relaxation of steel continues.
- After a long period a final value is reached i.e. $\mathbf{P_e = \beta P_i}$

Losses in Pre-Tensioning



Losses in Post-Tensioning

- The tendons are contained inside ducts, and the hydraulic jack is held directly against the member. During stressing operation, the tendons tend to get straightened and slide against the duct, thus resulting in the development of a frictional resistance. As a result, the stress in the tendon at a distance away from the jacking end will be smaller than that indicated by the pressure gauge mounted on the jack. This is known as **loss due to friction**.
- With regard to elastic shortening, there will be no loss of prestress if all the tendons are stressed simultaneously because the prestress gauge records the applied stress after the shortening has taken place.

Losses in Post-Tensioning

- However, if they are tensioned one after another in sequence, all tendons, except the last one, will lose stress due to elastic shortening of concrete caused by forces in the subsequent tendons.
- Once the stressed tendons are anchored, the time-dependent losses caused by shrinkage and creep of concrete and relaxation of steel begin.

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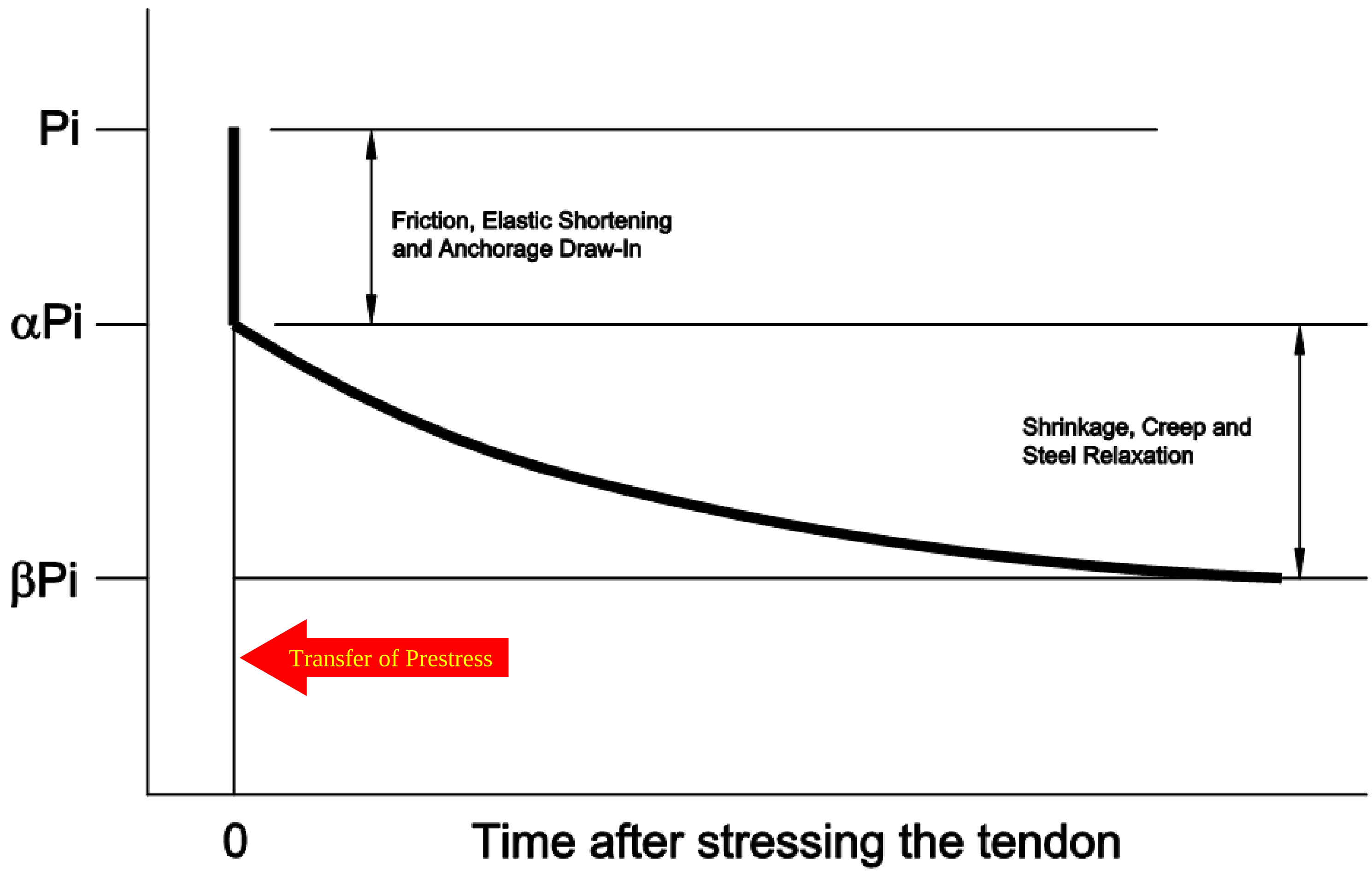
Losses in Post-Tensioning



Elastic Shortening

$\Delta f_p = m\, f_{co}$ ... (16)

$f_{co} = \left(f_{pi} / \left[m + A / A_{ps}(1 + e^2/r^2) \right] \right) - M_i e/I$ ...(17)

where:

Δf_p – loss of prestress due to elastic shortening of concrete

m – modular ratio = E_s/E_c

f_{co} – stress in concrete at the level of tendon

f_{pi} – intial stress in the tendon

e – eccentricity of tendon

Mi – moment due to self weight

r – radius of gyration of section = $(I/A)^{1/2}$

Since Mi & e varies along
the span, f_{co} also varies!
Use average f_{co}

Elastic Shortening

- For a post-tensioned member, where all the tendons are stressed simultaneously, $\Delta f_p = 0$
- For a post-tensioned member, where the tendons are stressed sequentially, $\Delta f_p = 0.5m f_{co}$
- For a pre-tensioned member, $\Delta f_p = m f_{co}$

Example 4-1

Determine the loss of prestress due to elastic shortening for the following problem:

- Span = 20m, Wsw = 9.97 kN/m
- A = 4.23 x 10^5 mm^2 ; I = 9.36 x 10^{10} mm^4
- f_{pi} = 1239 N/ mm^2 ; A_{ps} = 2850 mm^2
- m = 7.5;
- e at both ends = 0
- e at mid pan = 558 mm

Solution

$r = (9.36 \times 10^{10} / 4.23 \times 10^{5}/)^{1/2} = 471$ mm

At mid-span,

$M_i = 9.97 \times 20^2/8 = 498.5$ kNm

$f_{co} = 14.97$ N/mm^2

At supports,

$M_i = 0$

$f_{co} = 7.95$ N/mm^2

Therefore, $\Delta f_p = 0.5 \times 7.5 \times (14.97 + 7.95)/2 = 43$ N/mm^2

loss of prestress = (43/1239)*100 = 3.5 %

Friction

1. Curvature friction, which occur due to intended curvature of the cable path
2. Wobble friction, which is due to unintentional deviation between the centre lines of the tendon and the ducts

Friction

$$P(x) = P_i \, e^{(-(\mu x/r_{ps} + Kx))} \quad \text{..........(18)}$$

Where,

- x – distance from the start of curvature
- P_i – tendon force at the beginning of the curve
- $P(x)$ – tendon force at distance x from the start of curvature
- μ– coefficient of friction (clause 4.9.4.3)
- r_{ps} – radius of curvature (for parabolic curve = $L^2/8\delta$)
- K – profile coefficient (clause 4.9.3.3)

Example 4-2

Determine the loss of prestress due to friction at centre and the right-hand end if prestress is applied at the left-hand end. Given the following:

- Span = 20m, μ = 0.25 & K = 17 x 10^{-4} per metre
- f_{pi} = 1239 N/ mm^2 ; A_{ps} = 2850 mm^2
- e at both ends = 0
- e at mid pan = 558 mm

Solution

rps – (for parabolic curve = $L^2/8\ \delta$)

δ = Ys - Yms

$r_{ps} = 20^2 / (8 \times 0.558) = 89.61$ m

$P_i = 2850 \times 1239 \times 10^{-3} = 3531.2$ kN

At mid-span,

$P(x=10) = 3531.2 \times e^{[-(0.25 \times 10 / 89.61 + 17 \times 0.0001 \times 10)]}$

$= 3376.2$ kN

loss of prestress = (3531.2 – 3376.2) = 155 kN

= (155/3531.2) x 100 = 4.4 %

At the right end,

$P(x=20) = 3531.2 \times e^{[-(0.25 \times 20 / 89.61 + 17 \times 0.0001 \times 20)]}$

$= 3228.9$ kN

loss of prestress = (3531.2 – 3228.9) = 302.3 kN

= (302.3/3531.2) x 100 = 8.6 %

Anchorage Draw-In

- A small contraction during the process of transferring the tensioning force from the jack to the anchorage
- For a pre-tensioned member, this value can be compensated easily by initially over-extending the tendons by the calculated amount of the anchorage draw-in.
- The value of anchorage draw-in (δ_{ad}) depends on the anchorage system used. A typical value would be 5 mm.

Variation of Initial Prestress Force Along a Post-Tensioned Member

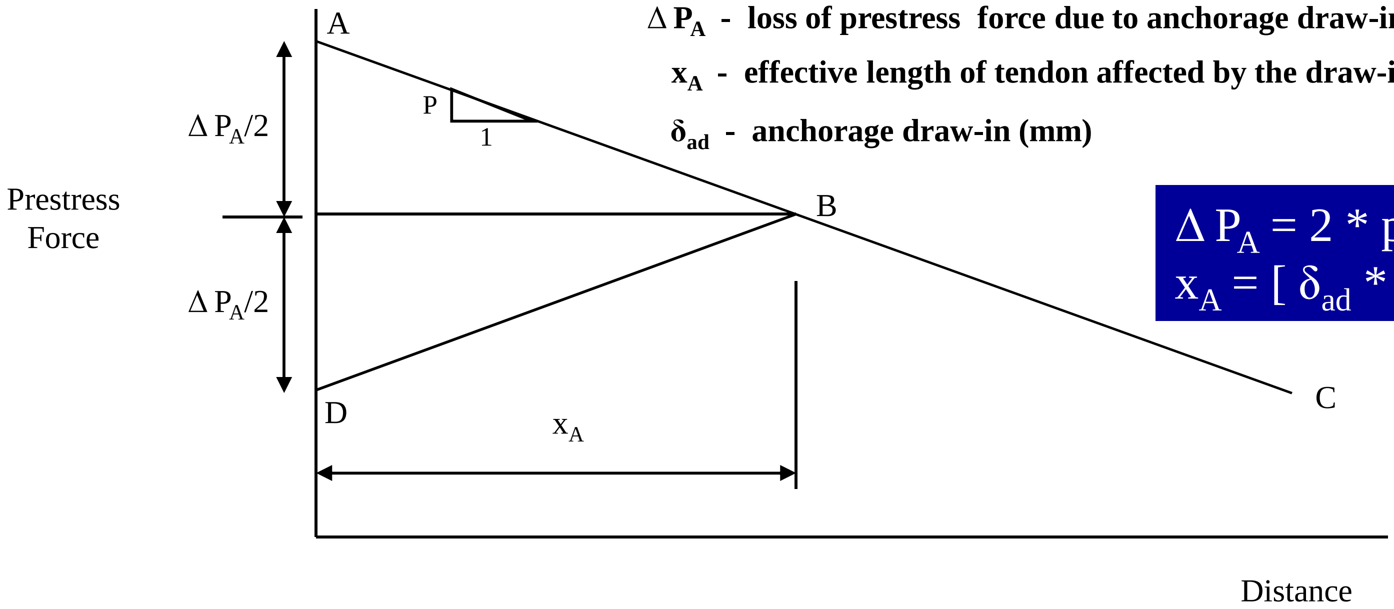
- Effect of elastic shortening ignored
- Total deviated angle is small, curves being approximated by straight lines

p - friction loss per metre (kN/m)

ΔP_A - loss of prestress force due to anchorage draw-in

x_A - effective length of tendon affected by the draw-in

δ_{ad} - anchorage draw-in (mm)



$\Delta P_A = 2 * p * x_A$……………(19)

$x_A = [\ \delta_{ad} * Es * Aps / p]^{1/2}$….(20)

Example 4-3

Determine the initial prestress force distribution along the beam if the anchorage draw-in is 5 mm. Given the following:

- Span = 20m, μ= 0.25 & K = 17 x 10^{-4} per metre
- f_{pi} = 1239 N/ mm^2 ; A_{ps} = 2850 mm^2
- e at both ends = 0
- e at mid pan = 558 mm
- E_s = 195 kN/mm^2

Solution

The friction loss per unit length near the anchorage is given by

$p = P_i \{1 - e^{[-(\mu/r_{ps} + K)]}\}$

$P_i = 3531.2$ kN ($= 2850 \times 1239 \times 10^{-3}$)

$p = 3531.2 \{1 - e^{[-(0.25/89.61 + 17 \times 0.0001)]}\} = 15.82$ kN/m

$x_A = (5 \times 195 \times 10^3 \times 2850/15.82)^{1/2} \times 10^{-3}$

$= 13.25$m

The loss of prestress force at the left-hand end is given by

$\Delta P_A = 2 \times 15.82 \times 13.25 = 419.3$ kN

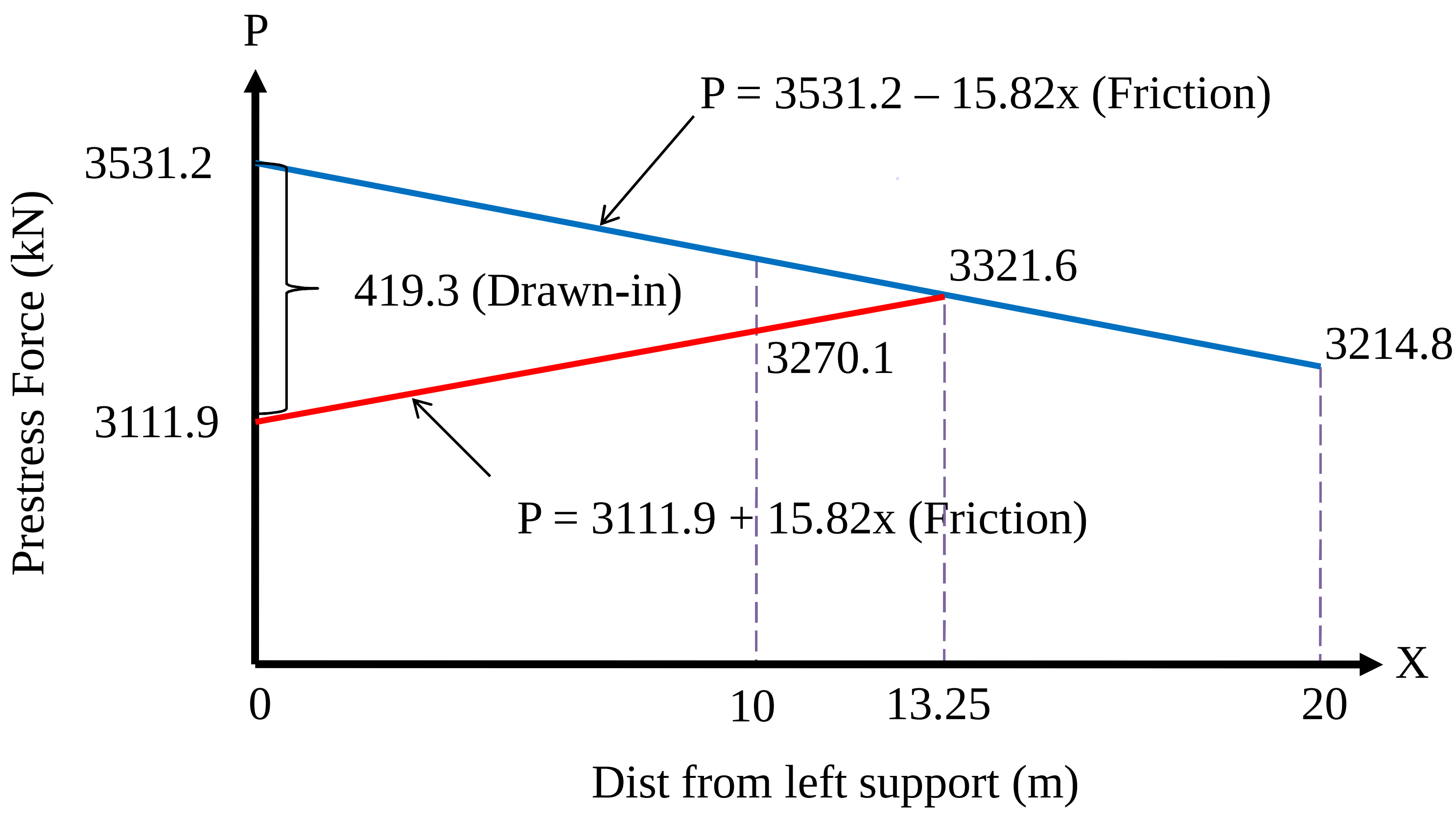
Prestress force at left-end = 3531.2 – 419.3 = 3111.9 kN

Prestress force at midspan = 3531.2 – 419.3 + (15.82x10) = 3270.1 kN

Prestress force at right-end = 3531.2 – (15.82 x 20) = 3214.8 kN

Pi after losses (due to ad + friction) **= (3111.9+3270.1+3214.8)/3=3198.9 kN**

Variation of Initial Prestress Force Along a Post-Tensioned Member



Example 4-4

Using all the previous examples, determine the total long-term prestress losses. Given the following:

- Assuming indoor conditions of exposure
- f_{pi} = 1239 N/ mm^2 ; E_{ci} = 25 kN/mm^2
- Prestressing tendons of low-relaxation steel
- Use initial load (Pi) = 0.7f_{pu}
- E_s = 195 kN/mm^2

Solution

Loss due to shrinkage

$\Delta f_p = \varepsilon_{sh} \times E_s = 300 \times 10^{-6} \times 195 \times 10^3 = 59$ N/mm^2

Loss due to creep

Assuming transfer at 28 days, $\varphi = 1.4$

From (17), $f_{co} = \left(f_{pi} / \left[m + A / A_{ps}(1 + e^2/r^2) \right] \right) - M_i e/I$

Average f_{pi} = [(3111.9+3270.1+3214.8) x 10^3/3]/2850 = 1122 N/mm^2 (allowing for friction loss)

At midspan, f_{co} = 1122 / [7.5 + 4.23 x 10^5/(2850(1+588^2/471^2))]

- 498.5 x 10^6 x 558 / 9.36 x 10^{10} = 14.16 N/mm^2

At support, f_{co} = 1122 / [7.5 + 4.23 x 10^5/(2850)] = 7.20 N/mm^2

Average f_{co} = 0.5(14.16 + 7.20) = 10.68 N/mm^2

$\Delta f_p = (\phi / E_{ci}) \times f_{co} \times E_s$ = 1.4 x 10.68 x 195 x 10^3 / 25 x 10^3

= 117 N/mm^2

Solution

Loss due to relaxation

Δf_p = RF x RV x f_{pi})

RF = 1.5

RV=2.5%=0.025

Δf_p = 1.5 x 0.025 x 1239

= 46 N/mm^2

Total long-term losses (excluding elastic shortening)

= 59 + 117 + 46 = 222 N/mm^2

Or 17.9%