Introduction to Modeling / System Identification

Learning outcomes

- Student is expected to be able to:
  - List different types of models
  - Define/identify some terminologies
  - Explain certain assumptions
  - Describe types of model assessment
  - To compare and give examples for linear and nonlinear regression models

There are several types of models

Empirical model: linear and non linear regression model

Theoretical model: based on mass, momentum, and energy balances

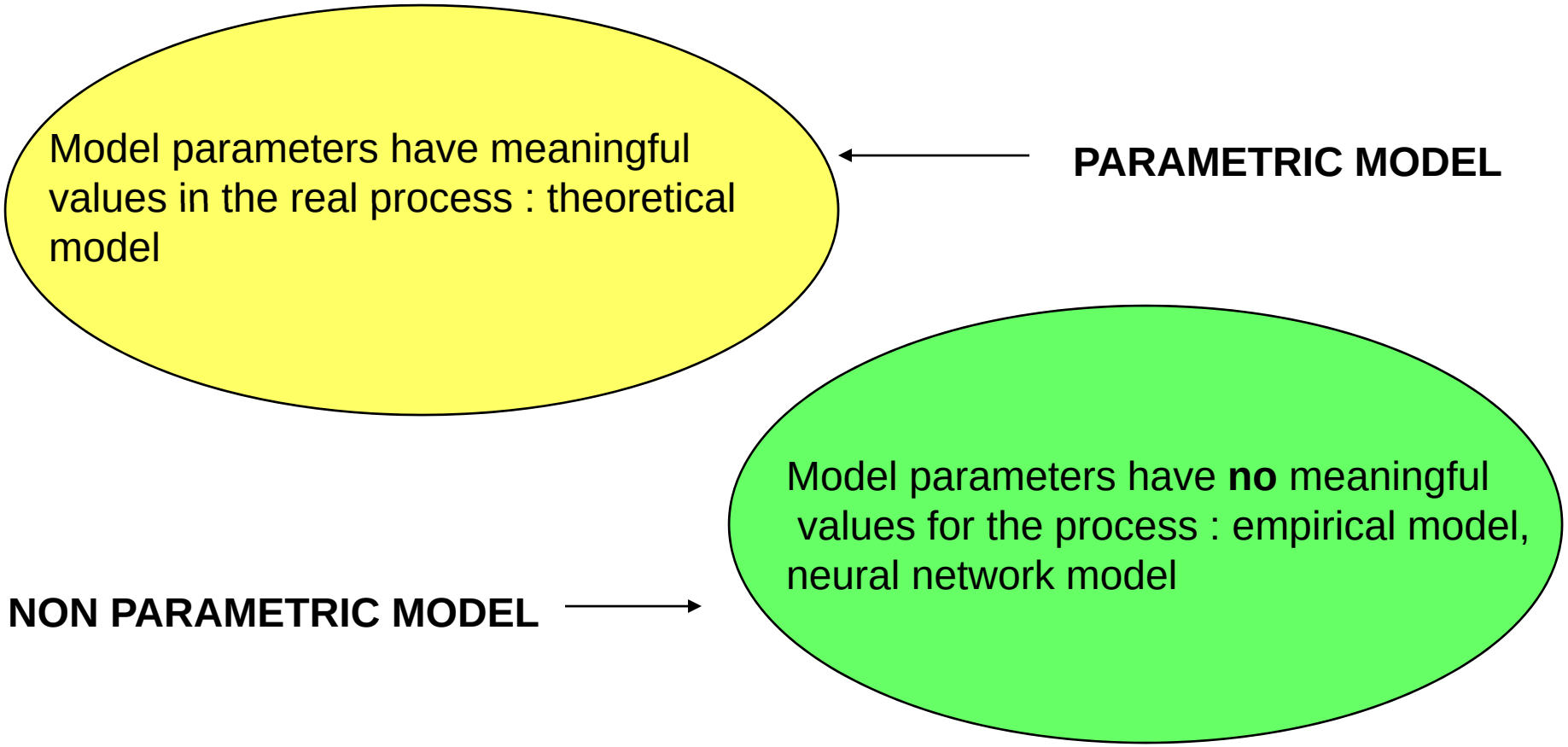
Semi-empirical model

Advanced model: neural network model etc

What is the Purpose of Modeling?

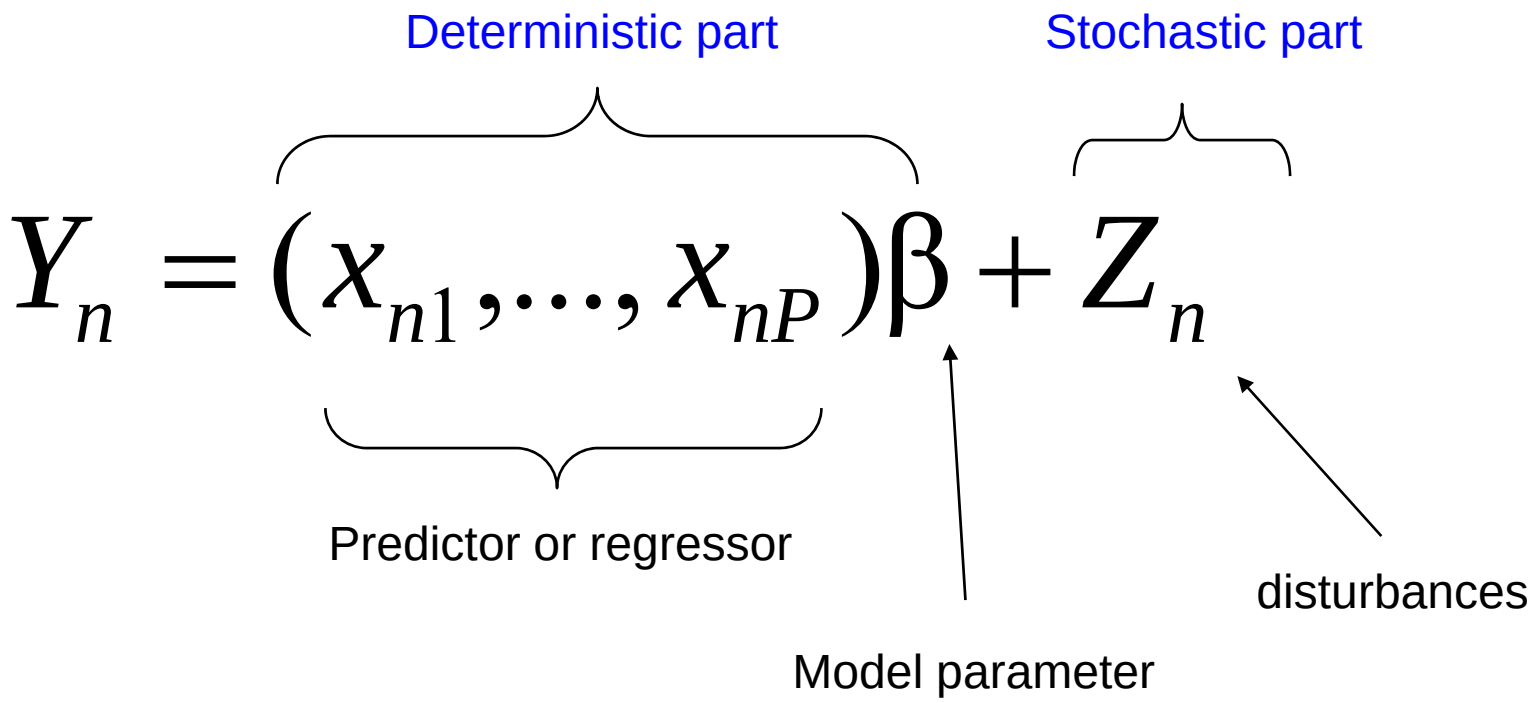
What is the reason for modeling a certain process?

They can be classified as Parametric & Non Parametric Models



Linear Regression Model

Xβ = expectation function for the regression model



Assumptions

1) The expectation function is correct
2) The response Y, is expectation function plus disturbance

   Constant variance. Corrective action is performed for non constant variance such as use weighted least squares or take a transformation of the response Y
3) The disturbance is independent of the expectation function

   Related to assumption 2 –constant variance

   All the important variables are included in the model

Assumptions

4. Each disturbance has a normal distribution. How to check?

   The assumption of normality may be checked by examining the residuals

5. Each disturbance has zero mean. How to check?

   The main implication of this assumption is that there is no systematic bias in the disturbances such could be caused by an unsuspected influential variable

Assumption

6. The disturbances have equal variances

   The validity of the assumption can be checked by plotting the residuals versus the fitted values

7. The disturbances are distributed independently

   The disturbance in different experiments are independent of one another – random error/bias, no systematic bias/error. What is random and systematic bias/error?

MODEL ASSESSMENT

Plotting residuals

- e versus time
  - –Curvilinear behaviour means the assumption of independence of the disturbances may be inappropriate
- e versus predicted values
  - –Can reveal outliers or general inadequacy in the form of the expectation function, and whether the assumption of constant variance is appropriate

$$e = y_n - \hat{y}_n$$

Predicted values or fitted values

MODEL ASSESSMENT

Normal probability plots

- If the expectation function is correct, normal probability plot of the residuals should be a fairly straight line
- The plots are also good for revealing outliers

NONLINEAR REGRESSION MODEL

Regressor or independent variables

$$Y_n = f(\mathbf{x}_n, \theta) + Z_n$$

Expectation function

Nonlinear models – at least one of the derivatives of the expectation function with respect to the parameters depends on at least one of the parameters.

NONLINEAR REGRESSION MODEL

EXAMPLE 1

$$f(t,\theta) = 60 + 70e^{-\theta t}$$

$$\frac{\partial f}{\partial \theta} = -70te^{-\theta t}$$

EXAMPLE 2

$$f(x,\theta) = \frac{\theta_1 x}{\theta_2 + x}$$

$$\frac{\partial f}{\partial \theta_1} = \frac{x}{\theta_2 + x}; \quad \frac{\partial f}{\partial \theta_2} = \frac{-\theta_1 x}{(\theta_2 + x)^2}$$

Activity in Class

Explain why people develop a model for certain process?

How a model can help people who plan to conduct an experiment?

How can you make sure the model represent the actual process?

How can you be confident with your data?