Number Systems

- Standard number systems
  - Decimal
  - Binary
  - Hexadecimal
  - Octal
  - Fractional number representation
- Binary Codes
  - BCD
  - Gray Codes
  - ASCII
- Representation of negative numbers
  - Sign magnitude
  - 1’s complement and 2’s complement
- Arithmetic operations using 2’s complement

Sign-Magnitude

- Left most bit is the sign bit, remaining bits are magnitude bits
- Examples:

0 0 0 1 1 0 0 1 => $+25_{10}$

Magnitude bits

Sign bit ‘0’ indicates positive number

1 0 0 1 1 0 0 1 => -25_{10}

Magnitude bits

Sign bit ‘1’ indicates negative number

Sign-Magnitude

- Find the equivalent decimal of the following sign-magnitude binary
  - 0100 0100 => $2^6 + 2^2$ => 68
  - 1011 0011 => $-(2^5 + 2^4 + 2^1 + 2^0)$ => -51
- Find the equivalent sign-magnitude binary of the following decimal numbers
  - 61 => $2^5 + 2^4 + 2^3 + 2^1 + 2^0$ => 0011 1011
  - -116 => $-(2^6 + 2^5 + 2^4 + 2^2)$ => 1111 0100

1's Complement

- Negative numbers are the complement of the positive numbers and vice versa
- Example:

1's complement: 0 0 0 1 1 0 0 1 => 25_{10}

How to get -25_{10}?

Taking the 1's complement,

1's complement: 1 1 1 0 0 1 1 0 => -25_{10}

1's Complement (cont.)

- Given 1100 1110 in 1's complement, how to find its equivalent decimal?

1's complement: 1 1 0 0 1 1 1 0

Taking the 1's complement,

1's complement: 0 0 1 1 0 0 0 1 => 49_{10}

(Standard binary)

Therefore, 1 1 0 0 1 1 1 0 is -49_{10}

1's Complement (cont.)

- Find the equivalent decimal of the following 1's complement binary:
  - 0010 1101 => $2^5 + 2^3 + 2^2 + 2^0$ => 45
  - 1111 0101 => 0000 1010 => $-(2^3 + 2^1)$ => -10
- Find the equivalent 1's complement binary of the following decimal:
  - 132_{10} => $2^7 + 2^2$ => 0 1000 0100
  - -84_{10} => $-(2^6 + 2^4 + 2^2)$ => -(0101 0100) => 1010 1011

2's Complement

- 1's complement incremented by 1
- Example:

2's complement: 0 0 0 1 1 0 0 1 => 25_{10}

How to get -25_{10}?

Taking the 1's complement,

1's complement: 1 1 1 0 0 1 1 0

Increment by 1

2's complement: 1 1 1 0 0 1 1 1 => -25_{10}

2’s Complement (cont.)

- Given 1100 1110 in 2’s complement, how to find its equivalent decimal?

2’s complement: 1 1 0 0 1 1 1 0

Taking the 1’s complement,

1’s complement: 0 0 1 1 0 0 0 1

Increment by 1

2’s complement: 0 0 1 1 0 0 1 0 => 50_{10}

Therefore, 1 1 0 0 1 1 1 0 is -50_{10}

2's Complement (cont.)

- Find the equivalent decimal of the following 2's complement binary:
  - 0010 1101 => $2^5 + 2^3 + 2^2 + 2^0$ => 45
  - 1111 0101 => 0000 1011 => $-(2^3 + 2^1 + 2^0)$ => -11
- Find the equivalent 2's complement binary of the following decimal:
  - 132_{10} => $2^7 + 2^2$ => 0 1000 0100
  - -84_{10} => $-(2^6 + 2^4 + 2^2)$ => -(0101 0100) => 1010 1100

Example

Fill in the following table

Decimal	Sign-Magnitude	1's Complement	2's Complement
+19	0001 0011	0001 0011	0001 0011
-19	1001 0011	1110 1100	1110 1101

Why is there 3 standards in representing negative numbers?

⇒There are limitations in sign-magnitude and 1's complement representation, therefore, 2's complement representation is the standard today

⇒Limitation of sign magnitude and 1's complement

⇒There are two representations of zero

Digit Group

Terminology for groups of bits

Digit Group	Amount of Bit
Quad-Word	64
Double-Word	32
Word	16
Byte	8
Nibble	4
Bit	1

Example

- What does the byte $0101\ 0000_2$ means?
  - 50 (hexadecimal)
  - 50 (BCD)
  - 120 (octal)
  - 80 (decimal equivalent of binary)
  - 0111 1000 (Gray Code equivalent)
  - Character ‘P’ (ASCII equivalent)
  - 80 (decimal equivalent of sign-magnitude)
  - 80 (decimal equivalent of 1’s complement)
  - 80 (decimal equivalent of 2’s complement)

2's Complement Arithmetic

- We will look only at Byte addition and subtraction of 2's complement number
- In Byte 2's complement, the largest number that can be represented is 0111 1111 (127) and the lowest is 1000 0000 (-128)
- If addition or subtraction exceeds 127 or -128, then the result will be incorrect
- This condition of incorrect result is called overflow

2’s Complement Arithmetic (cont.)

Range of Byte 2’s complement

2’s complement		Decimal
0111 1111	=>	127
0111 1110	=>	126
0111 1101	=>	125
…	…	…
0000 0010	=>	2
0000 0001	=>	1
0000 0000	=>	0
1111 1111	=>	-1
1111 1110	=>	-2
…	…	…
1000 0011	=>	-125
1000 0010	=>	-126
1000 0001	=>	-127
1000 0000	=>	-128

When adding/subtracting Byte 2’s complement number, make sure the result is within range

2's Complement Addition

		1	1	1	1	1	1			
		0	0	0	0	1	1	1	1	=> 15 + (-6)
+		1	1	1	1	1	0	1	0	Result will be valid
	1	0	0	0	0	1	0	0	1	=> 9

Discard carry

		1	1	1	1					
		0	1	1	1	1	1	0	1	=> 125 + 58
+		0	0	1	1	1	0	1	0	Result will be invalid
	0	1	0	1	1	0	1	1	1	How to make it valid?

Add additional sign-bit => Result = 183

2’s Complement Subtraction

- Take the 2’s complement of the subtrahend, then do addition
- Example: 8 – 3
  - Take 2’s complement of 3 => 1111 1101
  - Do addition (now its 8 + (-3))

		1	1	1	1				
		0	0	0	0	1	0	0	0
+		1	1	1	1	1	1	0	1
	1	0	0	0	0	0	1	0	1

=> 8 – 3 = 5

Discard carry