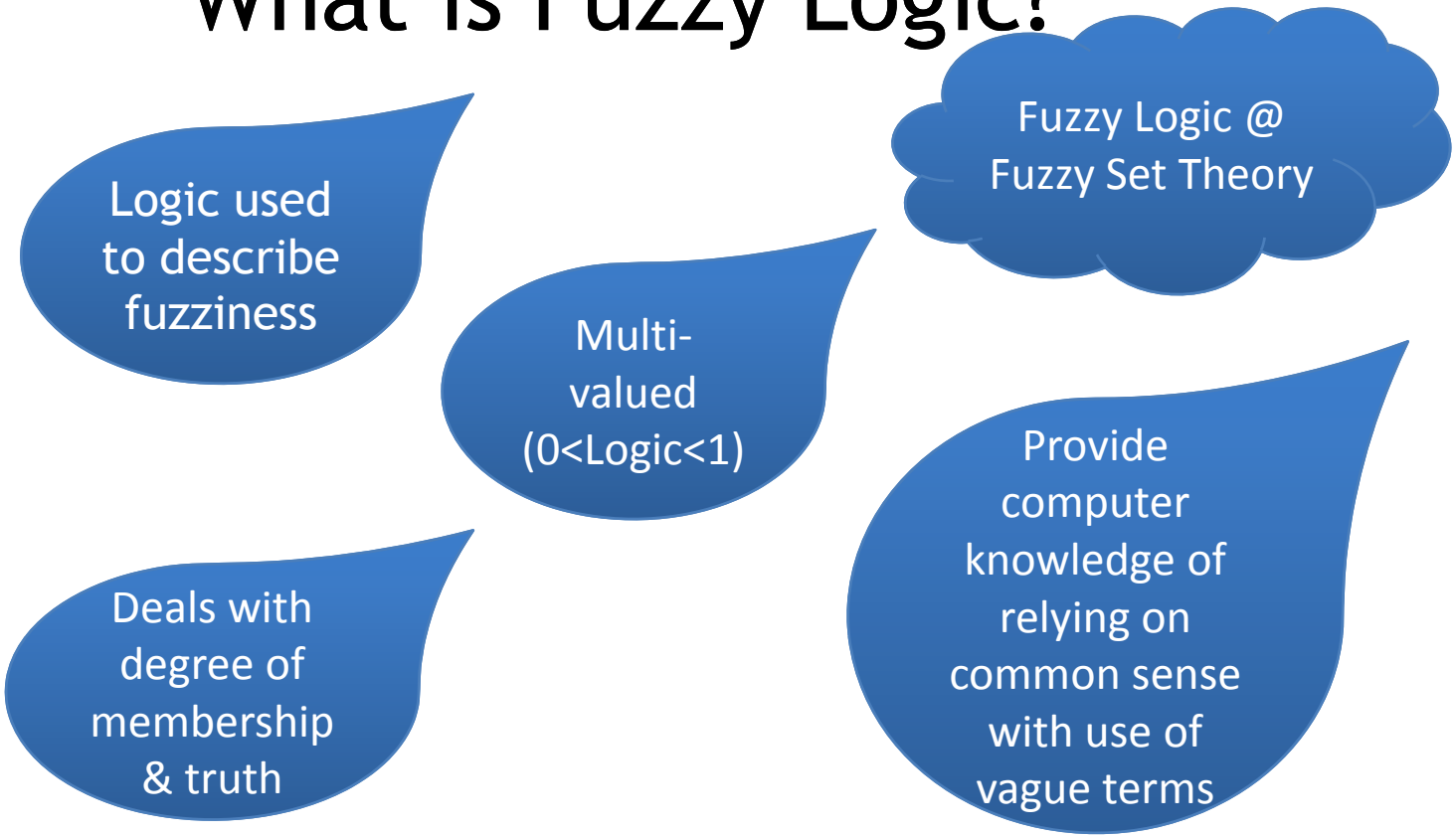
FUZZY EXPERT SYSTEMS

ROSLINA RASHID
Prepared by
AMIRA MOHD NASIB
MENAKA NARAYANASAMY

What is Fuzzy Logic?



A set of mathematical principles for knowledge representation based on degrees of membership rather than classical

Differences Between Conventional Logic & Fuzzy Logic

Boolean @ Conventional Logic	Fuzzy Logic
• Uses sharp distinctions ➢Exp: max height for short =170cm ✓ 170.5cm is considered tall	• No distinct values ➢Exp: John is very tall
• Draw lines between members of a class & non-members	•Do not distinguish members of a class & non-members
• Can be absurd ➢ Is 170cm really short?	• Avoids absurdities ➢ Reflects on how people think ➢ Creates a model based on ✓Sense of words ✓Decision making ✓Common sense
• Has only two values (0 or 1)	• Has an extended range of values between 0 and 1
• Possibility of having “True” or “False” only	• Possibility of having a little “Truth” in “False”

History of Fuzzy Logic

Jan Lukasiewicz (1930s)

- Introduced extended range of logic (0 < possibility < 1)
- Lead to inexact reasoning technique (possibility theory)

Max Black (1937)

- Published "Vagueness: an exercise in logical analysis
- Vagueness=matter of probability
- Defined 1st simple fuzzy set & outlined basic ideas of fuzzy set operations

Lotfi Zadeh (1965)

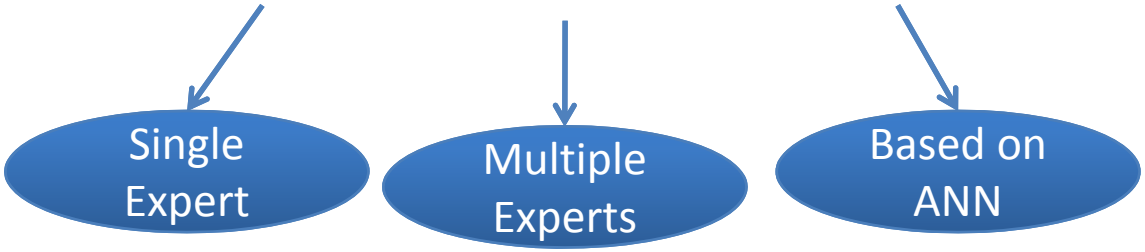
- Published "Fuzzy Sets"
- Extended work on possibility theory into a formal system of mathematical logic
- Introduced new concept for applying natural language terms
- Represented new logic & manipulated fuzzy terms into Fuzzy Logic

Fuzzy Set versus Crisp Set

Fuzzy Set	Crisp Set
• Set with fuzzy boundaries	• Set with distinct boundaries
•Principle of dichotomy X = universe of discourse x = elements $_A(x) : X \longrightarrow [0,1]$ (membership function) If x totally in A, $_A(x) = 1$ x not in A, $_A(x) = 0$ x partly in A, $0 < _A(x) < 1$	• Principle of dichotomy X = classical (crisp) set x = elements $f_A(x) : X \longrightarrow 0,1$ (characteristic function) If $x \in A$ value = 1 $x \in A$ value = 0
• Element belongs to a fuzzy set with a certain degree of membership ➢ Maybe partly true @ partly false	• Logic cannot represent vague concepts
• Degree is a real number in the interval [0,1]	

Representing Fuzzy Set

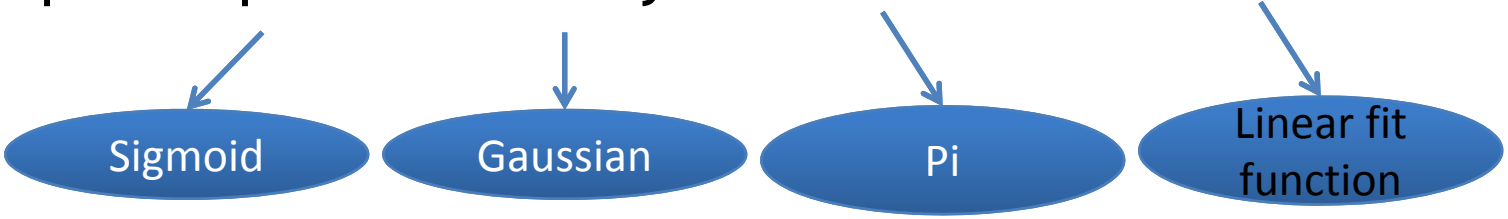
Step 1: Membership function determined



Step 2: Produce a fuzzy set

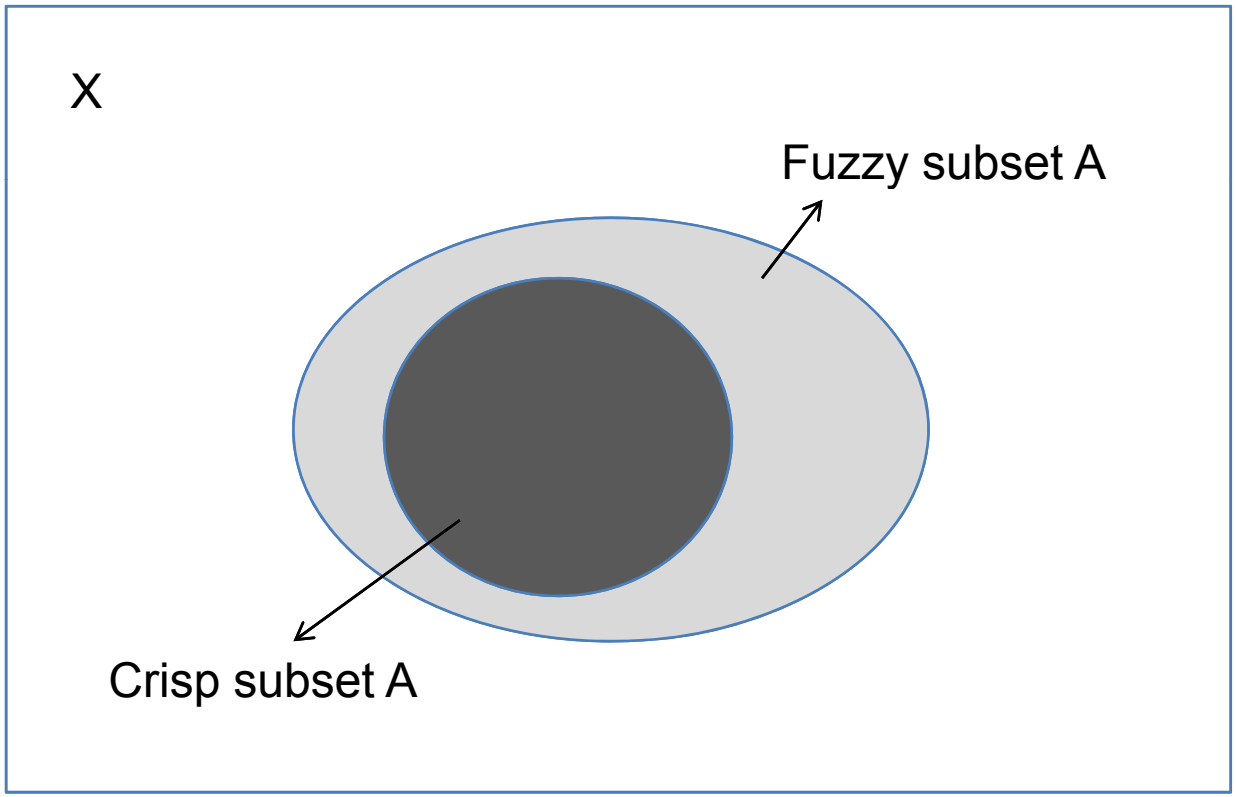
Step 3: Assume universe of discourse X = reference super set

Step 4: Express the fuzzy set as a function



Step 5: Map the elements of the set to their degree of membership

Crisp and Fuzzy Subset



Linguistic Variables and Hedges

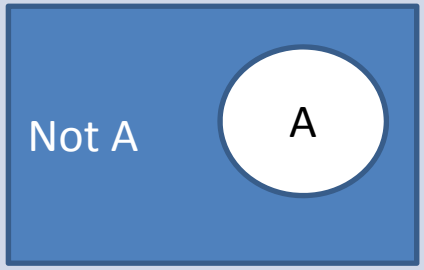
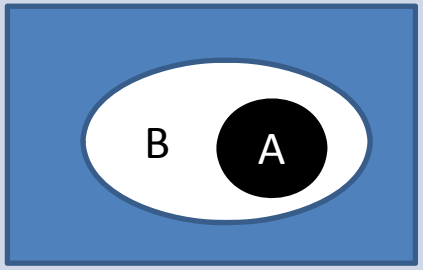
- Linguistic variable
  - Fuzzy variable
  - Used in fuzzy rules
  - Exp: for speed (*very slow, slow, medium, fast, very fast*)
- Hedges
  - Fuzzy set qualifiers
  - Carried by linguistic variable
  - Modifies the shape of fuzzy sets
  - Exp: (*very, somewhat, quite, slightly, more or less)*

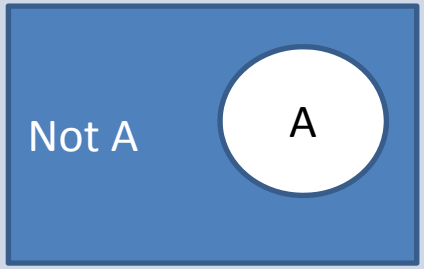
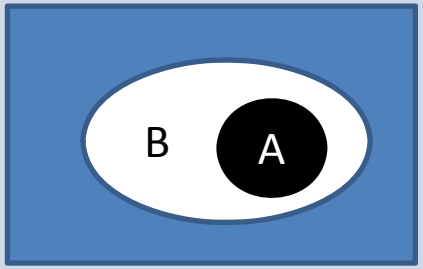
- Acts as operations
  - *Very* performs concentrations (creates a new subset)
  - *More or less* performs dilation (expands the set)
- Breaks down continuums into fuzzy intervals
- Causes fuzzy sets to overlap
- Helps to reflect human thinking
- Used as:
  - All-purpose modifiers
  - Truth-values
  - Probabilities
  - Quantifiers
  - Possibilities

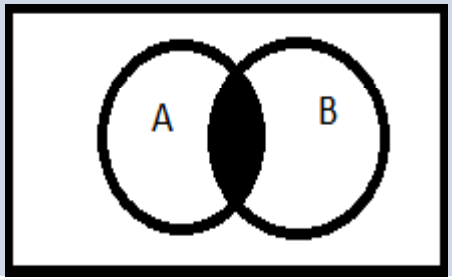
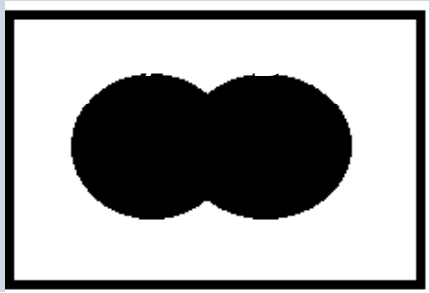
Operations Of Fuzzy Set

On 19th century: Georg Cantor describes how CRISP can be interact

INTERACTIONS = OPERATIONS

	COMPLEMENT	CONTAINMENT
Operation on classical set		
Crisp set	WHO does NOT BELONG to the set?	Which sets BELONG TO WHICH other sets?
Fuzzy set	HOW MUCH do elements not belong to the set?	Which sets BELONG to other sets?
Comments	• An opposite of the set • If A is the fuzzy set, the complement of A is as follow: $\mu_{\bar{A}}(x) = 1 - \mu_A(x)$	• A set can contain other set • the smaller set: subset • Crisp set: All elements in a subset is fully belong to a larger set which gives membership values is 1 • Fuzzy set: membership values of each element I subset is less than to the larger set

	COMPLEMENT	CONTAINMENT
Operation on classical set		
Crisp set	WHO does NOT BELONG to the set?	Which sets BELONG TO WHICH other sets?
Fuzzy set	HOW MUCH do elements not belong to the set?	Which sets BELONG to other sets?
Comments	• An opposite of the set • If A is the fuzzy set, the complement of A is as follow: $\mu_{\neg A}(x) = 1 - \mu_A(x)$	• A set can contain other set • the smaller set: subset • Crisp set: All elements in a subset is fully belong to a larger set which gives membership values is 1 • Fuzzy set: membership values of each element I subset is less than to the larger set

	INTERSECTION	UNION
Operation on classical set		
Crisp set	Which element belongs to both sets?	Which element belongs to either set?
Fuzzy set	How much of the elements is in both sets?	How much of the element is in either set?
Comments	• Fuzzy: An element may partly belong to both sets with different membership value $\mu_{A\cap B}(x) = \min[\mu_A(x), \mu_B(x)] = \mu_A(x) \cap \mu_B(x)$ $x \in X$ Where (choose the lowest of membership value)	• Reverse of the intersection • Union is the largest membership value of the element in either set $\mu_{A\cup B}(x) = \max[\mu_A(x), \mu_B(x)] = \mu_A(x) \cup \mu_B(x)$ $x \in X$ Where

Crisp and fuzzy sets have SAME PROPERTY.
And crisp can be considered just a special case of fuzzy set!

PROPERTIES	
Commutativity	$A \cup B = B \cup A\,;\; A \cap B = B \cap A$
Associativity	$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$
Distributivity	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Idempotency	$A \cup A = A\,;\; A \cap A = A$
Identity	$A \cup \emptyset = A$ $A \cap X = A$ $A \cap \emptyset = \emptyset$ $A \cup X = X$

PROPERTIES	
Involution	-A (-A) = A
Transivity	If (A ⊆ B) ∩ (B c C) then (A c C)
De Morgan's Laws	$-(A \cap B) = -A \cup -B$ $-(A \cap B) = -A \cap -B$

- Variety of fuzzy sets can be obtain from the existing one

Ex; fuzzy set A (tall men) and fuzzy set B (short men), thus fuzzy set C **(not very tall men and not very short men**) and set D **(not very very tall and not very very short men**) can be derived by using..

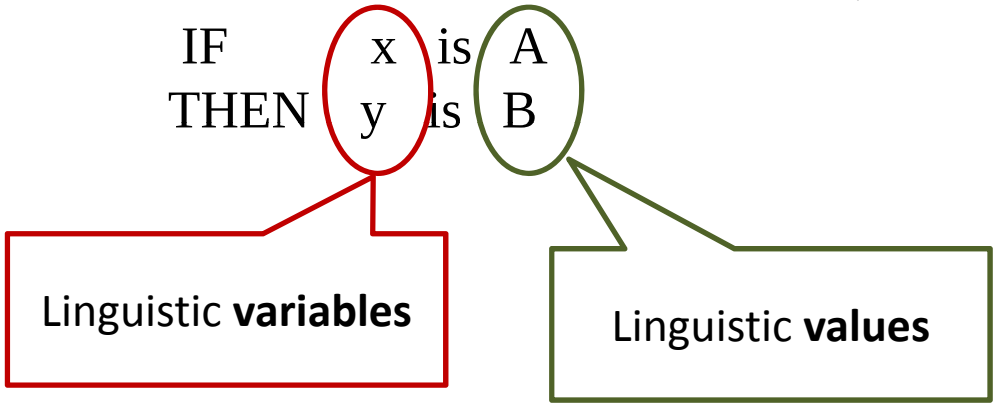
$$\mu_C(X) = [1 - \mu_A(x)^2] \cap [1 - (\mu_B(x))^2]$$
$$\mu_D(X) = [1 - \mu_A(x)^4] \cap [1 - (\mu_B(x))^4]$$

Fuzzy Rules

What??

As conditional statement in the form:

IF x is A
THEN y is B



CLASSICAL RULES

Rule 1

IF speed is > 100

THEN Stopping_distance is long

Rule 2

IF speed is < 40

THEN stopping_distance is short

- Speed can vary between 0-220km/h
- But linguistic variable (stopping_distance) can only take either long OR short
- Classical rules are expressed in black-and-white languange/Boolean logic

FUZZY RULES

CUT at least 90% rules

Rule 1

IF speed is fast

THEN Stopping_distance is long

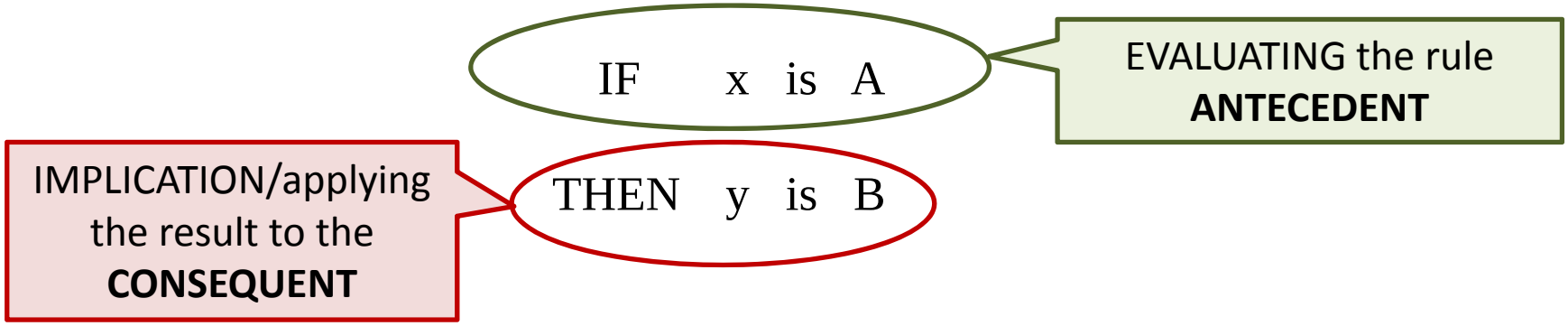
Rule 2

IF speed is slow

THEN stopping_distance is short

- Linguistic variable speed has the range (0-220km/h) which the value includes slow, medium and fast
- Linguistic variable of stopping_distance between (0-300km) include sets of short, medium and long

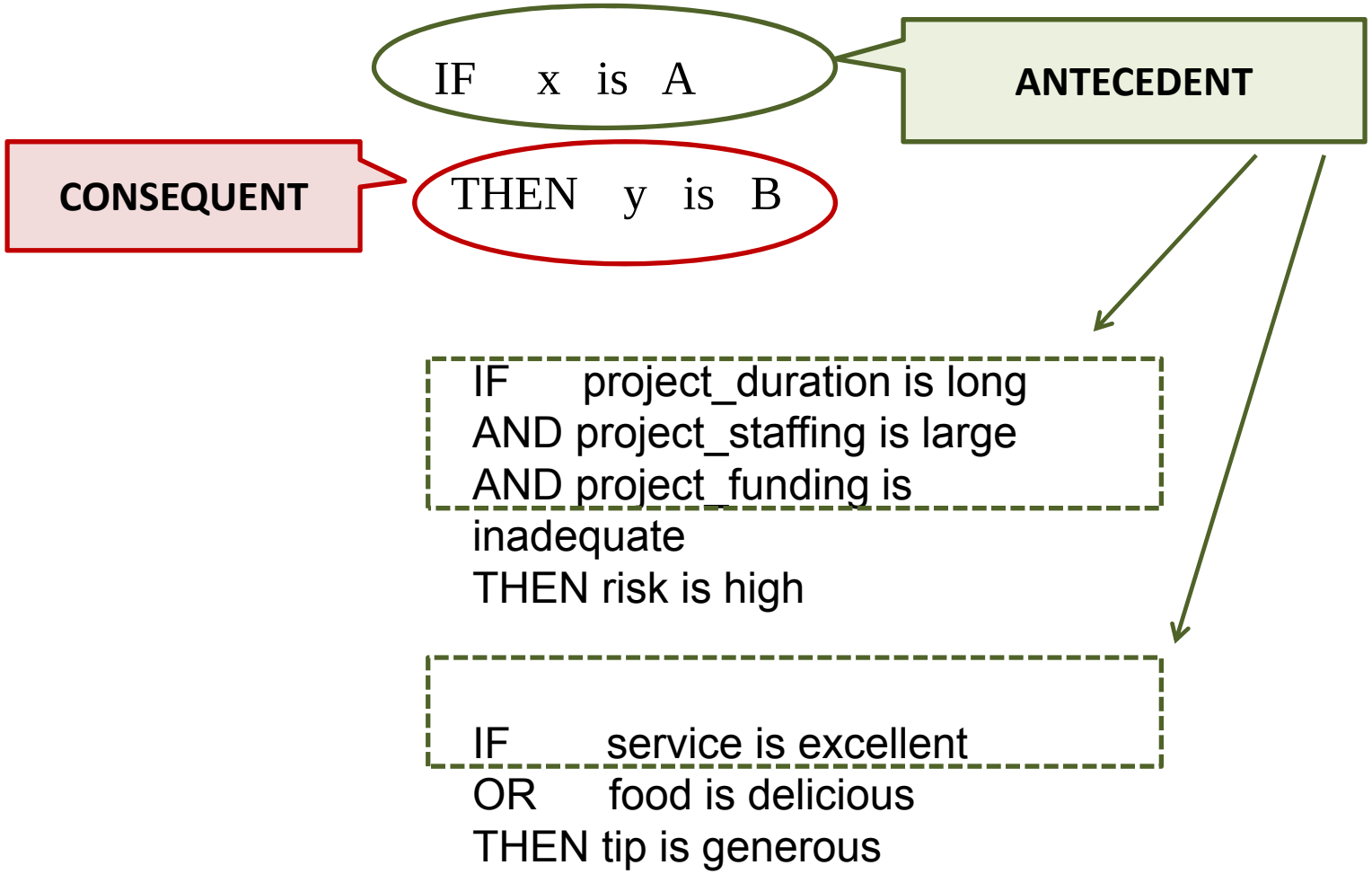
HOW TO REASON with fuzzy logic??



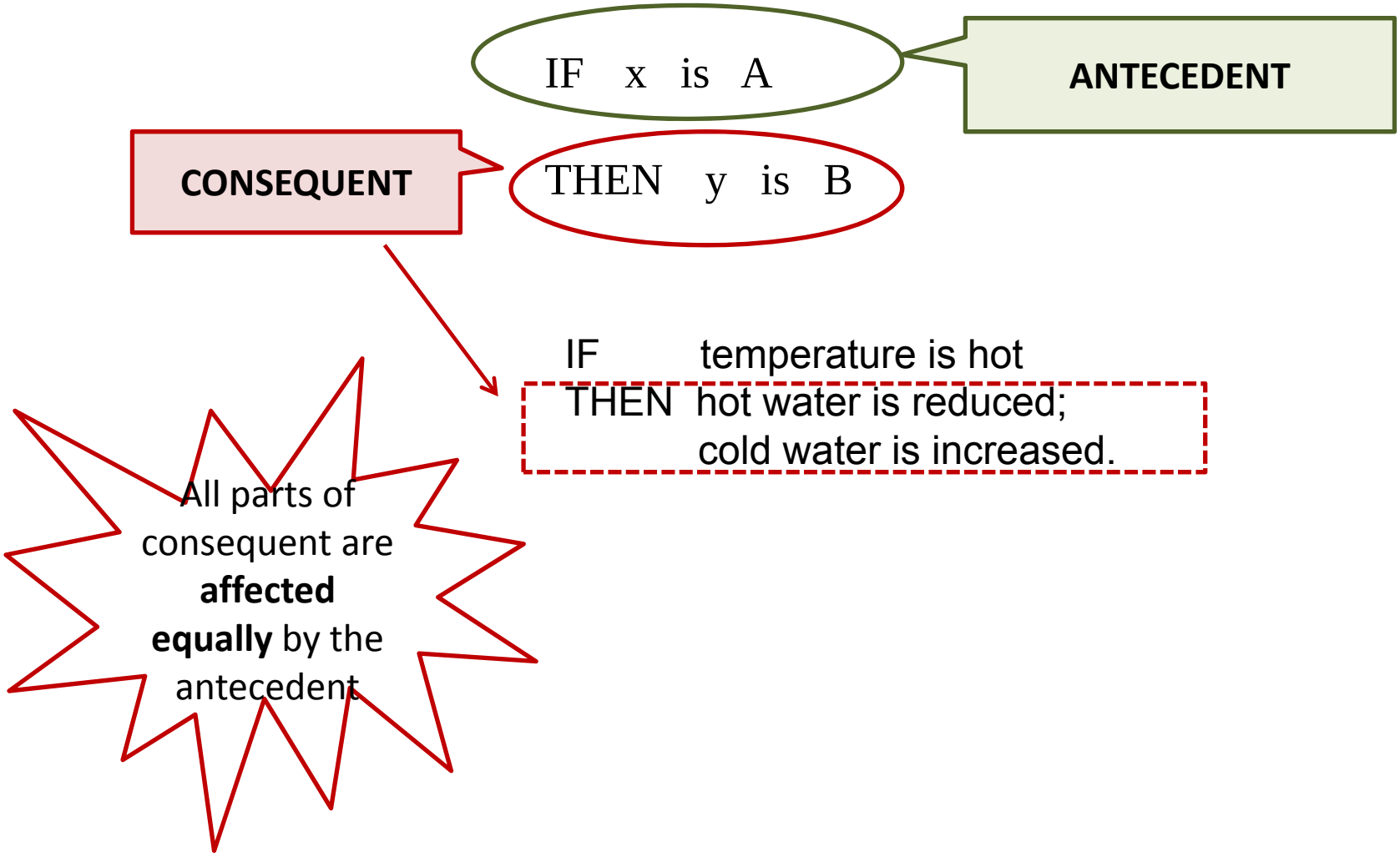
MONOTOMIC SELECTION
(Cox, 1994)

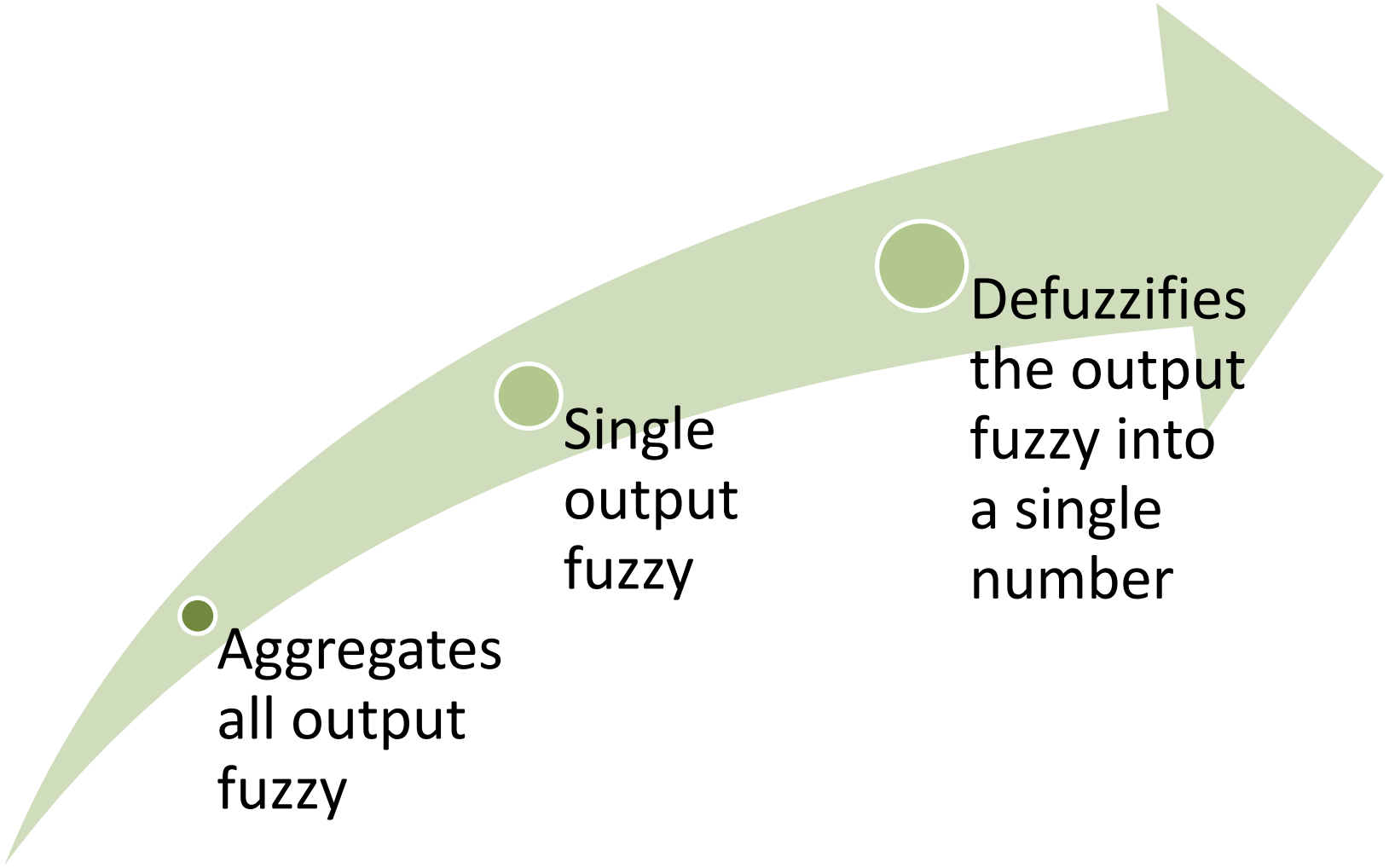
The **value of truth membership grade** of the rule consequent can be **estimated** directly **from** a corresponding truth membership grade in **the antecedent**.

Can antecedent or a consequent have multiple parts??



Can antecedent or a consequent have multiple parts??




Aggregates all output fuzzy
Single output fuzzy
Defuzzifies the output fuzzy into a single number