EQUILIBRIUM OF RIGID BODIES

TWO DIMENSIONAL

Equilibrium of a particle

$$\sum F_x = 0 \quad \sum F_y = 0$$

Equilibrium of a rigid body

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_A = 0 \text{ (where } A \text{ is any point)}$$

Identify all relevant forces by drawing the Free–Body Diagram (FBD) of the rigid body.

FREE BODY DIAGRAM

A free body diagram, also called a force diagram is a pictorial representation often used by physicists and engineers to analyze the forces acting on a body of interest. A free body diagram shows all forces of all types acting on this body. Drawing such a diagram can aid in solving for the unknown forces or the equations of motion of the body. Creating a free body diagram can make it easier to understand the forces, and torques or moments, in relation to one another and suggest the proper concepts to apply in order to find the solution to a problem.

Procedure for drawing the FBD

- Draw the boundary of the chosen subset and detach/ separate it from all other bodies,
- For body with a mass, put the weight $= mg$ acting at the centre of gravity (G) of the body in the vertically downwards direction,
- Input all <u>external</u> forces acting on the body,

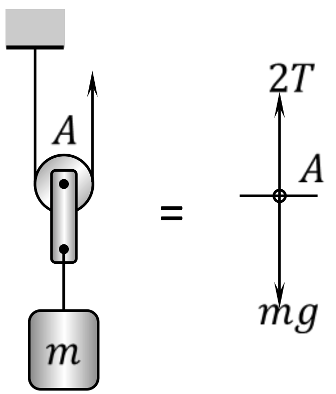
  For systems involving cables, the force of the cable *must* act outwards of the body, i.e. the cable must be in tension.
- place the reaction/s where the body touches or connected to a different rigid body,
- The FBD should also include dimensions for the process of taking moment

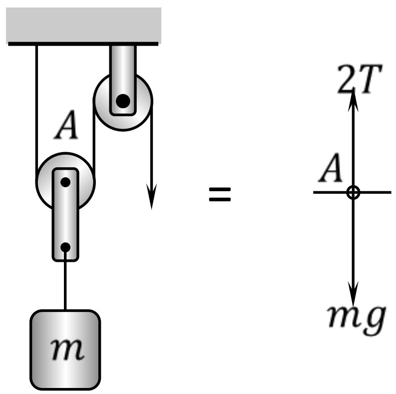
External Forces

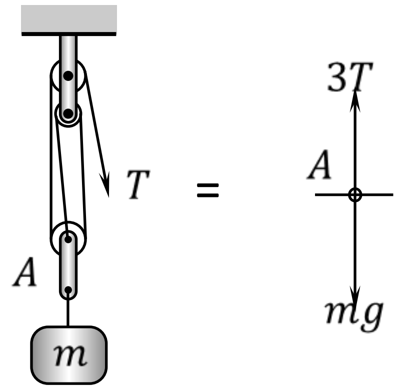
Special Notes – Cables/ wires/ ropes and pulleys

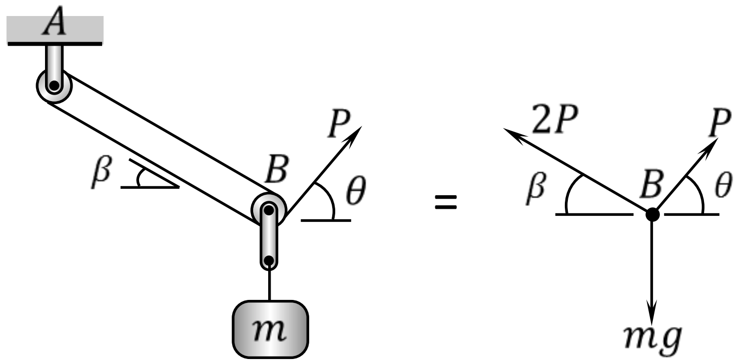
- All cables are assumed to be inextensible.
- All forces acting from cables must direct outwards from point of analysis, i.e. in tension.
- When a cable passes a pulley, the tension is the same as long as it is the same cable.
- Pulleys are assumed to be smooth except stated otherwise.
- Dimensions of a pulley are usually neglected in calculations except stated otherwise.

Examples


A
m
=
2T
A
mg


A
m
=
2T
A
mg


T
A
m
=
3T
A
mg


A
β
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P
θ
m
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2P
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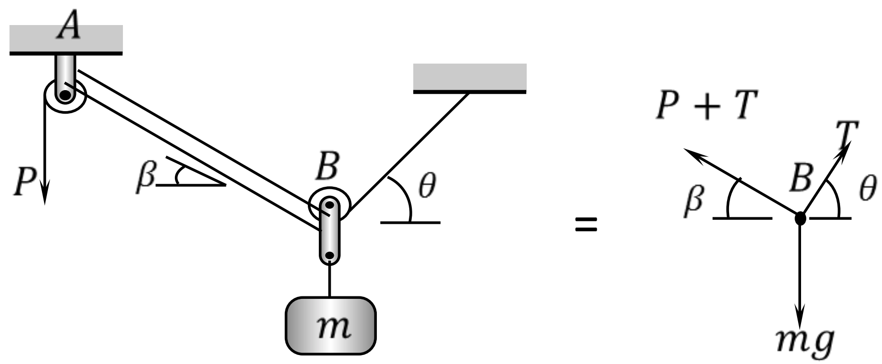
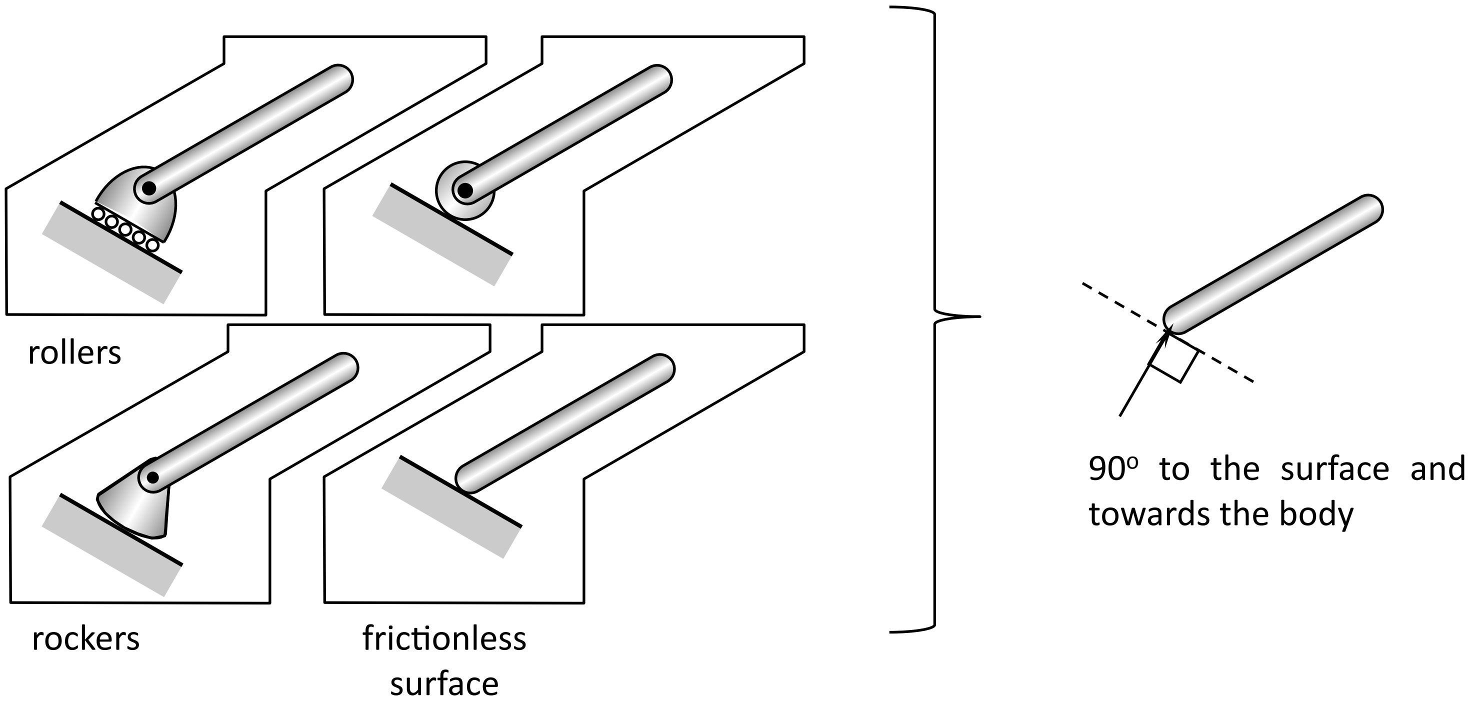

A
P
β
B
θ
m
=
P + T
T
β
B
θ
mg

Reactions at Supports and Connections (2D)

A force with a known line of action

Rollers, Rockers, Frictionless surface.

Perpendicular to the surface and must point towards the free body.



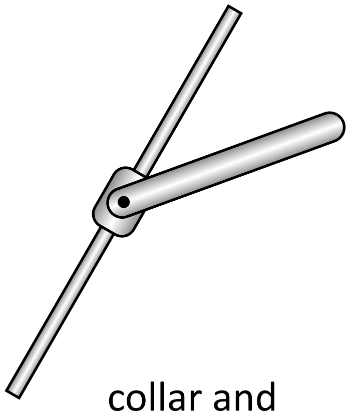
Reactions at Supports and Connections (2D)

A force with a known line of action

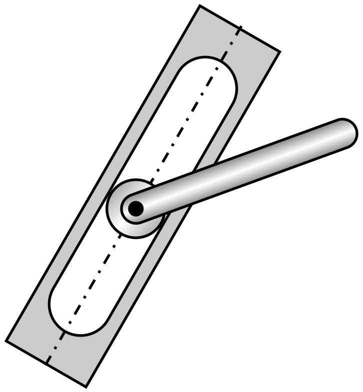
Collars on frictionless rod, Frictionless slot, Double track rollers.

(reversible single track rollers and rockers).

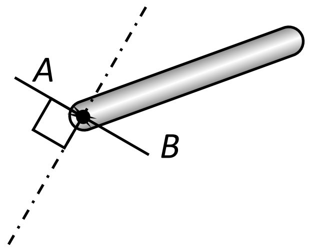
Perpendicular to the rod/ slot and can be directed either way (NOT both).



collar and
frictionless rod



frictionless slot

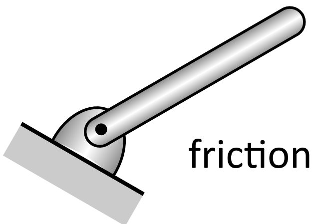


either A or B,
NOT both

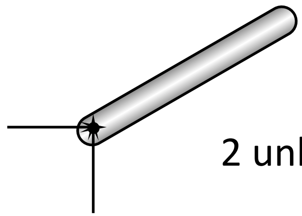
Two force components, involving 2 unknowns

Frictionless pin.

Usually represented by their x and y components that are normally assumed *safely*.



frictionless pin

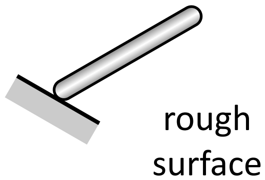


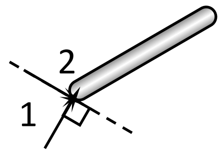
2 unknowns

Two force components, involving 2 unknowns

Rough surface.

One perpendicular to the surface and must point towards the free body and the other 90^o to it (tangent to the surface) that can be directed either way (not both).





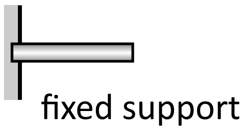
2 unknowns

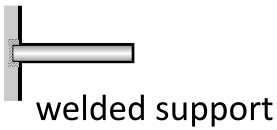
1. 90^o to the surface and towards the body.
2. Either upwards or downwards, tangent to the incline.

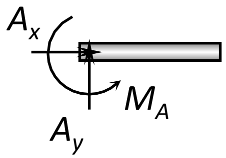
Two force components and one couple, involving 3 unknowns

Fixed Support, Welded

The force components are usually represented by their x and y components that are normally assumed *safely*. The sense of the couple can be arbitrarily assumed, the sign of the answer will indicate the correct direction.



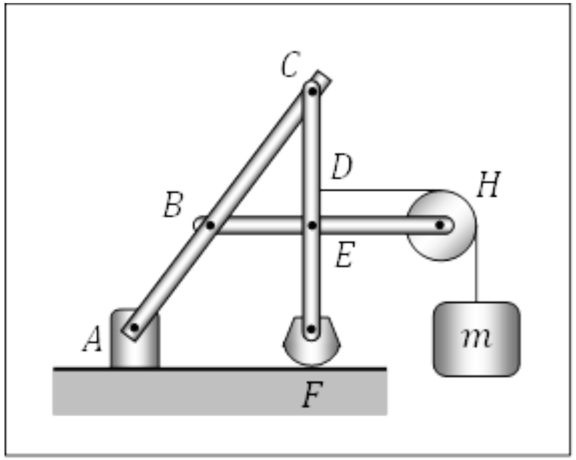


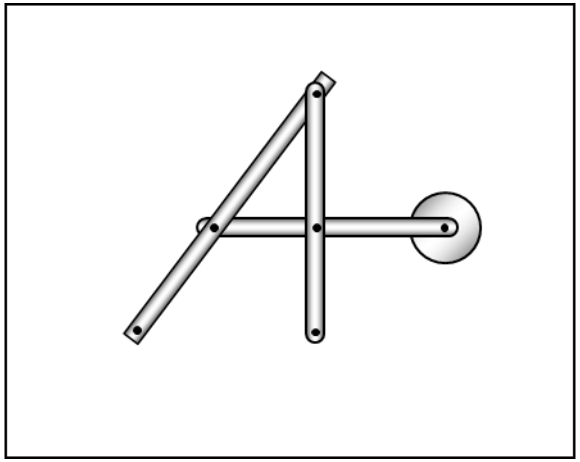


3 unknowns

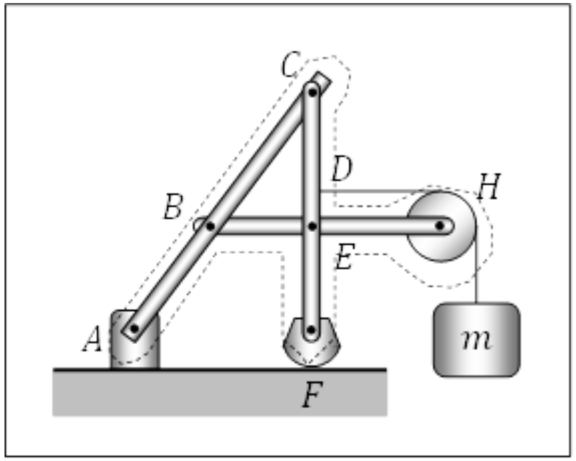
EXAMPLE 1

Complete the Free Body Diagrams. Mass of bodies is negligible unless stated by m kg. All contacting surfaces are smooth.





Draw the boundary of the required subset.



Check for

1. Mass within the subset:

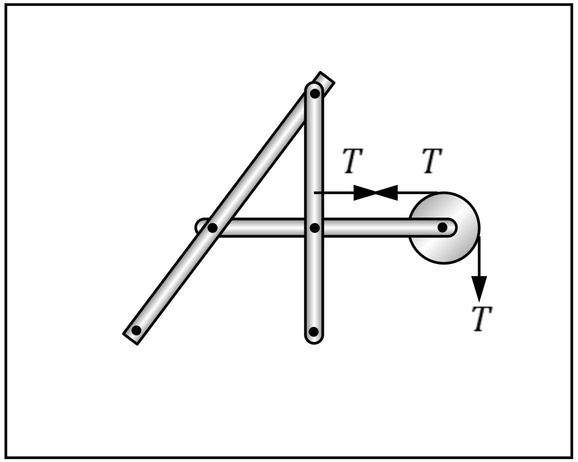
   Not relevant.

2. External force:

   The cable, where the tension $T = mg$.

   The cable tension at D cancels the tension at H.

   Tension must act outwards of the body.

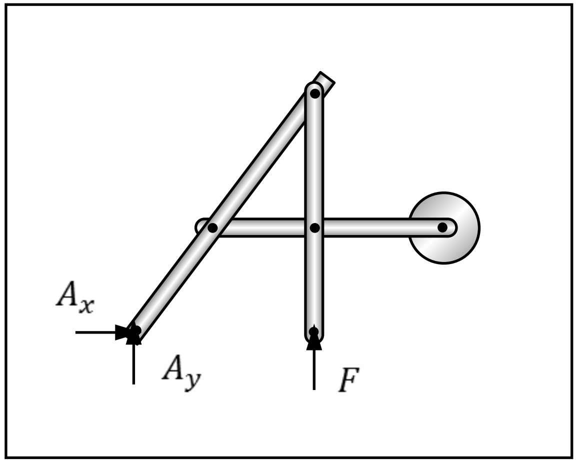


3. Reaction at the rocker at F:

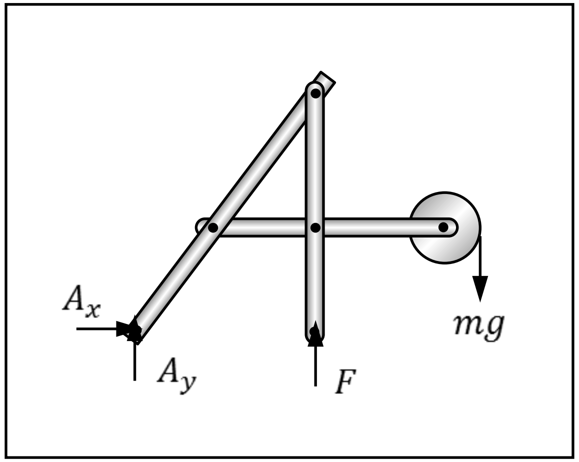
   A normal force perpendicular to the surface and towards the body.

   Reaction at pin A:

   Two unknown forces, A_x and A_y.

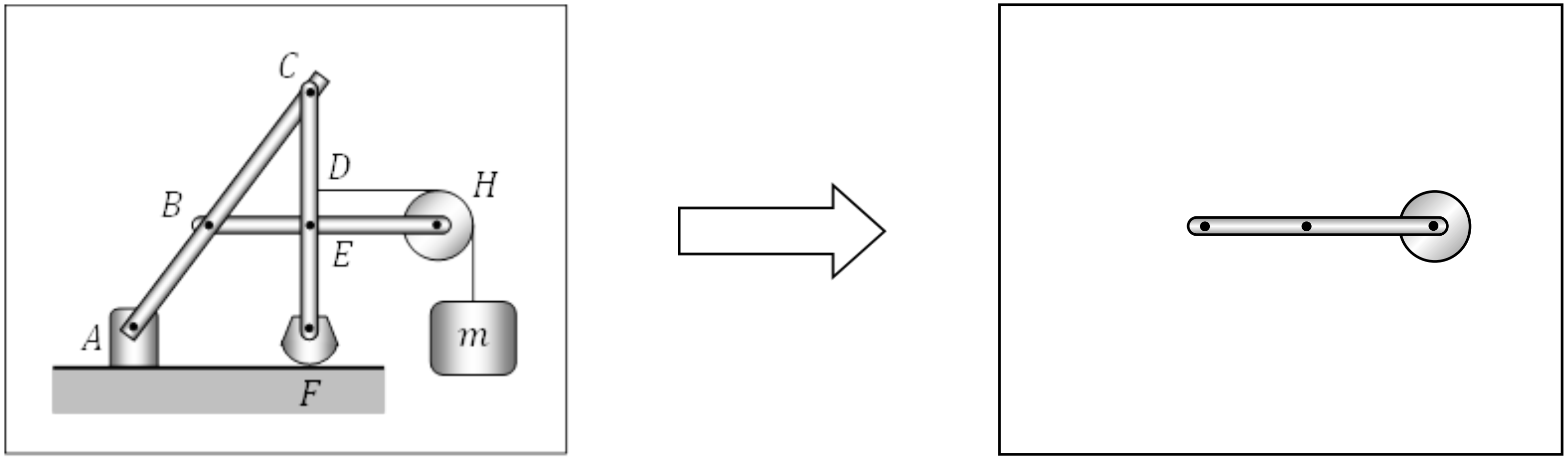


The complete FBD

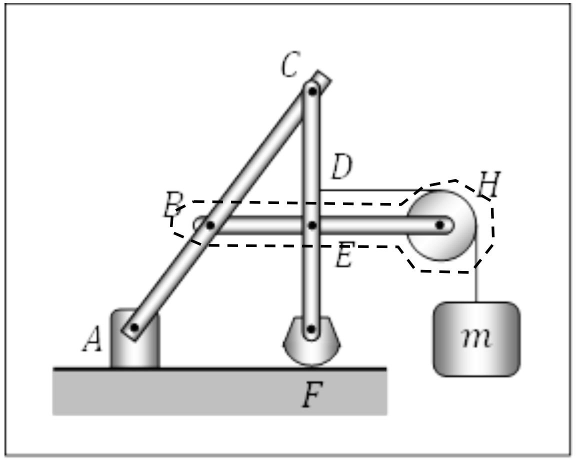


EXAMPLE 2

Complete the Free Body Diagrams. Mass of bodies is negligible unless stated by m kg. All contacting surfaces are smooth.



Draw the boundary of the required subset.

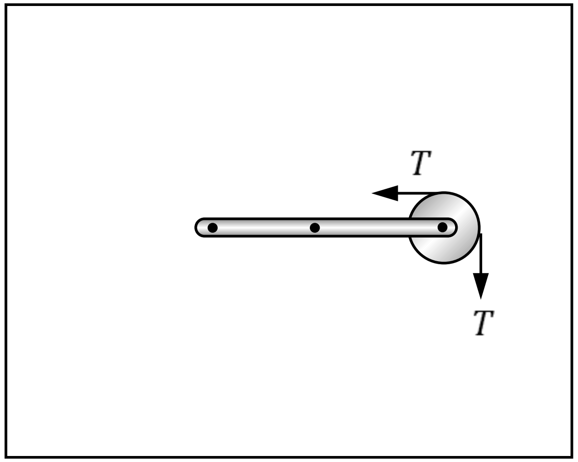


Check for

1. Mass within the subset: Not relevant.
2. External force:

   The cable, where the tension $T = mg$.

   Tension must be outwards of the body.

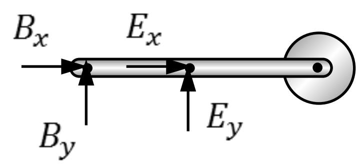


3. Reaction at pin B:

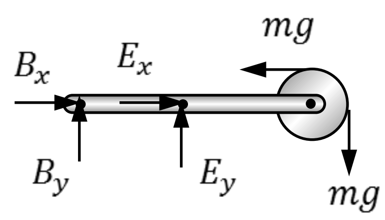
   Two unknown forces, B_x and B_y.

   Reaction at pin E:

   Two unknown forces, E_x and E_y.

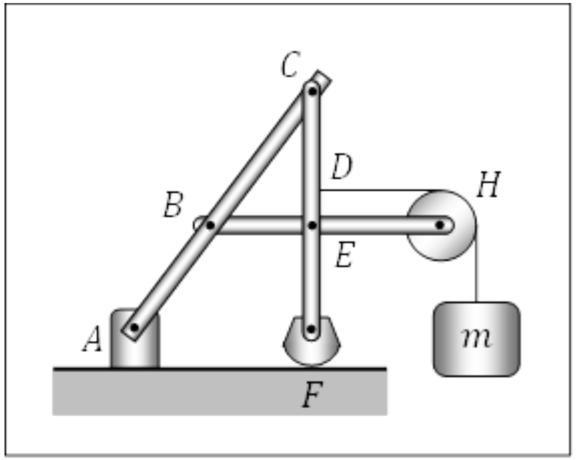


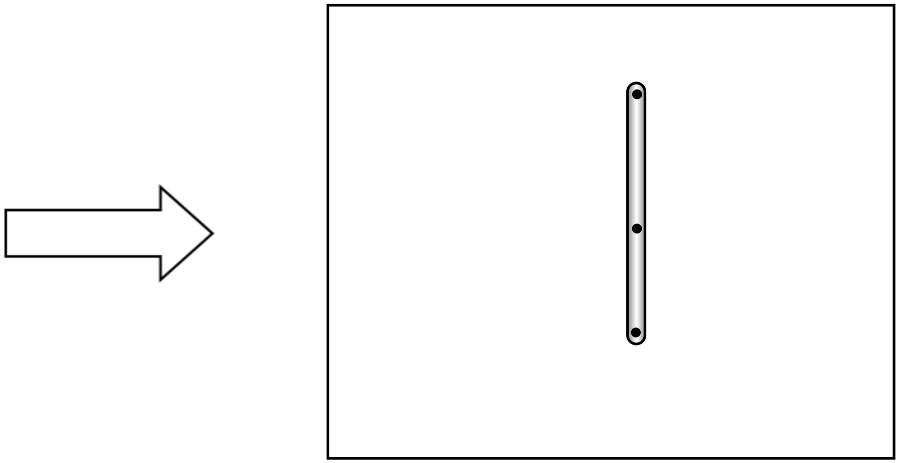
The complete FBD



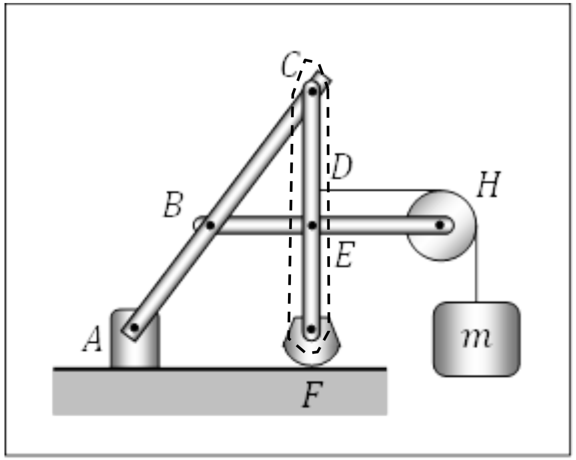
EXAMPLE 3

Complete the Free Body Diagrams. Mass of bodies is negligible unless stated by m kg. All contacting surfaces are smooth.





Draw the boundary of the required subset.

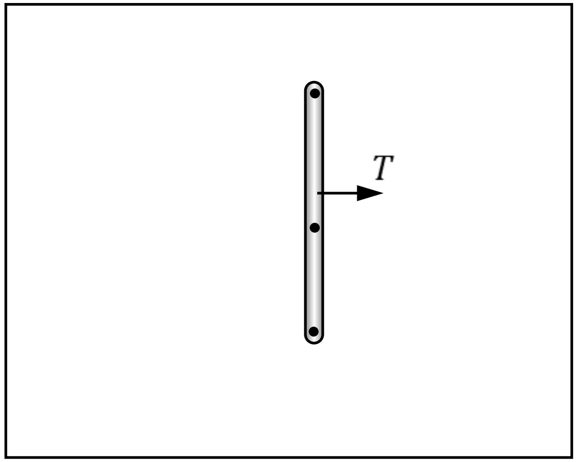


Check for

1. Mass within the subset: Not relevant.
2. External force:

   The cable, where the tension $T = mg$.

   Tension must act outwards of the body.



3. Reaction at pin B:

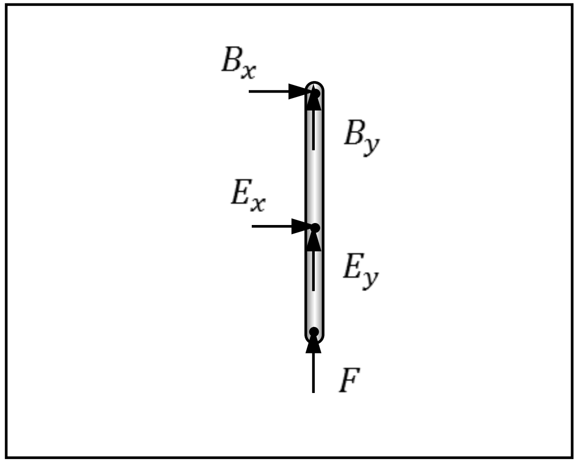
   Two unknown forces, B_x and B_y.

   Reaction at pin E:

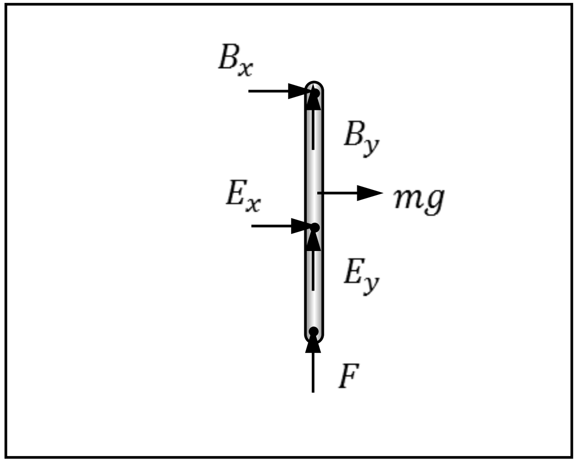
   Two unknown forces, E_x and E_y.

   Reaction at the rocker at F:

   A normal force perpendicular to the surface and towards the body.

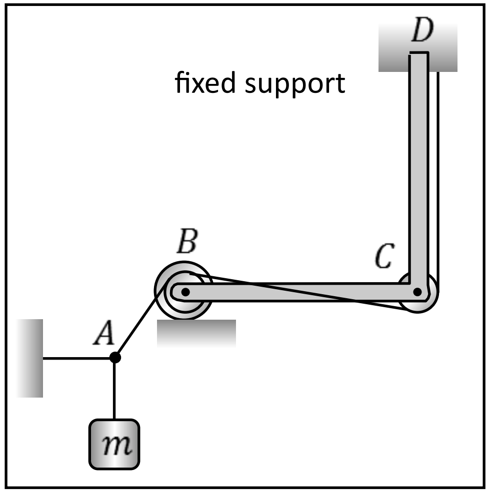


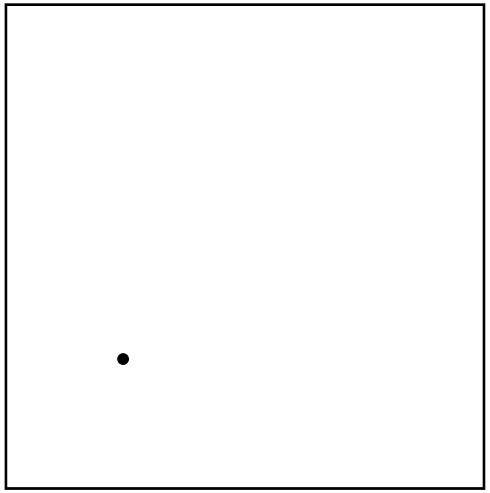
The complete FBD

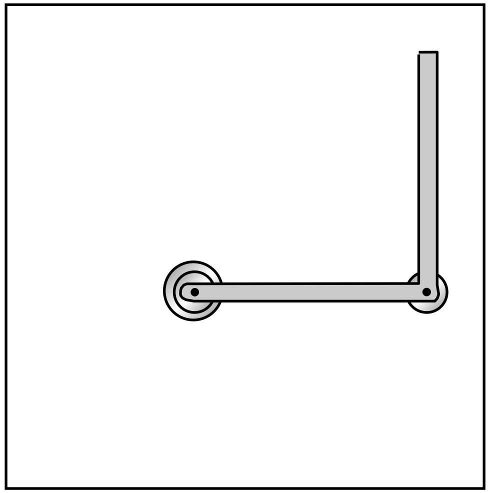


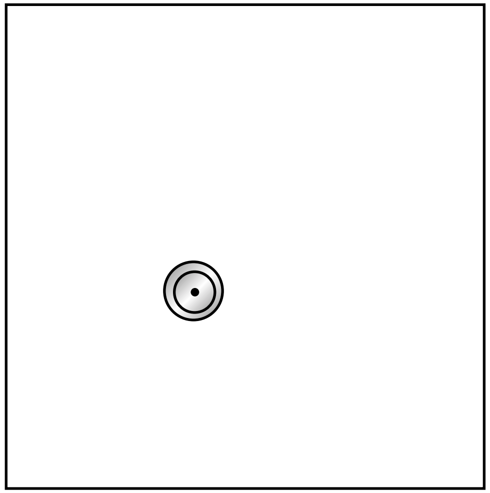
QUESTION 1

Complete the Free Body Diagrams. Mass of bodies is negligible unless stated by m kg. All contacting surfaces are smooth.



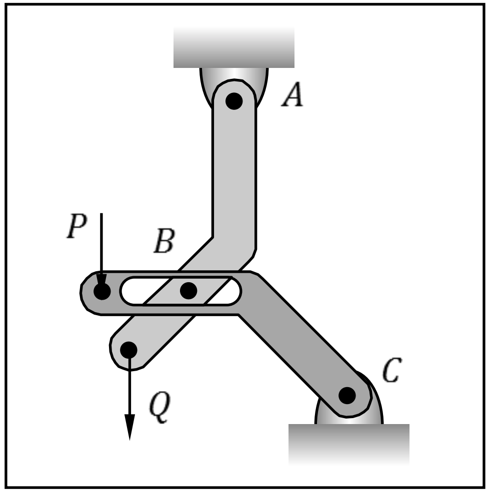


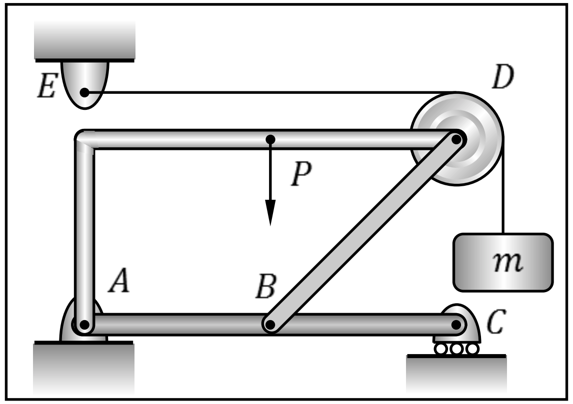


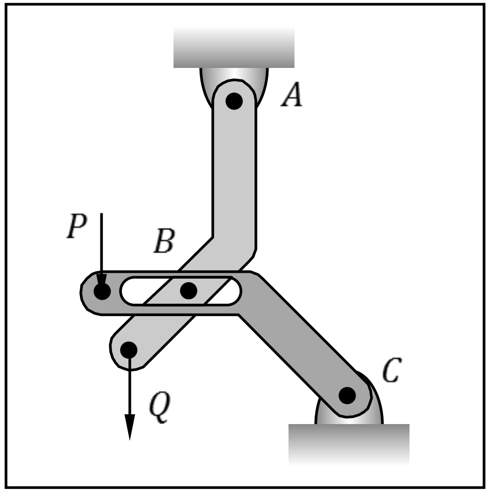


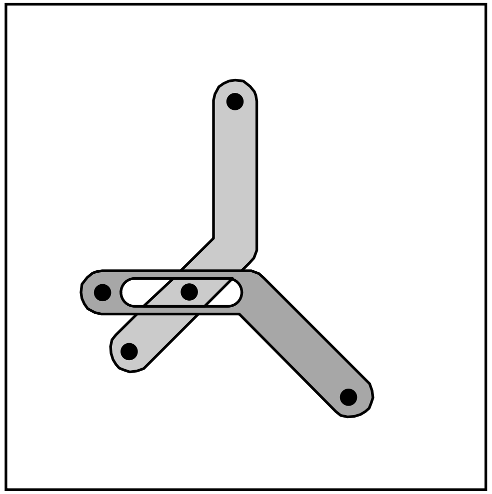
QUESTION 2

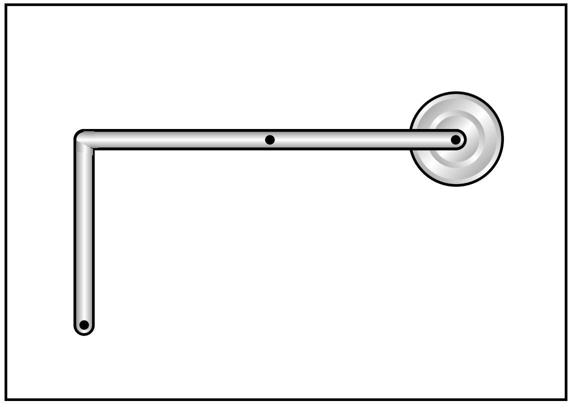
Complete the Free Body Diagrams. Mass of bodies is negligible unless stated by m kg. All contacting surfaces are smooth.

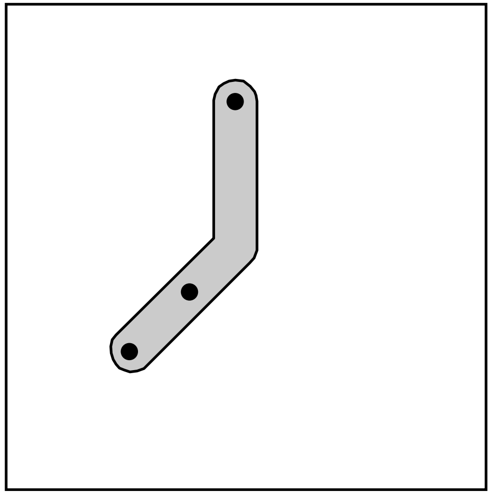






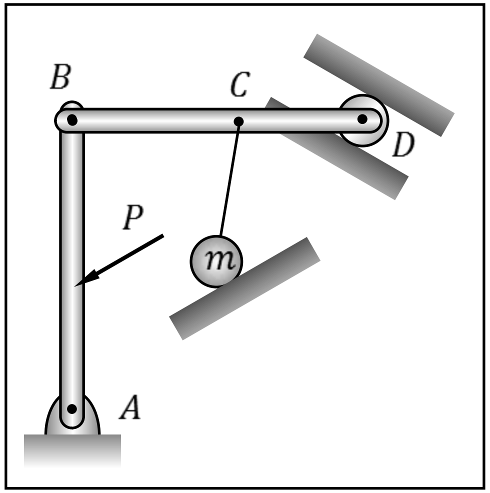


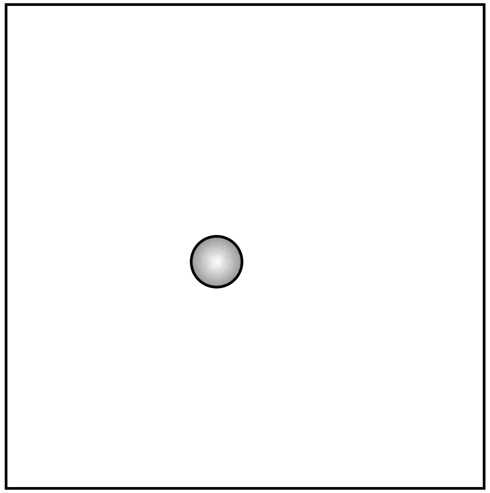


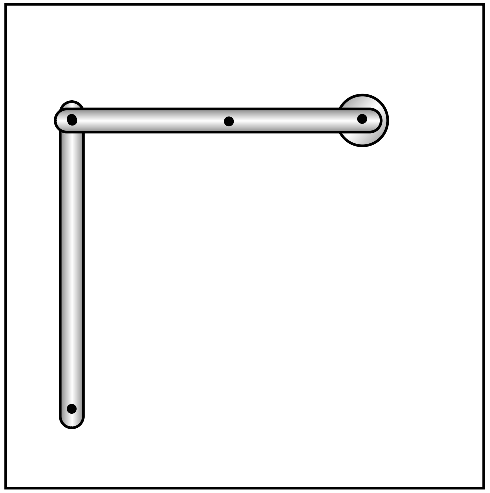


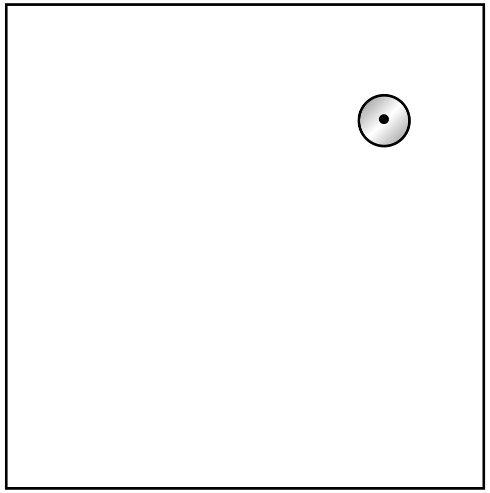
QUESTION 3

Complete the Free Body Diagrams. Mass of bodies is negligible unless stated by m kg. All contacting surfaces are smooth.



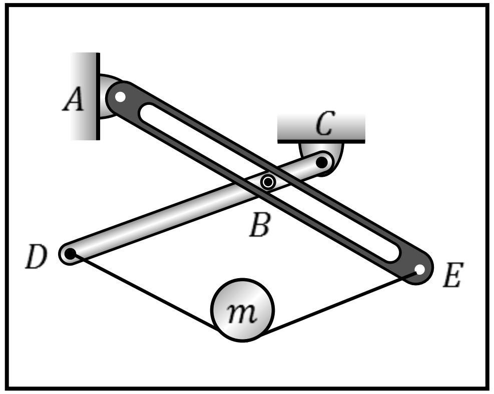


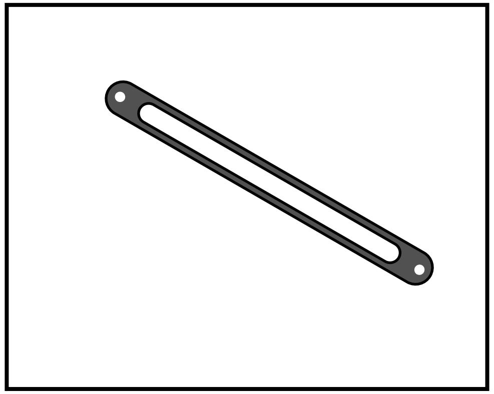


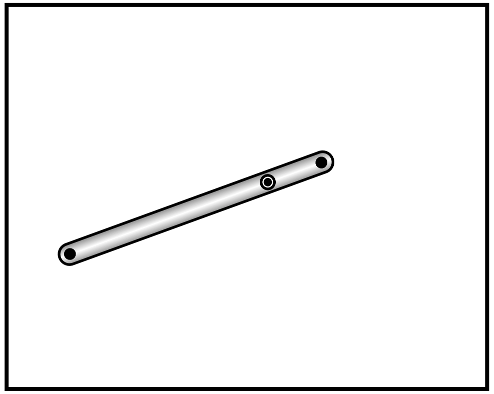


QUESTION 4

Complete the Free Body Diagrams. Mass of bodies is negligible unless stated by m kg. All contacting surfaces are smooth.

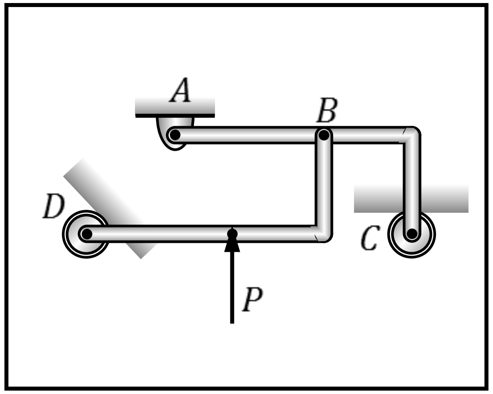


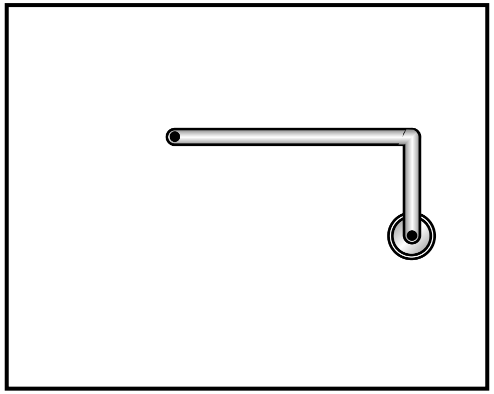


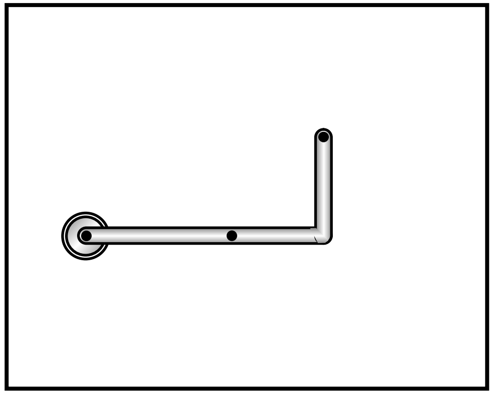


QUESTION 5

Complete the Free Body Diagrams. Mass of bodies is negligible unless stated by m kg. All contacting surfaces are smooth.

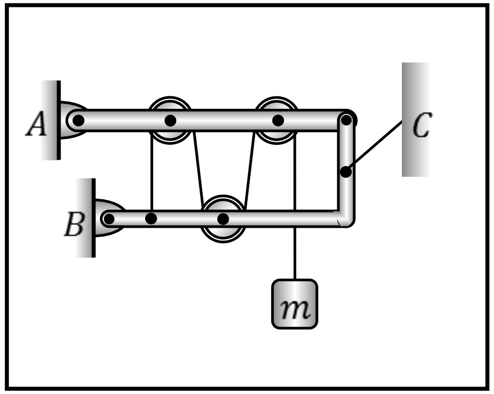


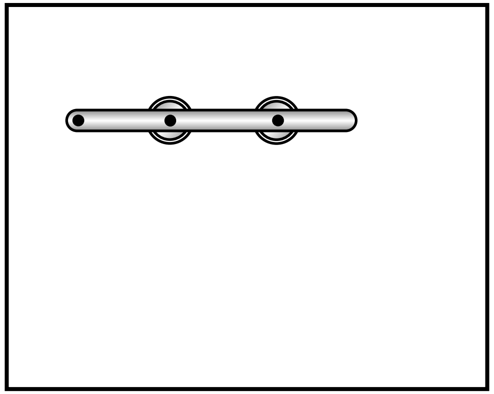


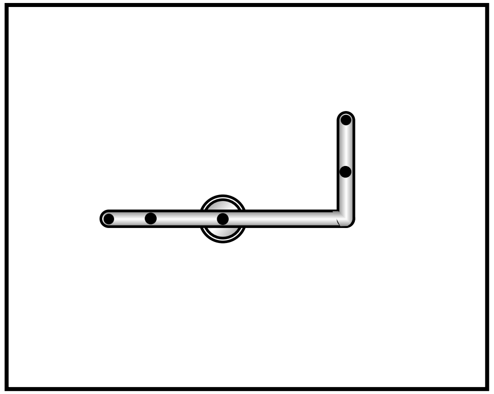


QUESTION 6

Complete the Free Body Diagrams. Mass of bodies is negligible unless stated by m kg. All contacting surfaces are smooth.

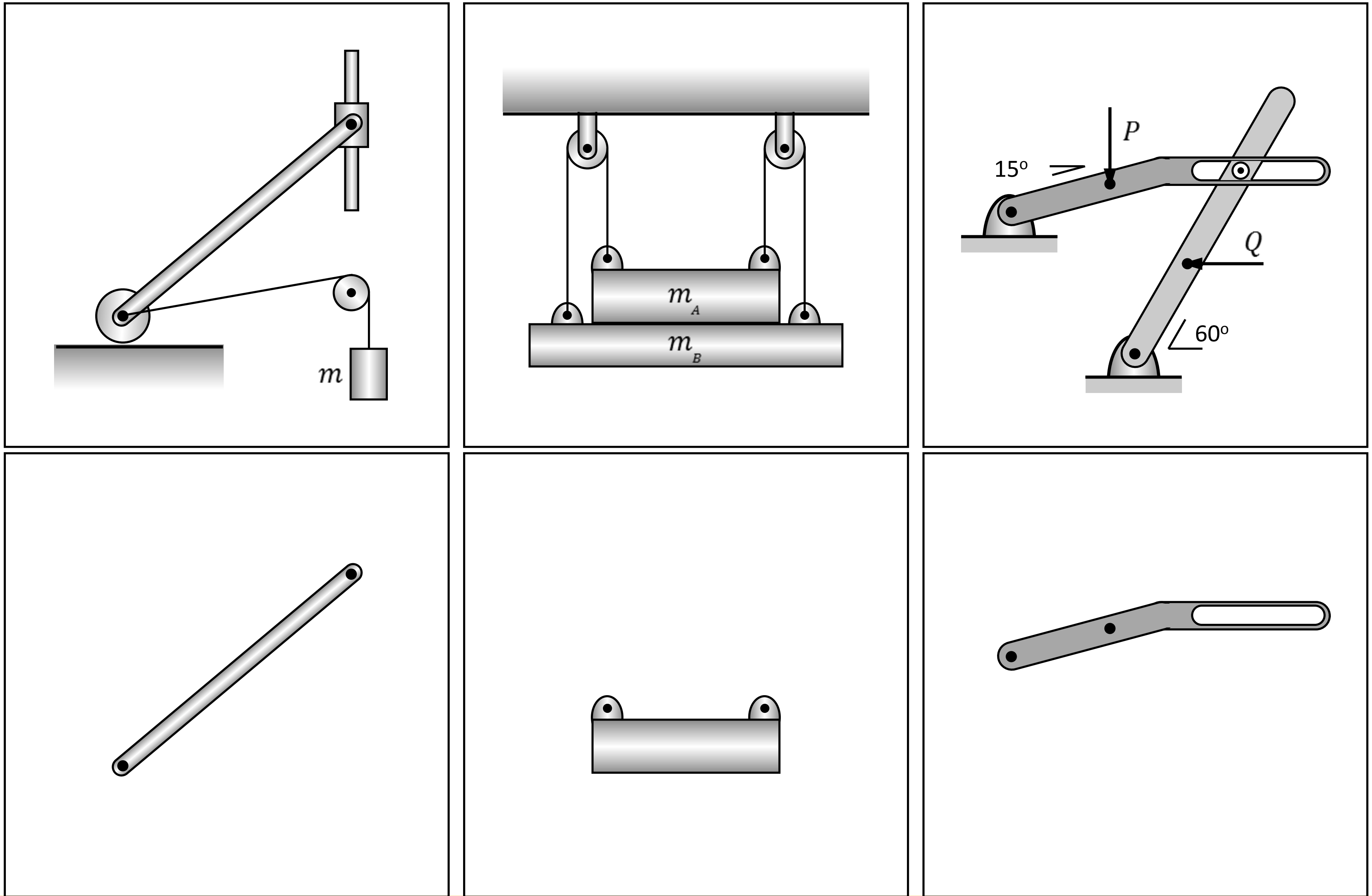






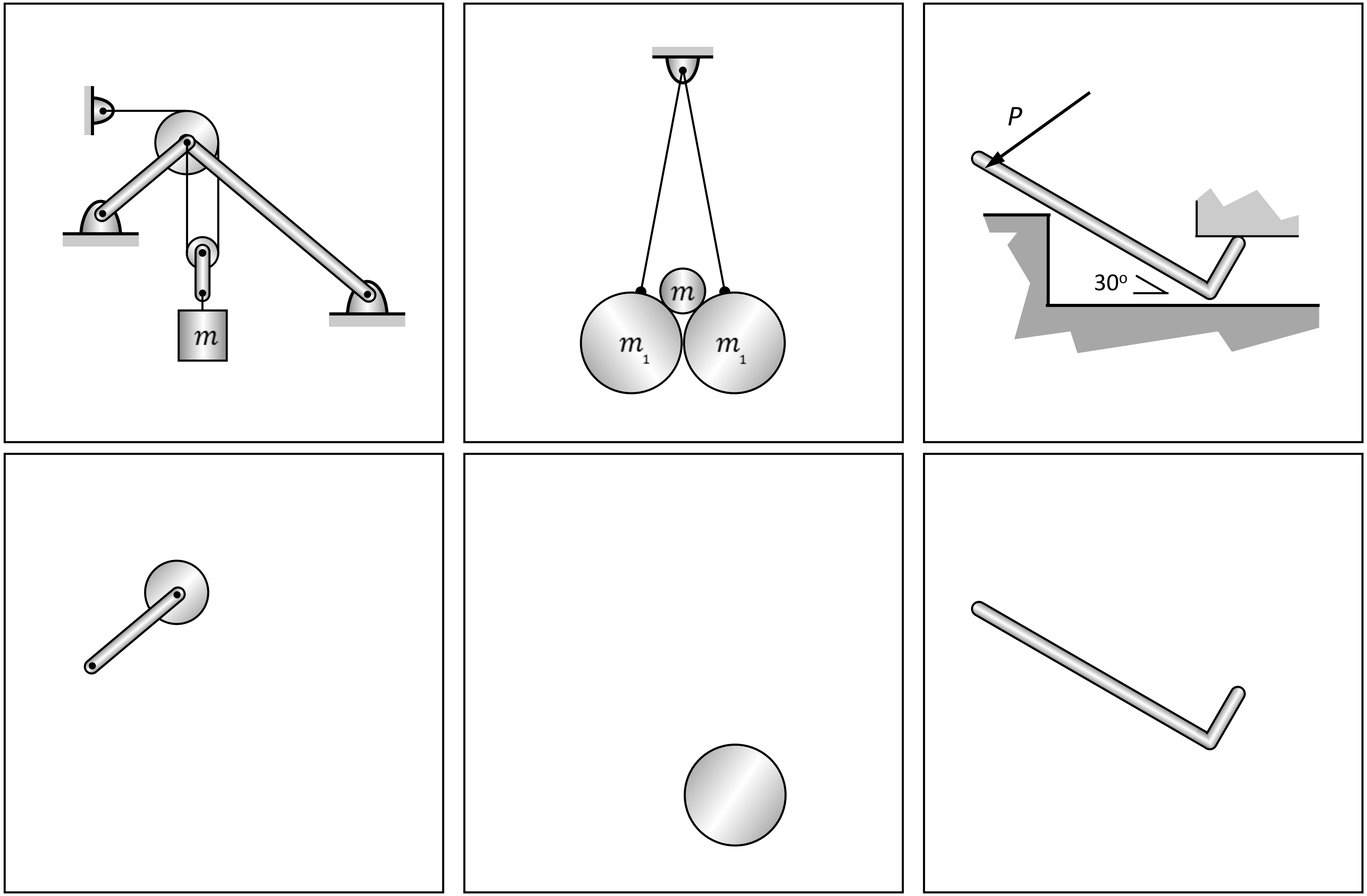
QUESTION 7

Complete the Free Body Diagrams. Mass of bodies is negligible unless stated by m kg. All contacting surfaces are smooth.



QUESTION 8

Complete the Free Body Diagrams. Mass of bodies is negligible unless stated by m kg. All contacting surfaces are smooth.



EQUILIBRIUM OF RIGID BODIES

TWO DIMENSION THE CALCULATIONS

Equilibrium of a particle

$$\sum F_x = 0 \quad \sum F_y = 0$$

Equilibrium of a rigid body

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_A = 0 \text{ (where } A \text{ is any point)}$$

Important

The FBD *must* be sketched for EVERY solution.

Moment can be taken from ANY point, even from points outside the body.

The three equations used may not necessarily be

$$\sum F_x = 0,\ \sum F_y = 0 \text{ and } \sum M_A = 0$$

Moments can be taken twice

$$\sum F_x = 0,\ \sum M_A = 0 \text{ and } \sum M_B = 0$$

or $$\sum F_y = 0\ ,\ \sum M_A = 0 \text{and } \sum M_B = 0$$

or even thrice

$$\sum M_A = 0,\ \sum M_B = 0 \text{ and } \sum M_C = 0$$

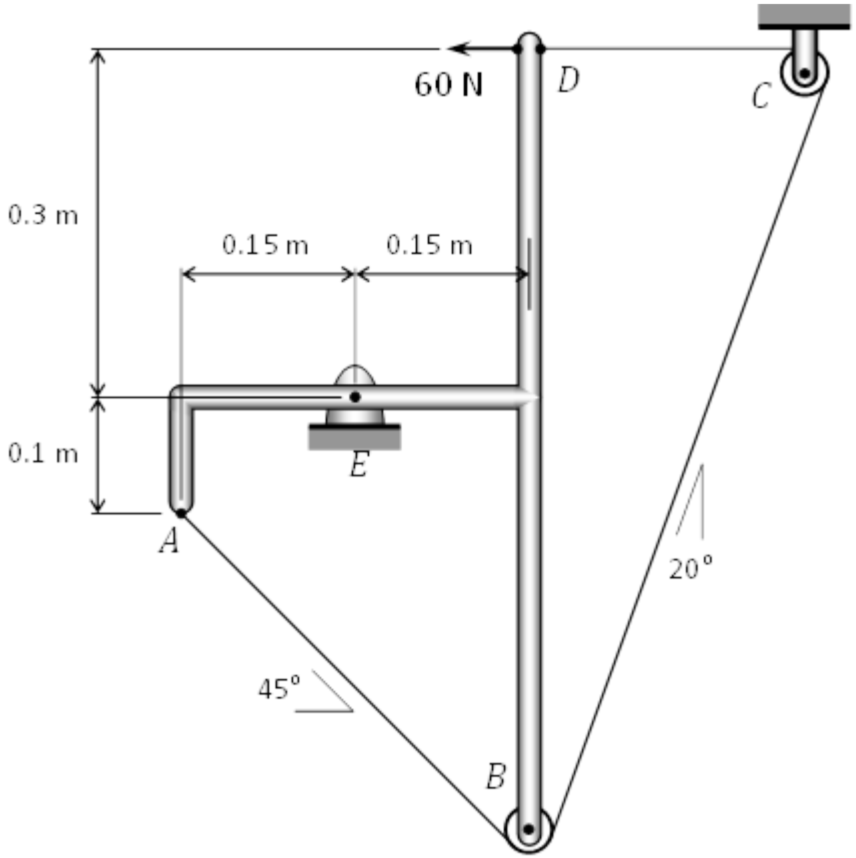
Strategy

The most common approach is to find moment about the point where there are two unknowns, thus directly solving for the other unknown.

The better strategy (where applicable) is to find an equation that can directly solve for one of the unknowns. The other equations can then be used to check earlier calculations.

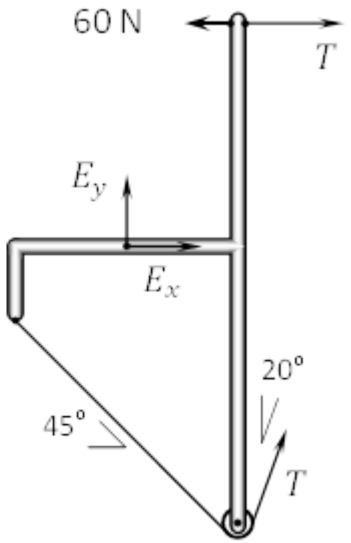
EXAMPLE 1

Determine tension in the cable and the reaction at E. Pulley diameter and mass of the rigid body can be neglected.



SOLUTION

Draw the FBD



<u>Observations</u>

Directions for the pin reaction at B are arbitrarily assumed.

The outwards tension at A cancelled out the outwards tension at B.

There are 3 unknowns, namely T, E_x and E_y.

Using the three basic equations to solve the problem gives,

(+↺) $\sum M_E = 0$

60(0.3) – T (0.3) + T cos 20°(0.15) + T sin 20°(0.4) = 0

18 – 0.3 T + 0.141 T + 0.137 T = 0

0.022 T = 18

T = 818.2 N

(+→) $\sum F_x = 0$

E_x – 60 + T + T sin 20° = 0

E_x = 60 – 818.2 – 818.2 sin 20° = – 1038 N

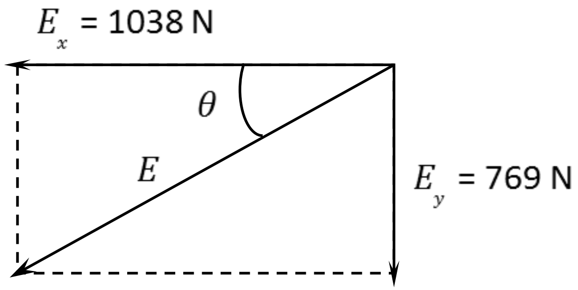
∴ E_x = 1038 N ←

(+↑) $\sum F_y = 0$

E_y + T cos 20° = 0

E_y = – 818.2 cos 20°= – 769 N

∴ E_y = 769 N ↓



$$E = \sqrt{1038^2 + 769^2} = 1292 \text{ N}$$

$$\theta = \tan^{-1}\frac{769}{1038} = 36.5^o$$

To check

$(+\circlearrowleft)\ \sum M_B = 0$

60 (0.7) – T (0.7) – E_x (0.4) – E_y (0.15) = 0

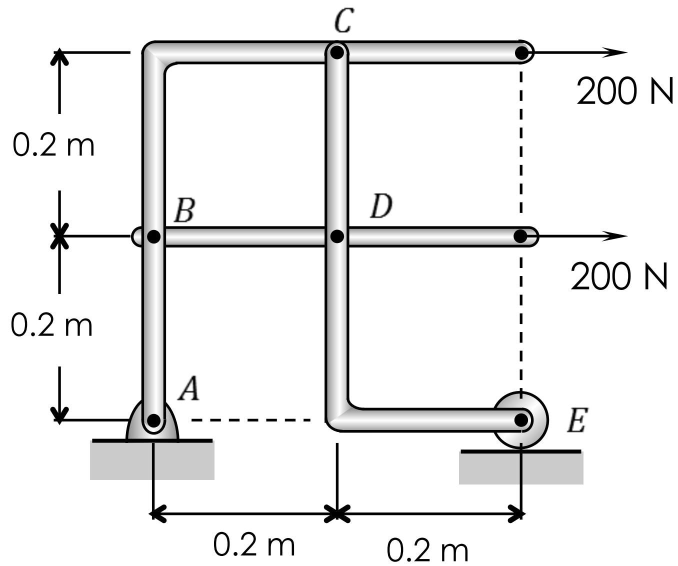
60 (0.7) – T (0.7) – E_x (0.4) – E_y (0.15) = 0

42 – 818.2 (0.7) – 1038 (0.4) – 769 (0.15) = 0

42 – 572.6 + 415.2 + 115.4 = 0 checked

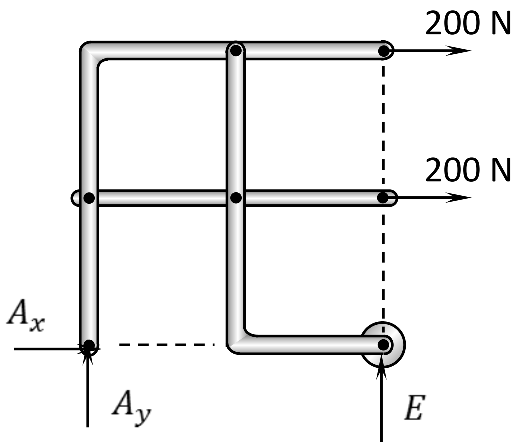
EXAMPLE 2

Determine the reaction at A and E.
All contacting surfaces are smooth.



SOLUTION

Draw the FBD



Observations

Directions for the pin reaction at A are arbitrarily assumed.

There are 3 unknowns: E, A_x and A_y.

There are 2 points where two unknowns meet,

A, where A_x and A_y meet, and

E, where E and A_x meet.

Solving for E

$(+\circlearrowleft)\ \sum M_A = 0$

$E\ (0.4) - 200\ (0.4) - 200\ (0.2) = 0$

$E = 300$ N $\uparrow$

Solving for A_y

$(+\circlearrowleft)\ \sum M_E = 0$

$-A_y\ (0.4) - 200\ (0.4) - 200\ (0.2) = 0$

$A_y = -300$ N

$\therefore \quad A_y = 300$ N $\downarrow$

Solving for A_x

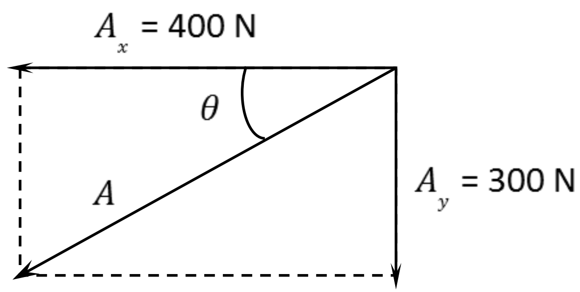
$(+\rightarrow)\ \sum F_x = 0$

$A_x + 200 + 200 = 0$

$A_x = -400$ N

$\therefore \quad A_x = 400$ N $\leftarrow$

The reaction at A



$$A = \sqrt{400^2 + 300^2} = 500\ \text{N}$$

$$\theta = \tan^{-1}\frac{300}{400} = 36.87^o$$

To check

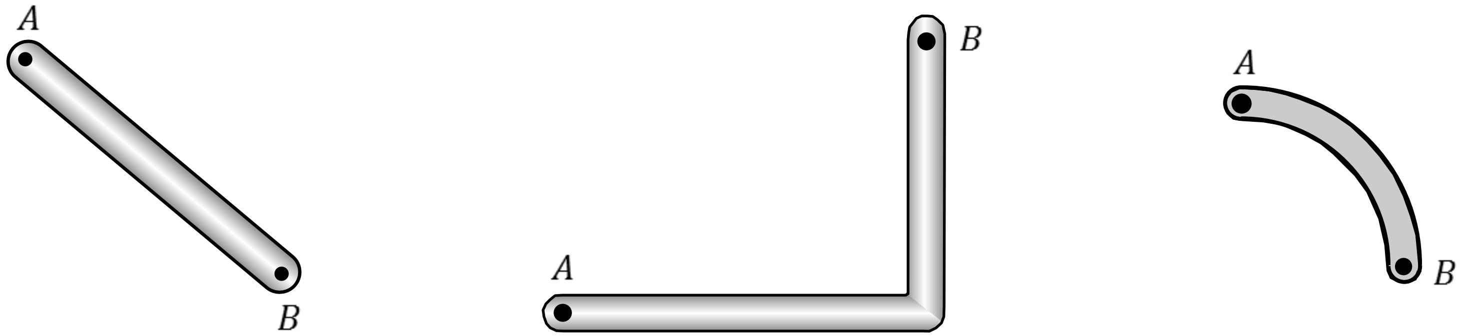
$(+\uparrow)\ \sum F_y = 0$

$A_y + E = 0$

$-300 + 300 = 0$ checked

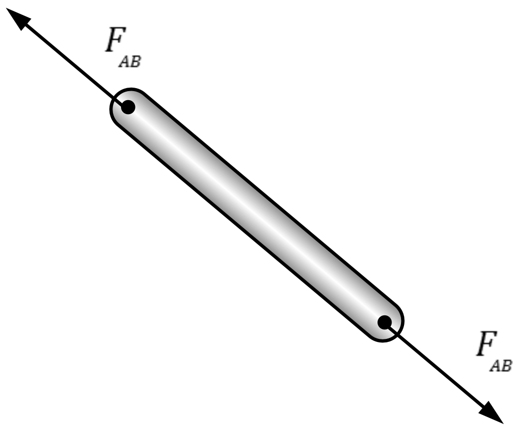
TWO FORCE MEMBER

Definition: A rigid body of negligible mass with only two forces acting on it.

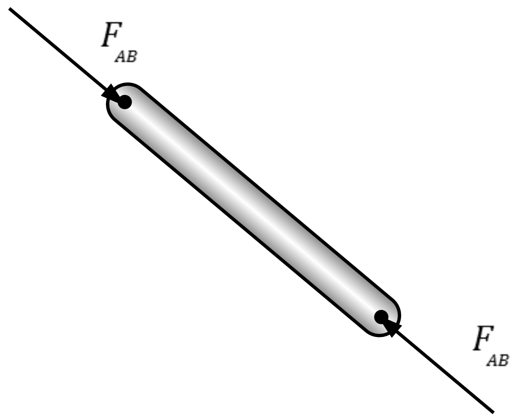


For a two force member, the two forces must have the same magnitude, the same line of action, and opposite sense. The two possibilities are

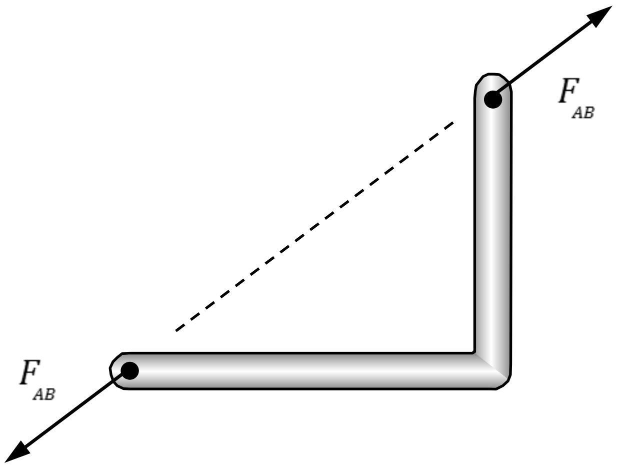
<u>Body in tension</u>

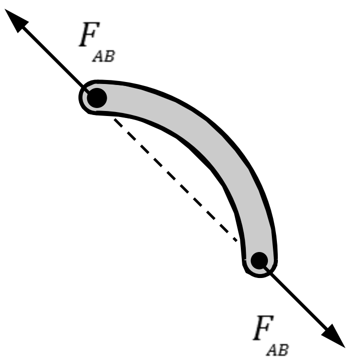


<u>Body in compression</u>

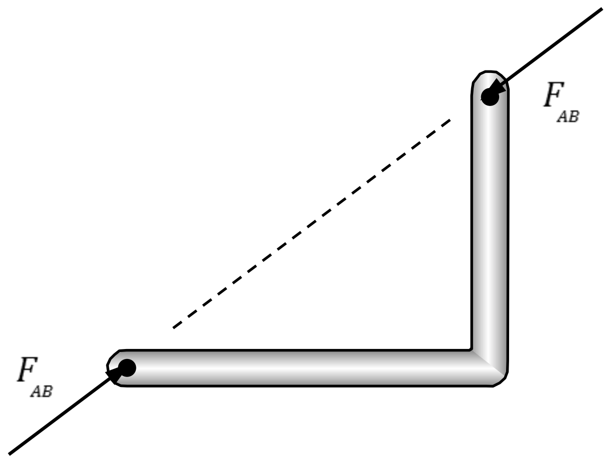


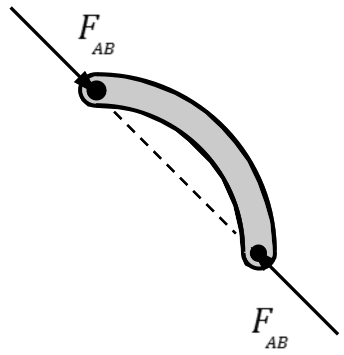
Body in tension





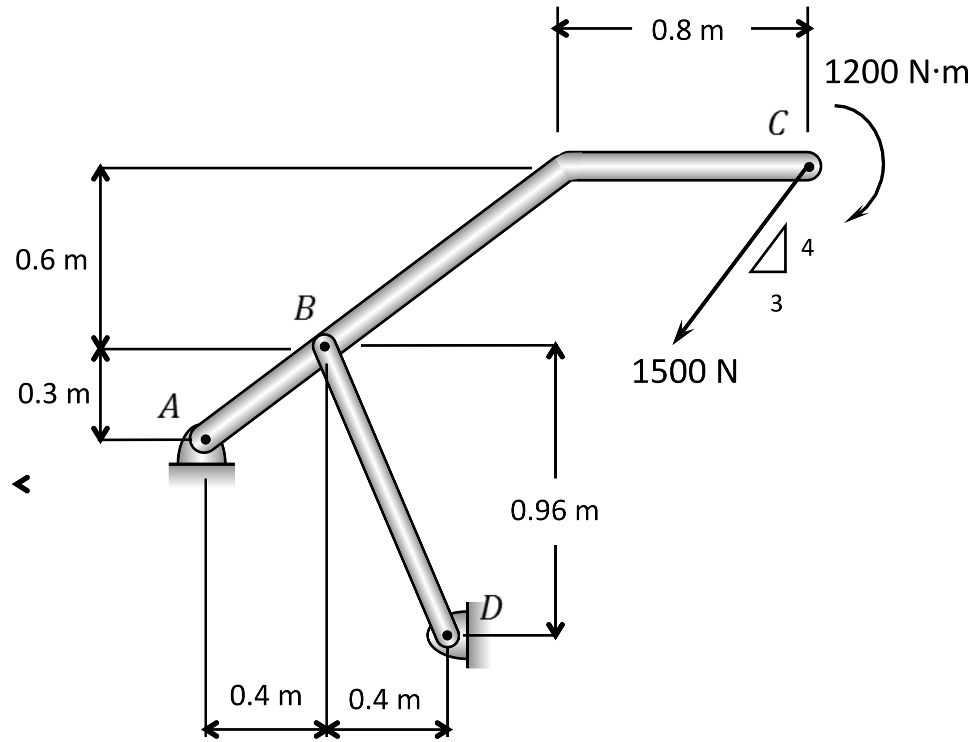
Body in compression





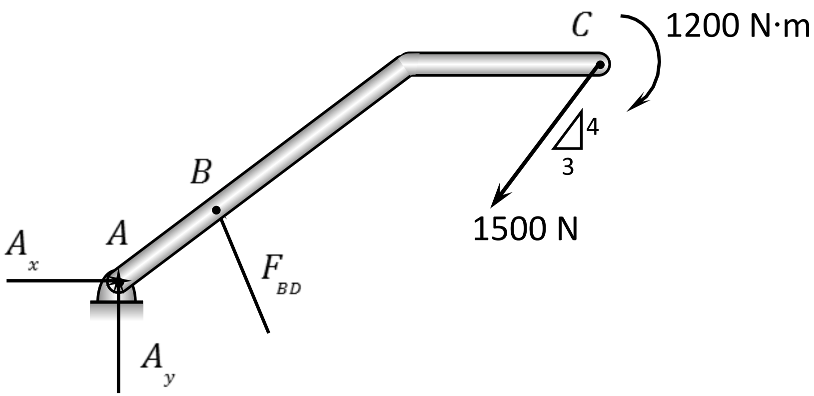
EXAMPLE 2

Determine the reaction at A and D for the system to maintain equilibrium.



SOLUTION

FBD



$(+\circlearrowright)\ \sum M_A = 0$

$1200 + 1500\,(\frac{4}{5})\,(2) - 1500\,(\frac{3}{5})\,(0.9) - F_{BD}\,(\frac{12}{13})(0.4) - F_{BD}\,(\frac{5}{13})(0.3) = 0$

$1200 + 2400 - 810 - (\frac{4.8}{13})\,F_{BD} - F_{BD}\,(\frac{1.5}{13}) = 0$

$2790 - (\frac{6.3}{13})\,F_{BD} = 0$

$F_{BD} = 5757\text{ N}$ ↖ 22.62°

$(+\rightarrow)\ \sum F_x = 0$

$A_x - 1500\,(\frac{3}{5}) - F_{BD}\,(\frac{5}{13}) = 0$

$A_x - 900 - 2214 = 0$

$A_x = 3114\text{ N} \rightarrow$

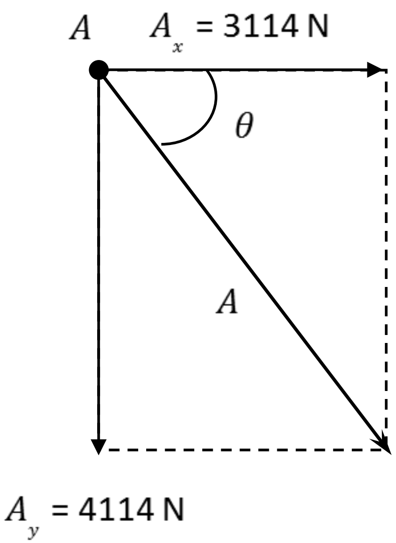
$(+\uparrow)\ \sum F_y = 0$

$A_y - 1500(\frac{4}{5}) + F_{BD}\,(\frac{12}{13}) = 0$

$A_y - 1200 + 5314 = 0$

$A_y = -4114\text{ N}$

$\therefore\quad A_y = 4114\text{ N} \downarrow$



$A = \sqrt{3114^2 + 4114^2} = 5160\text{ N}$

$\theta = \tan^{-1}\frac{4114}{3114} = 52.9^\circ$

Check

$(+\circlearrowright)\ \sum M_B = 0$

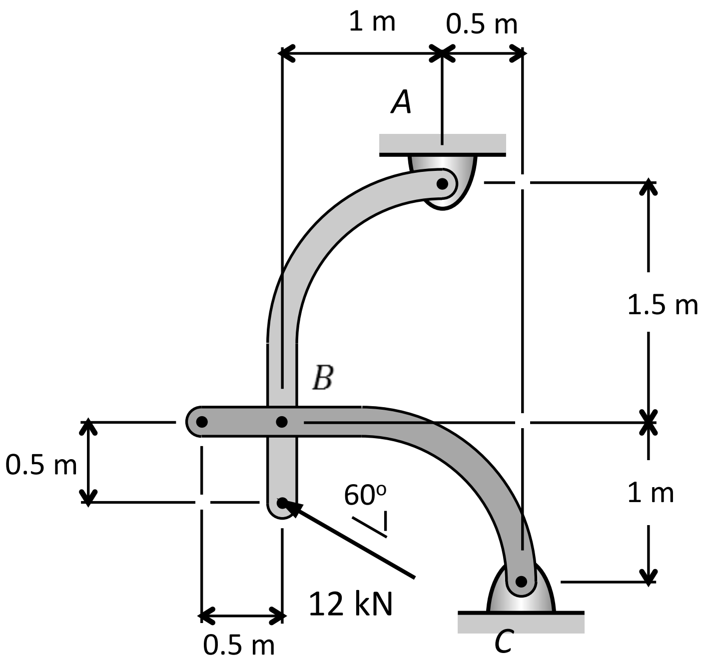
$1200 + 1500(\frac{4}{5})(1.6) - 1500(\frac{3}{5})(0.6) + A_y\,(0.4) - A_x\,(0.3) = 0$

$1200 + 1920 - 540 + (-4114)(0.4) - 3114(0.3) = 0$

$2580 - 1646 - 934 = 0$ checked

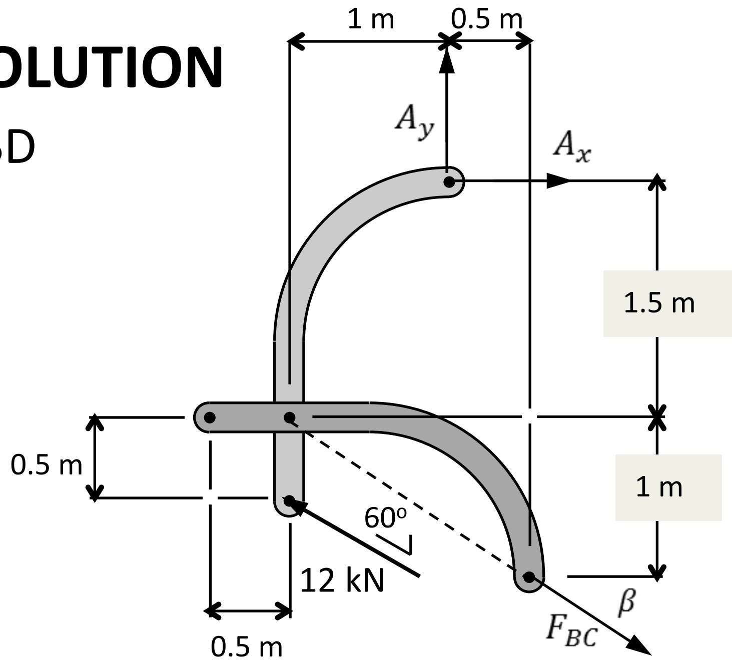
EXAMPLE 2

Determine the reaction at A and C.



SOLUTION

FBD



β

$$\beta = \tan^{-1}\frac{1}{1.5} = 33.7^o$$

$(+\circlearrowright)\ \sum M_A = 0$

$12\cos 60^o\ (1) +$

$12\sin 60^o\ (2) + F_{BC}\sin 33.7^o(0.5) - F_{BC}\cos 33.7^o\ (2.5) = 0$

$6 + 20.8 + 0.277F_{BC} - 2.08F_{BC} = 0$

$F_{BC} = 14.86$ kN ↘ 33.7^o

$(+\rightarrow) \sum F_x = 0$

$A_x - 12\sin 60^o + 14.86\cos 33.7^o = 0$

$A_x - 10.39 + 12.36 = 0$

$A_x = -1.97$ kN

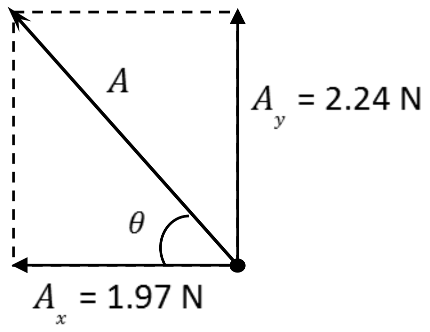
$\therefore \quad A_x = 1.97$ kN $\leftarrow$

$(+\uparrow) \sum F_y = 0$

$A_y + 12\cos 60^o - 14.86\sin 33.7^o = 0$

$A_y + 6 - 8.24 = 0$

$\therefore \quad A_y = 2.24$ kN $\uparrow$



$$A = \sqrt{1.97^2 + 2.24^2}$$

A = 2.98 kN

$$\theta = \tan^{-1}\frac{2.24}{1.97} = 48.7^o$$

A = 2.98 kN 48.7^o

check

(+↻) $\sum M_c = 0$

$$12\cos 60^o\,(1.5) - 12\sin 60^o\,(0.5) + A_x\,(2.5) + A_y(0.5) = 0$$

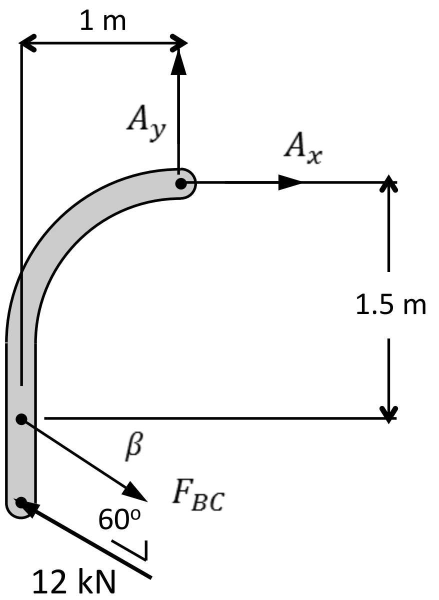
9 – 5.2 + (– 1.97)(2.5) + (2.24)(0.5) = 0

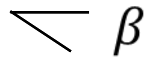
9 – 5.2 – 4.92 + 1.12 = 0 checked

The problem can also be solved by drawing the FBD for member AB as illustrated below.

SOLUTION

FBD





$$\beta = \tan^{-1}\frac{1}{1.5} = 33.7^o$$

$(+↻)\ \sum M_A = 0$

$12\cos 60^o\,(1) +$

$12\sin 60^o\,(2) - F_{BC}\sin 33.7^o(1) - F_{BC}\cos 33.7^o\,(1.5) = 0$

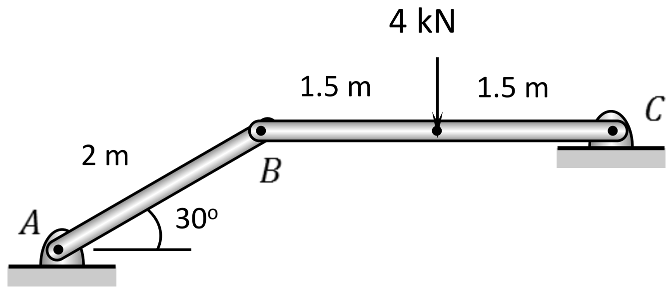
$6 + 20.8 - 0.555F_{BC} - 1.248F_{BC} = 0$

$F_{BC} = 14.86$ kN ↘ 33.7^o

Remaining parts of the solution are similar.

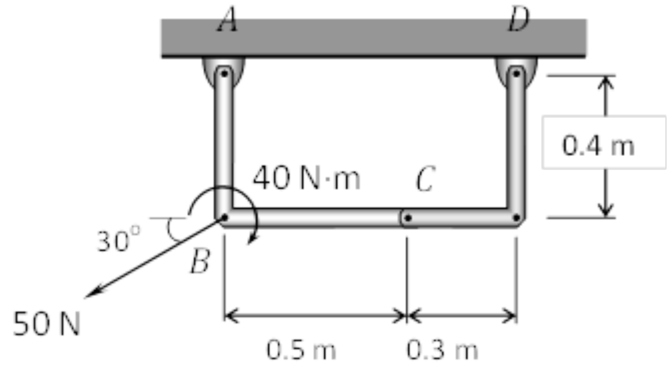
QUESTION 1

Determine the reaction at pins A and C to support the 4000 N load. Neglect the weight of rods AB and BC.



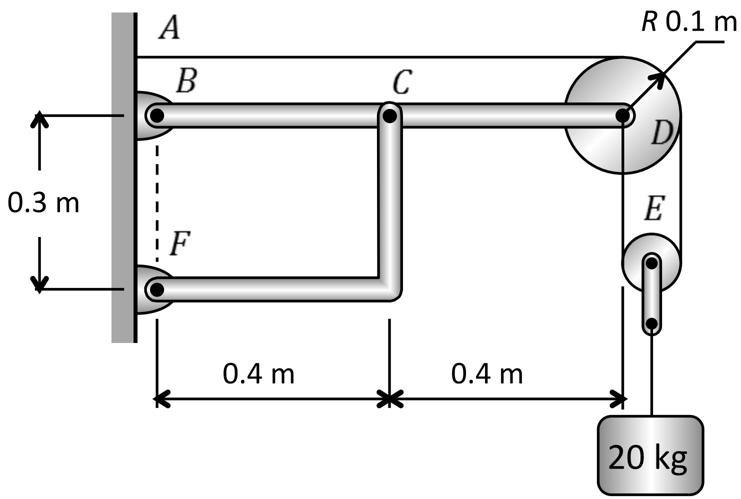
QUESTION 2

Determine the reaction at pins A and D.



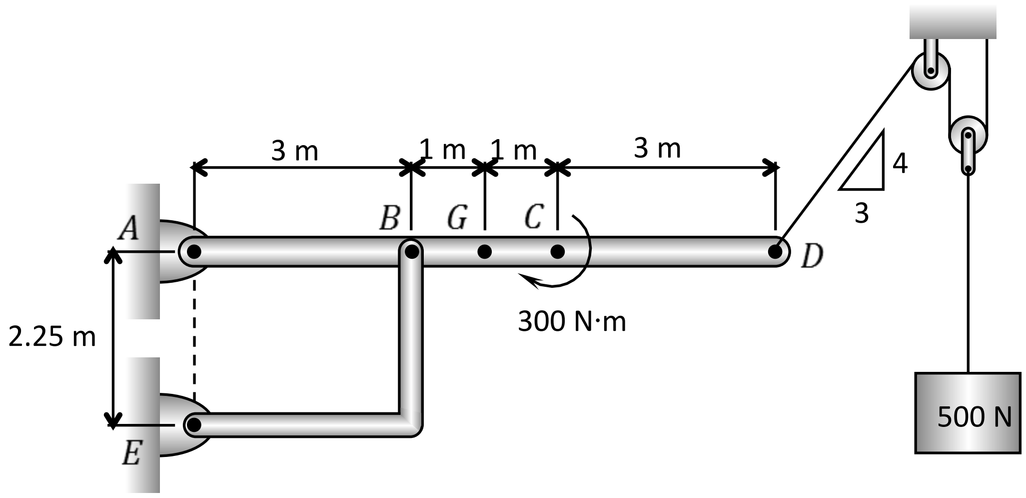
QUESTION 3

The structure shown is used to support the 20 kg mass. Determine the reaction at pins B and F.



QUESTION 4

Rigid body $ABCD$ of weight 100 N (acting at G) is used to support the 500 N load and the 300 N·m couple. Determine the reactions at A and E.



QUESTION 5

Determine reaction at B and C, and the angle β to maintain ABC in the vertical position.