Statics SKMM1203

Concurrent forces: Equilibrium (2D & 3D)

Faculty of Mechanical Engineering

Introduction

A particle can be subjected only to a system of concurrent forces, and the necessary and sufficient conditions for equilibrium can be expressed mathematically as:

$$R = \sum F = 0$$

where is the vector sum of all forces acting on the particle

Objectives:

- To determine required forces for equilibrium condition of concurrent force system
- To draw a free-body diagram for concurrent force system

Introduction

In particular, a particle is in equilibrium when the resultant of all forces acting on it equals zero. A particle is subjected to two forces as shown in Figure 3.1. It is in equilibrium condition if the two forces have same magnitude with opposite direction and act on the same line of action. If a particle is subjected to multiple loadings, equilibrium condition is achieved when the resultant of all the forces equals zero as demonstrated in Figure 3.2.

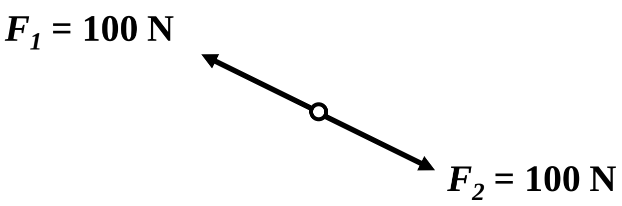


Figure 3.1

Introduction

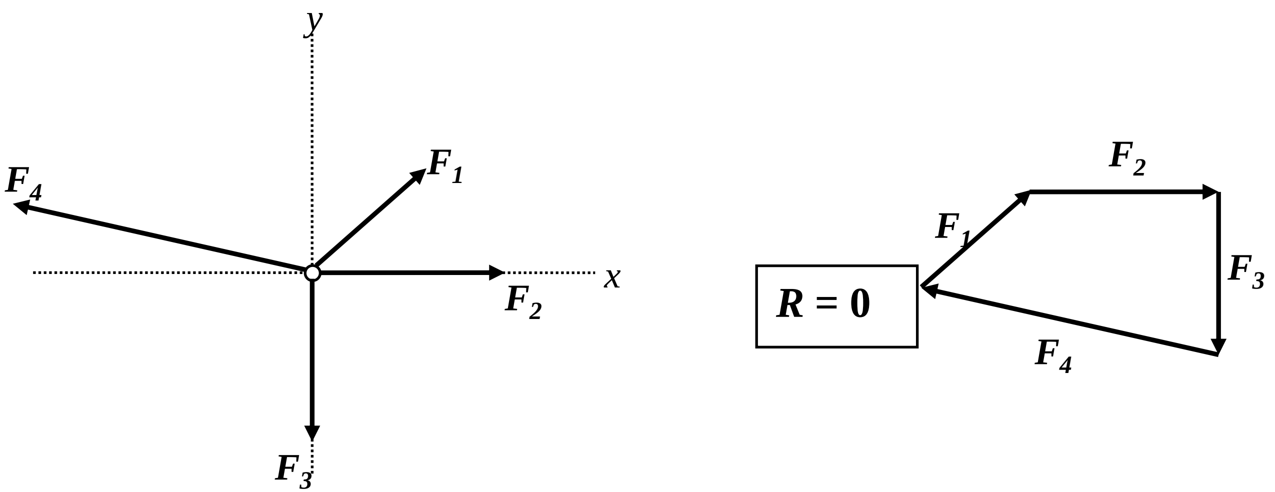


Figure 3.2

Two dimensional concurrent force system

For a system of coplanar (e.g *x-y* plane), concurrent forces can be written as

$$R = R_x i + R_y j$$
$$= \sum F_x i + \sum F_y j = 0$$

In scalar form, the equations become

$$R_x = \sum F_x = 0$$
$$R_y = \sum F_y = 0$$

That is, the sum of the rectangular components of the forces in any direction must be zero.

Three dimensional concurrent force system

For a three-dimensional system of concurrent forces, the equilibrium condition can be expressed as

$$R = \sum F = R_x i + R_y j + R_z k = 0$$

$$\sum F_x i + \sum F_y j + \sum F_z k = 0$$

This equation is satisfied only if

$$R_x = \sum F_x i = 0$$

$$R_y = \sum F_y j = 0$$

$$R_z = \sum F_z k = 0$$

In scalar form, these equations become

$$R_x = \sum F_x = 0$$

$$R_y = \sum F_y = 0$$

$$R_z = \sum F_z = 0$$

Concurrent force system

Solving an engineering mechanics problem usually requires identification of all external forces acting on a “body of interest”. A carefully prepared drawing or sketch that shows a “body of interest” separated from all interacting bodies is known as a *free-body diagram* (FBD). It is important that all forces acting on the body of interest be shown. The actual procedure for drawing a *free-body diagram* consists of three essential steps:-

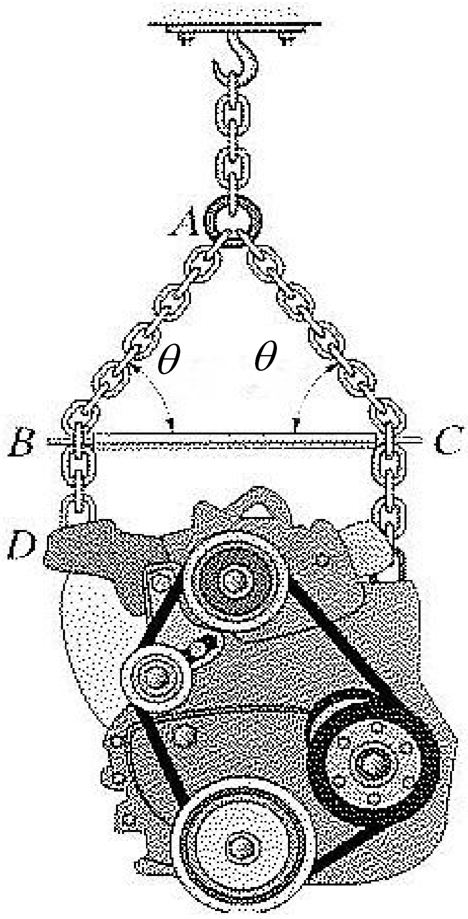
✓ Decide which body or combination of bodies is to be isolated and analyzed.

✓ Prepare a drawing or sketch of the outline of the isolated body selected.

✓ Represent all forces, known and unknown, that are applied by other bodies to the isolated body with vectors in their correct positions.

Example:

Q1. The weight of the engine is 4.4 kN and is suspended from a vertical chain at *A*. A second chain round the engine is attached at *A*, with a spreader bar between *B* and *C*. The angles, θ at B and *C* are 50º. Determine the tension in the chain *AB* and determine the compressive force acting in the spreader bar *BC*.



Solution:

$(+\uparrow)\Sigma F_y = 0$

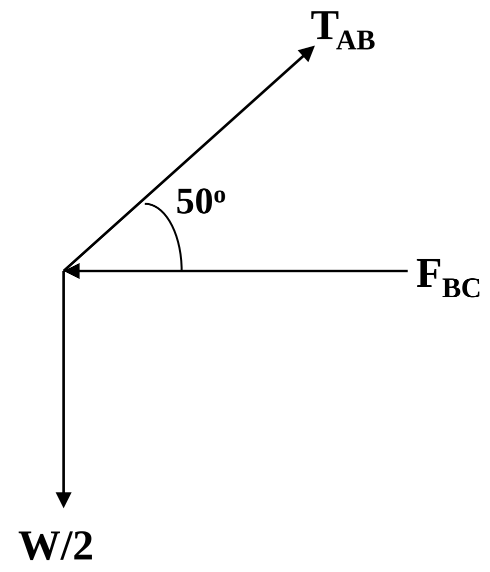
$T_{AB} \sin 50^o - 4.4/2 = 0$

$T_{AB} = 2.87$ kN

$(+\rightarrow)\Sigma F_x = 0$

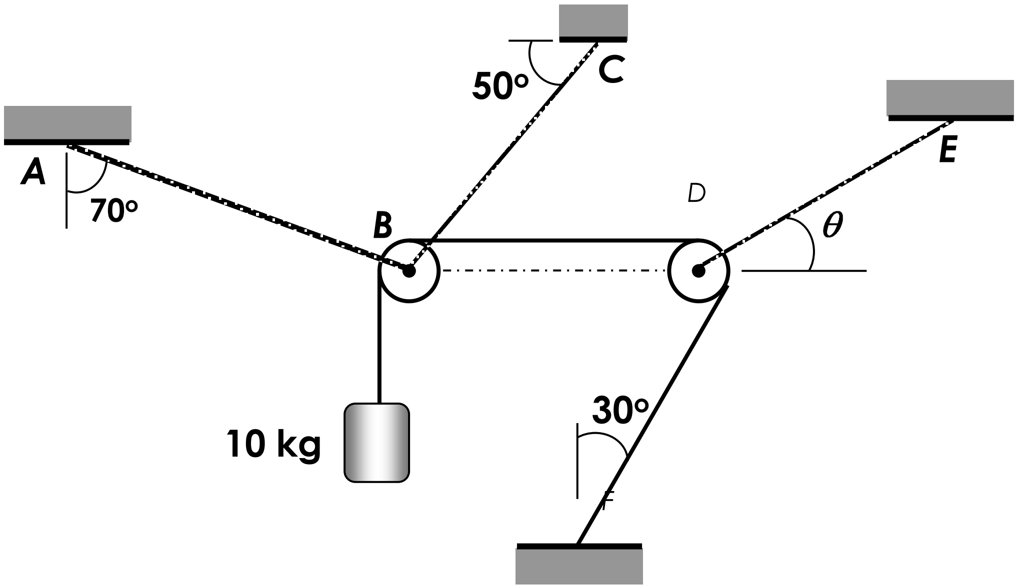
$2.87\cos 50^o - F_{BC} = 0$

$F_{BC} = 1.84$ kN



Example:

Q2. The 10 kg mass is supported by the cable system shown. Determine the tension in cables *AB*, *BC* and *DE*, and the angle θ.



Solution:

$(+\rightarrow)\Sigma F_x = 0$

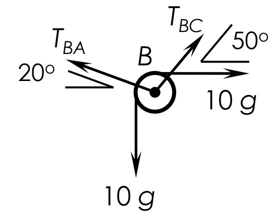
$10g + T_{BC}\cos 50^o - T_{AB}\cos 20^o = 0$
$98.1 + 0.643T_{BC} - 0.94T_{AB} = 0 \qquad (1)$

$(+\uparrow)\Sigma F_y = 0$
$-10g + T_{BC}\sin 50^o + T_{AB}\sin 20^o = 0$
$-98.1 + 0.766T_{BC} + 0.342T_{AB} = 0 \quad (2)$

–
(1)÷ 0.643 $\qquad 152.57 + T_{BC} - 1.462\, T_{AB} = 0$
(2)(2) ÷ 0.766 $\qquad -128.07 + T_{BC} + 0.446T_{AB} = 0$
$280.64 = 1.908T_{AB}$
$T_{AB} = 147.1 \text{ N}$

input into (1)

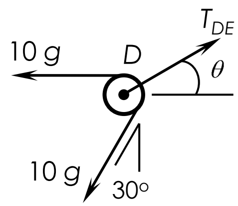
$98.1 + 0.643T_{BC} - 0.94(147.1) = 0$
$T_{BC} = 62.48 \text{ N}$

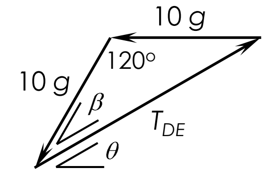


cosine rule
$T_{DE}{}^2 = (10g)^2 + (10g)^2 - 2(10g)(10g)\cos 120^o$
$T_{DE} = 169.9 \text{ N}$

$\beta = 30^o$ since the triangle is isosceles

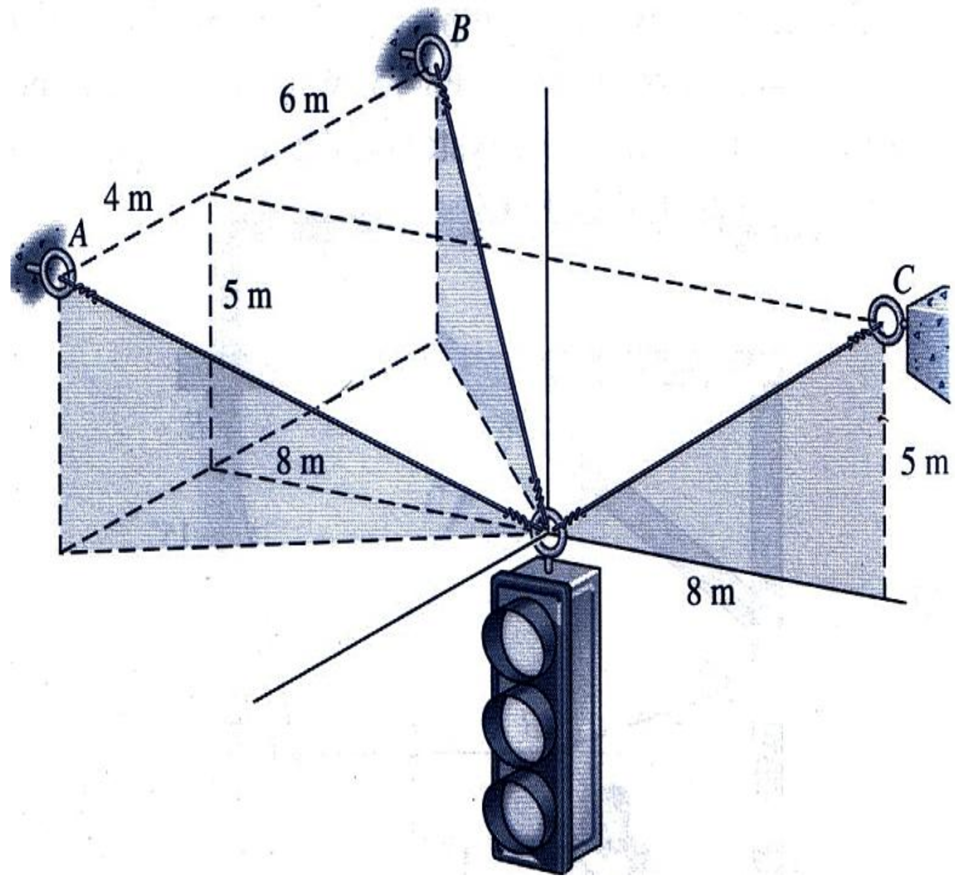
$\therefore \quad \theta = 30^o$





Example:

Q3. A 100 kg traffic light is supported by a system of cables as shown in this figure. Determine the tensions in each of the three cables



Solution:

T_{OA} $dx = -8$ m $dy = 5$ m
$d = \sqrt{(-8)^2 + 5^2 + 4^2} = 10.25$ m
$dz = 4$ m

$\mathbf{T}_{OA} = (-8/10.25)\, T_{OA}\, \mathbf{i} + (5/10.25)\, T_{OA}\, \mathbf{j} + (4/10.25)\, T_{OA}\, \mathbf{k}$
$\mathbf{T}_{OA} = -\,0.78\, T_{OA}\, \mathbf{i} + 0.488\, T_{OA}\, \mathbf{j} + 0.39\, T_{OA}\, \mathbf{k}$

T_{OB} $dx = -8$ m
$dy = 5$ m $d = \sqrt{(-8)^2 + 5^2 + (-6)^2} = 11.18$ m

$dz = -6$ m

$\mathbf{T}_{OB} = (-8/11.18)\, T_{OB}\, \mathbf{i} + (5/11.18)\, T_{OB}\, \mathbf{j} + (-6/11.18)\, T_{OB}\, \mathbf{k}$

$\mathbf{T}_{OB} = -0.716\, T_{OB}\, \mathbf{i} + 0.447\, T_{OB}\, \mathbf{j} - 0.537\, T_{OB}\, \mathbf{k}$

T_{OC} $dx = 8$ m
$dy = 5$ m $d = \sqrt{8^2 + 5^2 + (0)^2} = 9.43$ m
$dz = 0$ m

$\mathbf{T}_{OC} = (8/9.43)\, T_{OC}\, \mathbf{i} + (5/9.43)\, T_{OC}\, \mathbf{j}$

$\mathbf{T}_{OC} = 0.848\, T_{OC}\, \mathbf{i} + 0.530\, T_{OC}\, \mathbf{j}$

100 kg 100 kg = $-\,100g\, \mathbf{j}$

i component

$-0.78T_{OA} - 0.716T_{OB} + 0.848T_{OC} = 0$ (1)

j component

$0.488T_{OA} + 0.447T_{OB} + 0.53T_{OC} - 100g = 0$ (2)

k component

$0.39T_{OA} - 0.537T_{OB} = 0$ (3)

$T_{OA} = 1.377T_{OB}$ (3a)

Substitute (3a) into (1) and (2)

Then, solve simultaneous equations for (1) and (2)

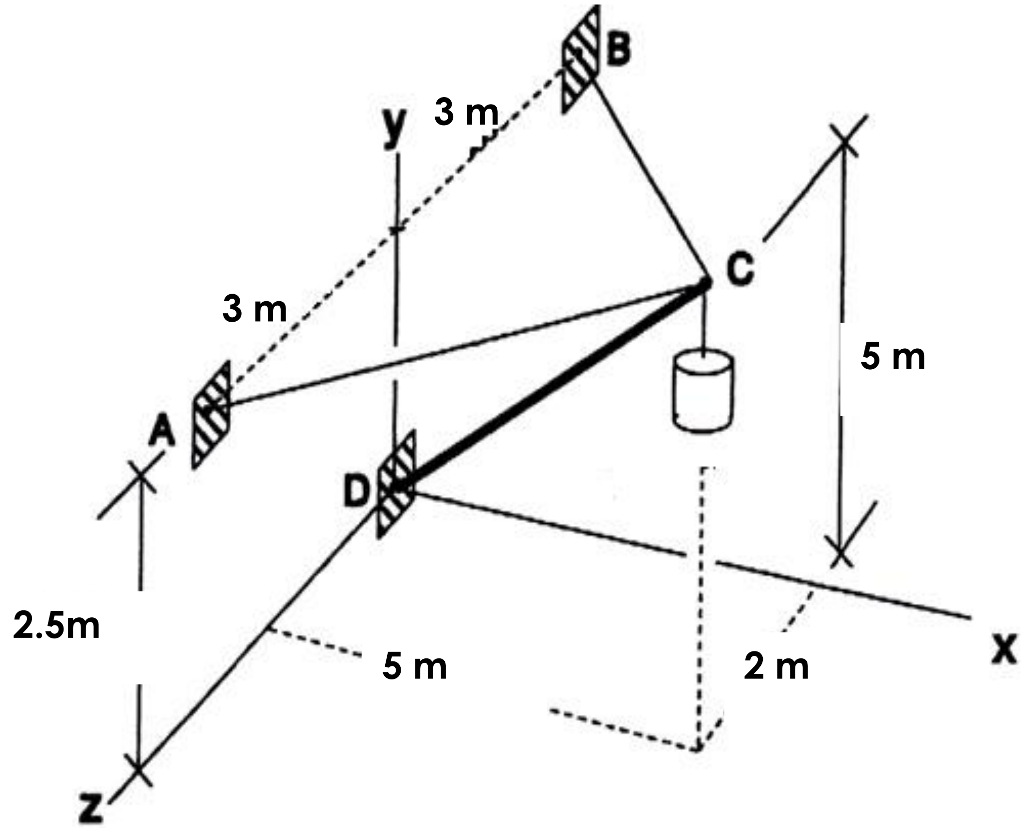
$T_{OA} = 603$ N

$T_{OB} = 439$ N

$T_{OC} = 925$ N

Example:

Q4. Determine the forces in cables *AC* and *BC*, and in the strut *CD* due to the weight, 10 kg hanging off point *C*



Solution:

T_{AC} $dx = -5$ m
$dy = -2.5$ m $d = \sqrt{(-5)^2 + (-2.5)^2 + 1^2} = 5.68$ m
$dz = 1$ m

$T_{AC} = (-5/5.68)\,T_{AC}\,\mathbf{i} + (-2.5/5.68)\,T_{AC}\,\mathbf{j} + (1/5.68)\,T_{AC}\,\mathbf{k}$
$T_{AC} = -\,0.88\,T_{AC}\,\mathbf{i} - 0.44\,T_{AC}\,\mathbf{j} + 0.18\,T_{AC}\,\mathbf{k}$

T_{BC} $dx = -5$ m
$dy = -2.5$ m $d = \sqrt{(-5)^2 + (-2.5)^2 + (-5)^2} = 7.5$ m

$dz = -5$ m

$T_{BC} = (-5/7.5)\,T_{BC}\,\mathbf{i} + (-2.5/7.5)\,T_{BC}\,\mathbf{j} + (-5/7.5)\,T_{BC}\,\mathbf{k}$

$T_{BC} = -0.67\,T_{BC}\,\mathbf{i} - 0.33\,T_{BC}\,\mathbf{j} - 0.67\,T_{BC}\,\mathbf{k}$

F_{CD} $dx = 5$ m
$dy = 5$ m $d = \sqrt{5^2 + 5^2 + 2^2} = 7.35$ m

$dz = 2$ m

$F_{CD} = 5/7.35\,F_{CD}\,\mathbf{i} + 5/7.35\,F_{CD}\,\mathbf{j} + 2/7.35\,F_{CD}\,\mathbf{k}$

$F_{CD} = 0.68\,F_{CD}\,\mathbf{i} + 0.68\,F_{CD}\,\mathbf{j} + 0.27\,F_{CD}\,\mathbf{j}$

50 kg 50 kg = $-\,50g\,\mathbf{j}$

i component

$-0.88T_{AC} - 0.67T_{BC} + 0.68\,F_{CD} = 0$ (1)

j component

$-0.44T_{AC} - 0.33T_{BC} + 0.68F_{CD} - 50g = 0$ (2)

k component

$0.18T_{AC} - 0.67T_{BC} + 0.27\,F_{CD} = 0$ (3)

Thus, solve simultaneous equations for (1), (2) and (3)

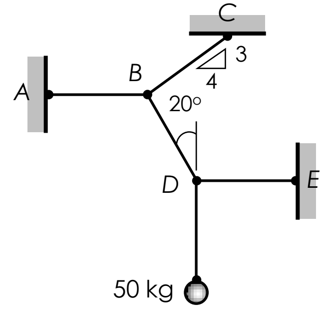
$T_{AC} = 56.49$ N

$T_{BC} = 74.07$ N

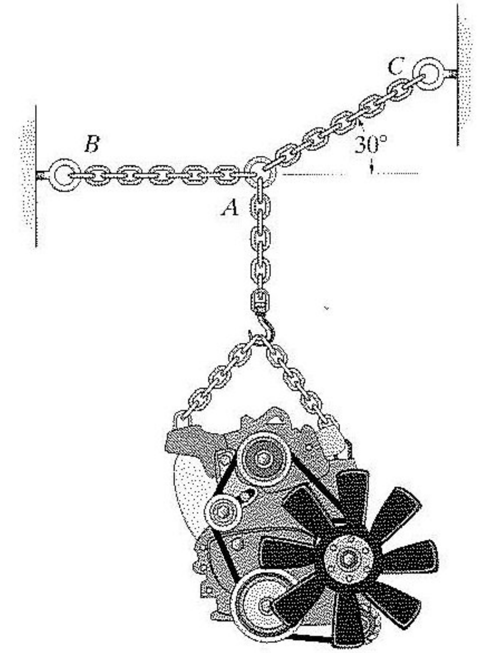
$T_{CD} = 145.97$ N

Practice Questions:

Q1. Determine the tension in each cable for the system to maintain equilibrium

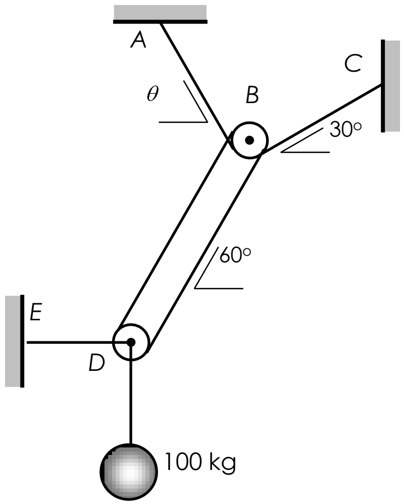


Q2. Determine the maximum weight in kN of the engine that can be supported without exceeding the maximum tension given of 4.5 kN in chain AB or 6.50 kN in chain AC.

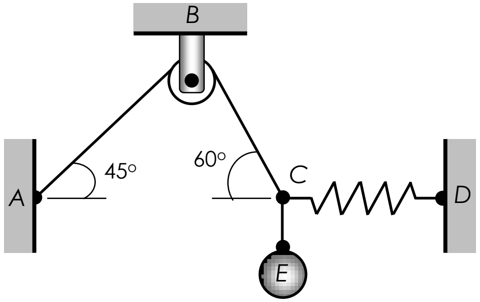


Practice Questions:

Q3. The 100 kg mass is supported by a cable and pulley system as shown. Determine the tension in each cable and the angle θ for the system to maintain equilibrium. (T_{BD} = 566.4 N, T_{DE} = 566.4 N, $T_{AB} = T_{BC}$ = 654 N, $\theta = 90^{o}$)

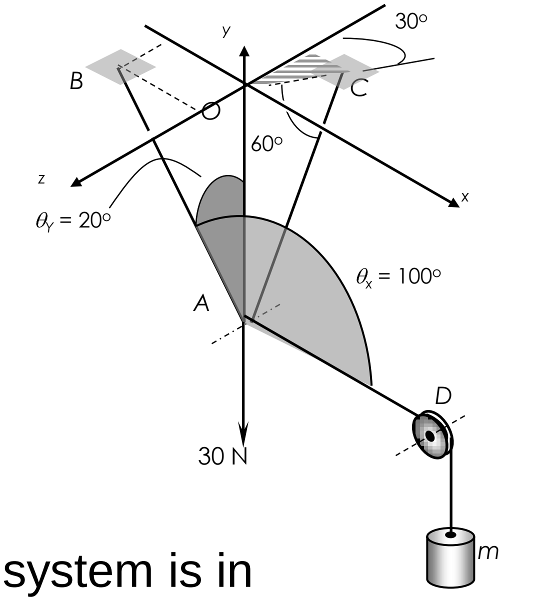


Q4. A 6 kg mass at *E* is supported as shown. Determine tension in the spring and cable *AB.*

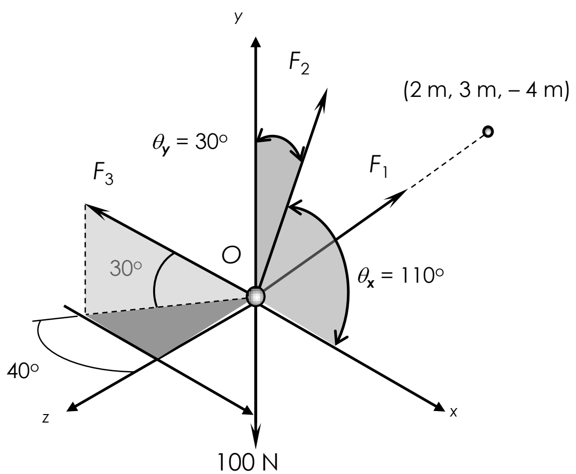


Practice Questions:

Q5. The mass *m* is maintained in equilibrium with the support of cables *AB* and *AC*, and a 30 N force at *A*. Cable *AD* is parallel to the *x*-axis. Determine the tension in cables *AB* and *AC*, and the mass *m*.

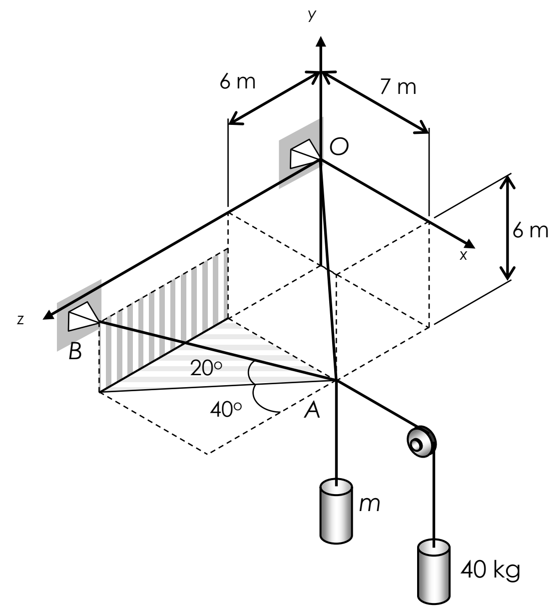


Q6. Determine the forces F_1, F_2, and F_3 so that the system is in equilibrium.



Practice Questions:

Q7. The mass *m* is supported at the position shown by cables *AO*, *AB* and the 40 kg mass. Determine the tension in cables *AO* and *AB*, and mass *m* in kg.



Q8. A weight of 500 N at point *A* is supported by a system of five cables as shown in this figure. Determine tension in each cable.

Points *C*, *O*, *A* and *B* are located on the same *x*–*z* plane while points *O*, *D* and *E* are located on the same *x*–*y* plane.