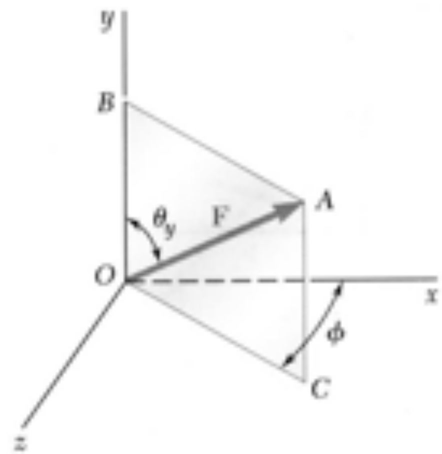
Statics SKMM1203

Resultant (3D): Rectangular component in Space

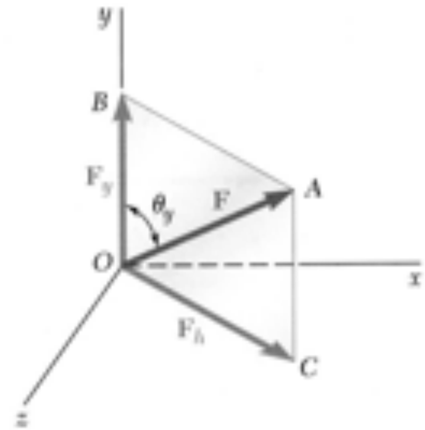
Faculty of Mechanical Engineering

Brief concept:



Brief Concept:

•The vector ***F*** is contained in the plane *OBAC*.



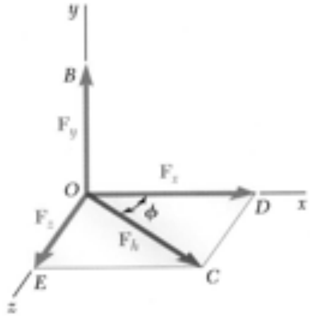
•Resolve ***F*** into horizontal and vertical components.

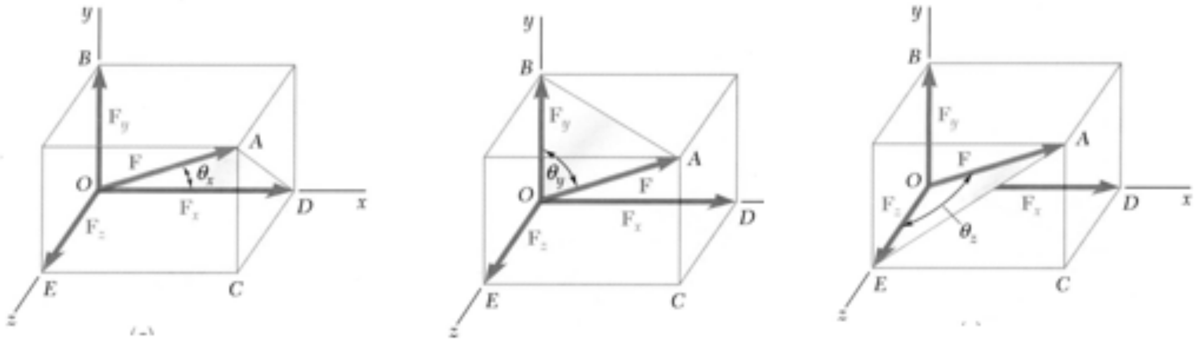
$$F_h = F \sin\theta_y \qquad F_y = F \cos\theta_y$$

Brief concept:

•Resolve $\boldsymbol{F_h}$ into rectangular components.

$$F_x = F_h \cos\phi$$
$$= F \sin\theta_y \cos\phi$$
$$F_y = F_h \sin\phi$$
$$= F \sin\theta_y \sin\phi$$





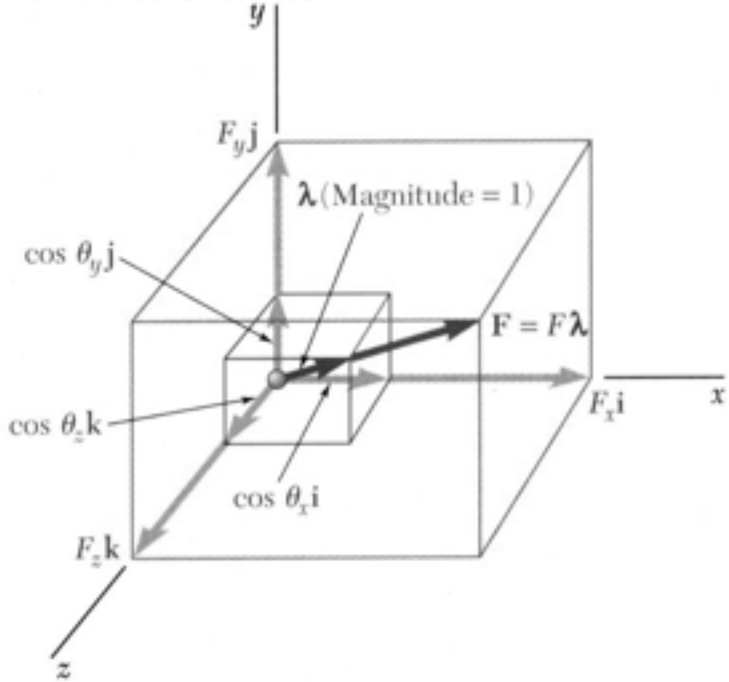
•With the angles between $\boldsymbol{F}$ and the axes,

$$F_x = F\cos\theta_x \quad F_y = F\cos\theta_y \quad F_z = F\cos\theta_z$$
$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$
$$= F\left(\cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}\right)$$
$$= F\vec{\lambda}$$
$$\vec{\lambda} = \cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}$$

Brief concept:

- $\vec{\lambda}$ is a unit vector along the line of action of $\vec{F}$

and $\cos\theta_x$, $\cos\theta_y$, and $\cos\theta_z$ are the direction cosines



Brief concept:

$$\vec{d} = \text{vector joining } M \text{ and } N$$

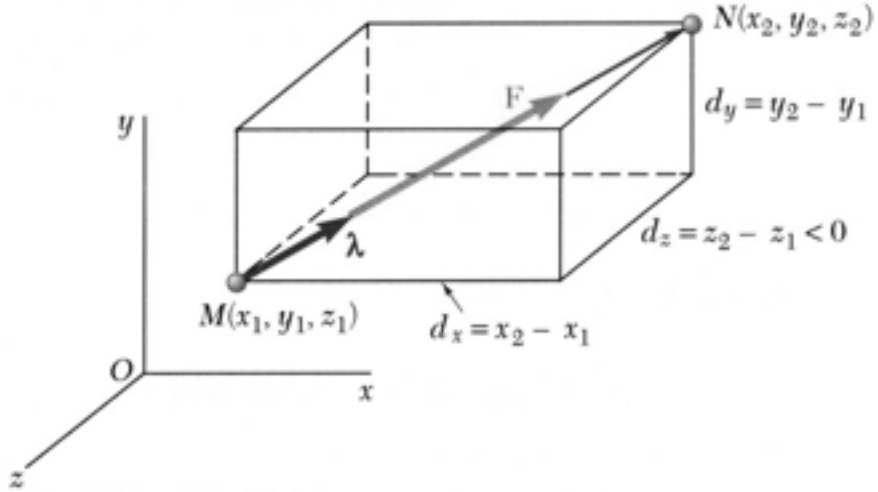
$$= d_x\vec{i} + d_y\vec{j} + d_z\vec{k}$$

$$d_x = x_2 - x_1 \quad d_y = y_2 - y_1 \quad d_z = z_2 - z_1$$

$$\vec{F} = F\vec{\lambda}$$

$$\vec{\lambda} = \frac{1}{d}\left(d_x\vec{i} + d_y\vec{j} + d_z\vec{k}\right)$$

$$F_x = \frac{Fd_x}{d} \quad F_y = \frac{Fd_y}{d} \quad F_z = \frac{Fd_z}{d}$$



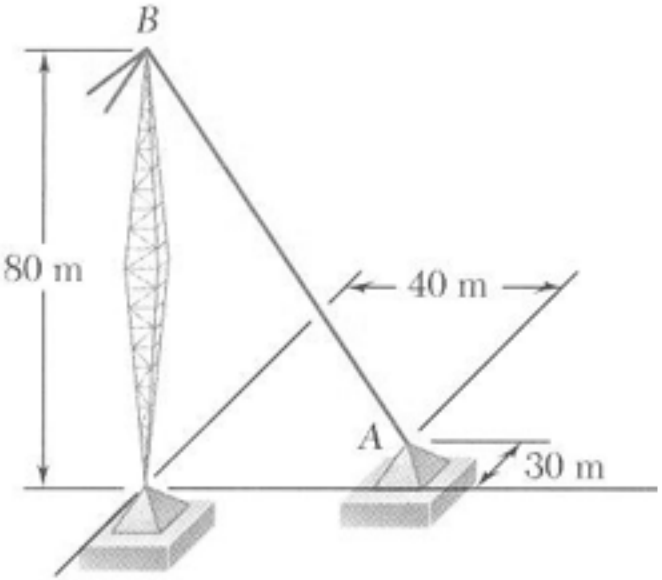
Examples:

Example:

Q1. If the tension in the guy wire is 2500 N. Determine:

a. Force components

b. Angles



A1. Determine the vector AB

a.

$$\overrightarrow{AB} = (-40\,\mathrm{m})\vec{i} + (80\,\mathrm{m})\vec{j} + (30\,\mathrm{m})\vec{k}$$

$$AB = \sqrt{(-40\,\mathrm{m})^2 + (80\,\mathrm{m})^2 + (30\,\mathrm{m})^2}$$

$$= 94.3\,\mathrm{m}$$

Examples:

$$\vec{\lambda} = \left(\frac{-40}{94.3}\right)\vec{i} + \left(\frac{80}{94.3}\right)\vec{j} + \left(\frac{30}{94.3}\right)\vec{k}$$
$$= -0.424\vec{i} + 0.848\vec{j} + 0.318\vec{k}$$

$$\vec{F} = F\vec{\lambda}$$
$$= (2500\text{ N})(-0.424\vec{i} + 0.848\vec{j} + 0.318\vec{k})$$
$$= (-1060\text{ N})\vec{i} + (2120\text{ N})\vec{j} + (795\text{ N})\vec{k}$$

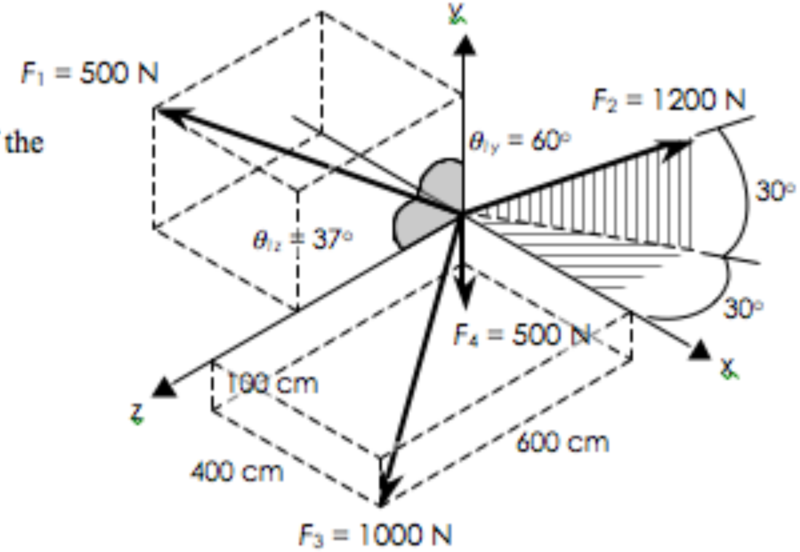
b. $$\vec{\lambda} = \cos\theta_x\,\vec{i} + \cos\theta_y\,\vec{j} + \cos\theta_z\,\vec{k}$$
$$= -0.424\vec{i} + 0.848\vec{j} + 0.318\vec{k}$$

$$\theta_x = 115.1^\circ$$
$$\theta_y = 32.0^\circ$$
$$\theta_z = 71.5^\circ$$

Examples:

Q 2

Determine the **i**, **j** and **k** components of the resultant of the four forces shown.



Examples:

A 2

F_1

$\cos^2\theta_{1x} + \cos^2\theta_{1y} + \cos^2\theta_{1z} = 1$

$\cos^2\theta_{1x} + \cos^2 60° + \cos^2 37° = 1$

$\theta_{1x} = 70.43°$ or $109.57°$

$\therefore\ \theta_{1x} = 109.57°$ since obtuse angle

$F_1 = 500\cos 109.57°\ \mathbf{i} + 500\cos 60°\ \mathbf{j} + 500\cos 37°\ \mathbf{k}$

$F_1 = -167.5\ \text{N}\ \mathbf{i} + 250\ \text{N}\ \mathbf{j} + 399\ \text{N}\ \mathbf{k}$

F_2

$F_2 = 1200\cos 30°\cos 30°\ \mathbf{i}$

$+ 1200\sin 30°\ \mathbf{j}$

$- 1200\cos 30°\sin 30°\ \mathbf{k}$

$F_2 = 900\ \text{N}\ \mathbf{i} + 600\ \text{N}\ \mathbf{j} - 519.6\ \text{N}\ \mathbf{k}$

F_3 $\left.\begin{array}{l} dx = 400 \\ dy = -100 \\ dz = 600 \end{array}\right\}$ $d = \sqrt{400^2 + 100^2 + 600^2} = 728$

$$F_3 = 1000\frac{400}{728}\mathbf{i} + 1000\frac{-100}{728}\mathbf{j} + 1000\frac{600}{728}\mathbf{k}$$

$F_3 = 549.5\ \text{N}\ \mathbf{i} - 137.4\ \text{N}\ \mathbf{j} + 824.2\ \mathbf{k}$

F_4

$F_4 = -500\ \text{N}\ \mathbf{j}$

i component:

$R_x = -167.5 + 900 + 549.5 = 1282\ \text{N}$

j component:

$R_y = 250 + 600 - 137.4 - 500 = 212.6\ \text{N}$

k component:

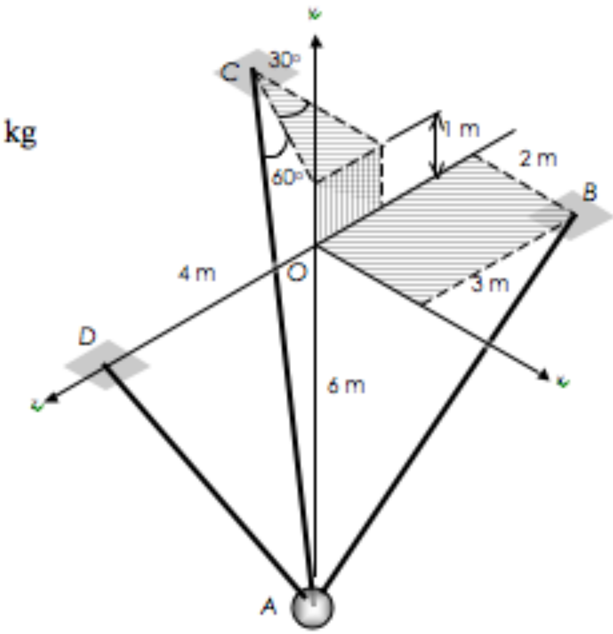
$399 - 519.6 + 824.2 = 703.6\ \text{N}$

$\therefore\ R = 1282\ \text{N}\ \mathbf{i} + 212.6\ \text{N}\ \mathbf{j} + 703.6\ \text{N}\ \mathbf{k}$

Examples:

Q 3

Determine the tension in all cables so that the 70 kg mass at *A* is maintained at the position shown.



Examples:

T_{AB} $\quad dx = 2$ m, $dy = 6$ m, $dz = -3$ m $\quad d = \sqrt{2^2 + 6^2 + 3^2} = 7$ m

$T_{AB} = (2/7)\, T_{AB}\, \mathbf{i} + (6/7)\, T_{AB}\, \mathbf{j} + (-3/7)\, T_{AB}\, \mathbf{k}$

$T_{AB} = 0.286\, T_{AB}\, \mathbf{i} + 0.857\, T_{AB}\, \mathbf{j} - 0.429\, T_{AB}\, \mathbf{k}$

T_{AC} $\quad T_{AC} = -T_{AC} \cos 60^\circ \cos 30^\circ\, \mathbf{I}$

(1)

$+ T_{AC} \sin 60^\circ\, \mathbf{j}$

$- T_{AC} \cos 60^\circ \sin 30^\circ\, \mathbf{k}$

$T_{AC} = -0.433\, T_{AC}\, \mathbf{i} + 0.866\, T_{AC}\, \mathbf{j} - 0.25\, T_{AC}\, \mathbf{k}$

T_{AD} $\quad dx = 0$ m, $dy = 6$ m, $dz = 4$ m $\quad d = \sqrt{6^2 + 4^2} = 7.21$ m

(3)

$T_{AD} = (6/7.21)\, T_{AB}\, \mathbf{j} + (4/7.21)\, T_{AB}\, \mathbf{k}$

$T_{AD} = 0.832\, T_{AD}\, \mathbf{j} + 0.555\, T_{AD}\, \mathbf{k}$

$= 0.9\, T_{AC}$

70 kg $\quad$ 70 kg $= -70g\, \mathbf{j}$

i component

$0.286\, T_{AB} - 0.433\, T_{AC} = 0$

$T_{AB} = 1.514\, T_{AC}$ (1a)

j component

$0.857\, T_{AB} + 0.866\, T_{AC} + 0.832\, T_{AD} - 70g = 0$ (2)

k component

$-0.429\, T_{AB} - 0.25\, T_{AC} + 0.555\, T_{AD} = 0$

input (1a) into (3)

$0.555\, T_{AD} = 0.429(1.514\, T_{AC}) + 0.25\, T_{AC}$

$T_{AD} = 1.621\, T_{AC}$ (3a)

input (1a) and (3a) into (2)

$0.857(1.514\, T_{AC}) + 0.866\, T_{AC} + 0.832(1.621\, T_{AC})$

$-70g = 0$

$1.297\, T_{AC} + 0.866\, T_{AC} + 1.349\, T_{AC} - 70g = 0$

$3.51\, T_{AC} - 70g = 0$

$T_{AC} = 195.6$ N

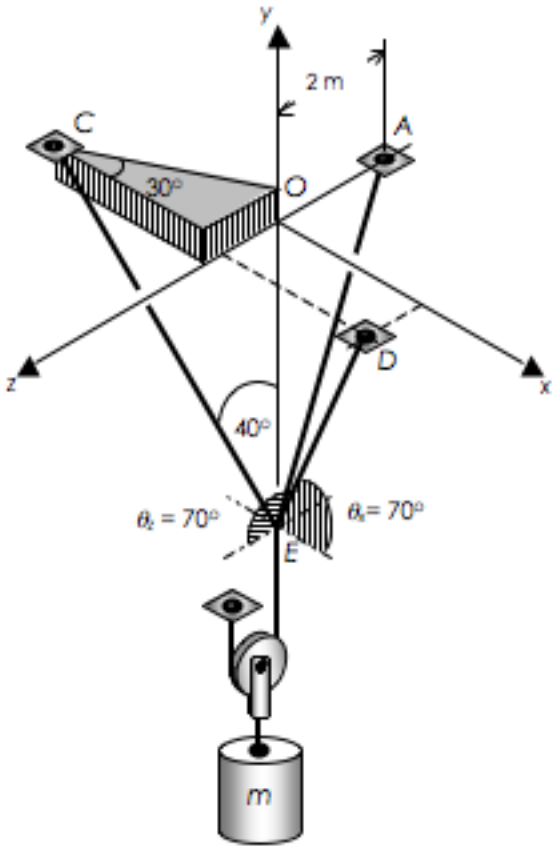
from (1a) $\quad T_{AB} = 296$ N

from (3a) $\quad T_{AD} = 317$ N

Examples:

Q 4

The system shown is in equilibrium. Determine the tension in cables *ED* and *EC*, and the mass *m* if the tension in cable *EA* is 500 N. *OE* = 5 m.



Examples:

$\bar{}mg = -\frac{1}{2}\, mg\, \mathbf{j}$

T_{EA}

$dx = 0$
$dy = 5$
$dz = -2$
$\Big\}$ $d = 52+(-2)2 = 5.39$

$T_{EA} = 50055.39\text{ N }\mathbf{j} + 500\text{-}25.39\text{ N }\mathbf{k}$

$T_{EA} = 463.8\text{ N }\mathbf{j} - 185.5\text{ N }\mathbf{k}$

T_{ED}

$\cos^2 70^\circ + \cos^2 \theta_y + \cos^2 70^\circ = 1$

$\theta_y = 28.9^\circ$

$T_{ED} = T_{ED}\cos 70^\circ\,\mathbf{i} + T_{ED}\cos 28.9^\circ\,\mathbf{j}$
$+ T_{ED}\cos 70^\circ\,\mathbf{k}$

$\therefore \quad T_{ED} = 344\text{ N}$

$T_{ED} = 0.342\,T_{ED}\,\mathbf{i} + 0.875\,T_{ED}\,\mathbf{j} + 0.342\,T_{ED}\,\mathbf{k}$

T_{EC}

$T_{EC} = -T_{EC}\sin 40^\circ\cos 30^\circ\,\mathbf{I}$

$(211) = 0$

$+ T_{EC}\cos 40^\circ\,\mathbf{j}$

$+ T_{EC}\sin 40^\circ\sin 30^\circ\,\mathbf{k}$

$T_{EC} = -0.557\,T_{EC}\,\mathbf{i} + 0.766\,T_{EC}\,\mathbf{j}$
$+ 0.321\,T_{EC}\mathbf{k}$

$\Sigma F = 0$

i component

$0.342\,T_{ED} - 0.557\,T_{EC} = 0$

$T_{ED} = 1.63\,T_{EC}$

k component

$-185.5 + 0.342\,T_{ED} + 0.321\,T_{EC} = 0$

$-185.5 + 0.342\,(1.63\,T_{EC}) + 0.321\,T_{EC} = 0$

$T_{EC} = 211\text{ N}$

j component

$-\frac{1}{2}\,mg + 463.8 + 0.875\,T_{ED} + 0.766\,T_{EC} = 0$

$-4.905\,m + 463.8 + 0.875\,(344) + 0.766$

$m = 189\text{ kg}$

Examples:

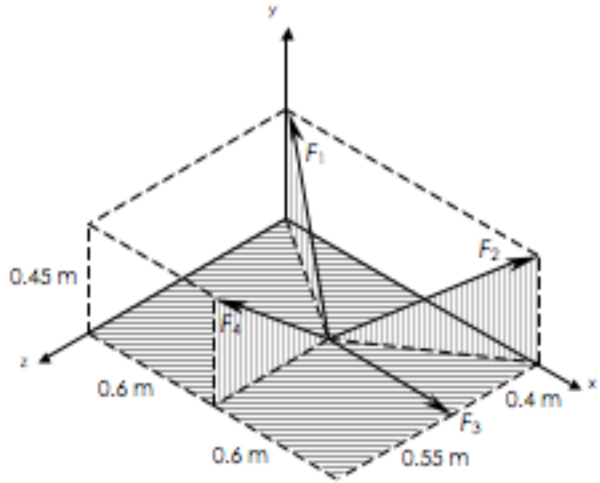
Practice:

PQ 1

F_1 = 510 N, F_3 = 100 N and F_4 = 645 N, determine

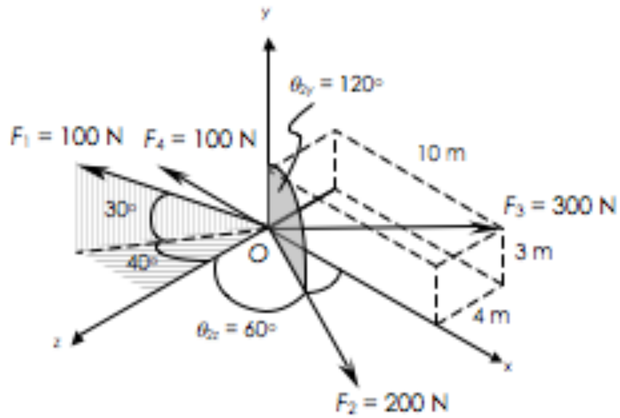
a. F_2, if component along the x-axis of the resultant of F_1 and F_2 equals 90 N

b. the resultant of the whole system.



PQ 2

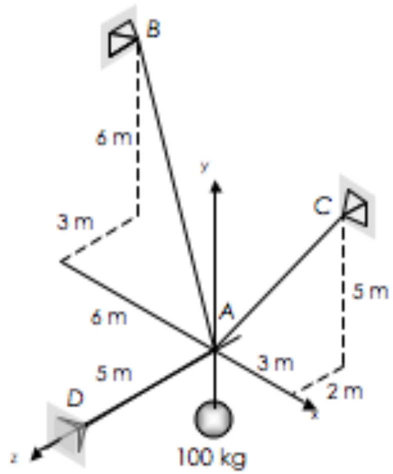
Four forces; F_1, F_2, F_3 and F_4 act on point O as shown. Determine the resultant of the system in terms $R = R_x\,\mathbf{i} + R_y\,\mathbf{j} + R_z\,\mathbf{k}$.



Examples:

PQ 3

Determine the tension in cables *AB*, *AC* and *AD* to support the 100 kg mass.



PQ 4.

A hot air balloon is held in position by three cables. Determine the tension in cables *AC* and *AD*, and the upwards thrust *F* if the tension in cable *AB* is 6600 N AC = 10 m.