OPENCOURSEWARE

Structural Steel and Timber Design SAB3233

Topic 7
Laterally unrestrained beams

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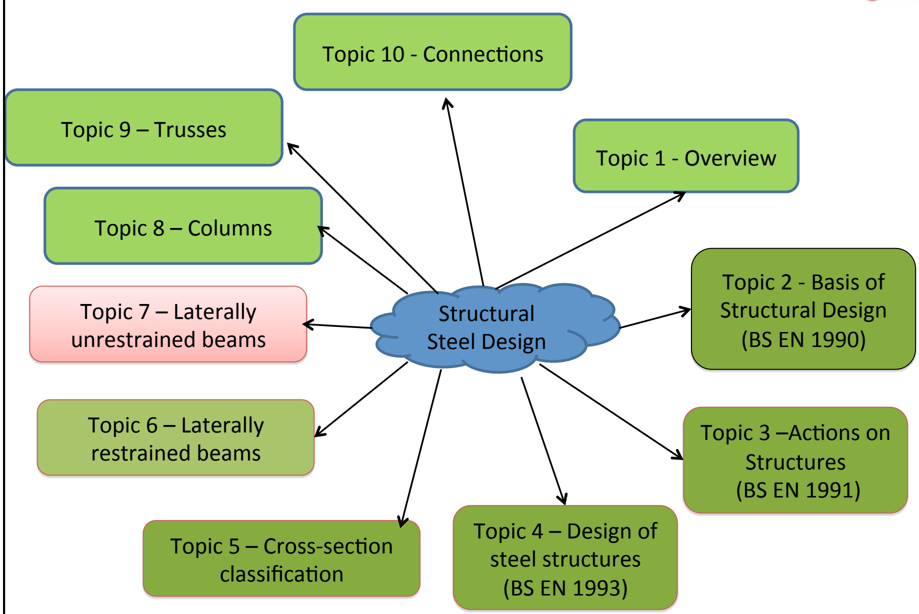
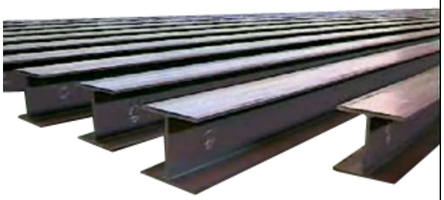
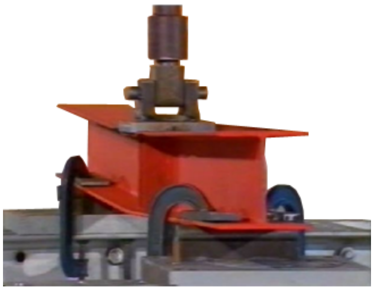
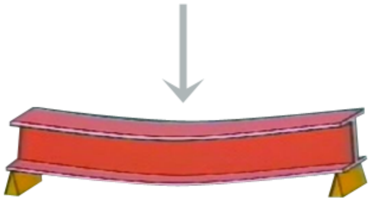
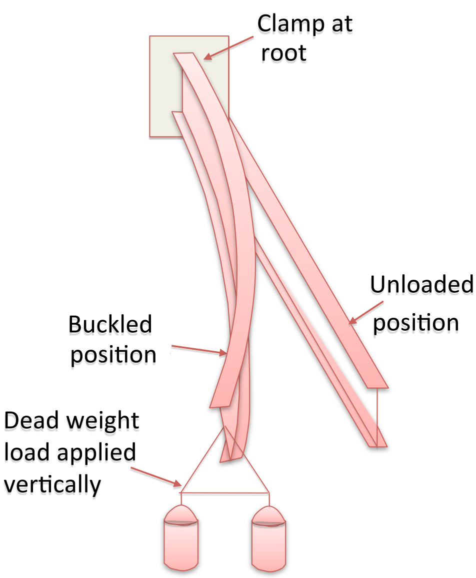


Beam is a member predominantly subject to bending. A beam is a structural member which is subject to transverse loads, and accordingly must be designed to withstand shear and moment. Generally, it will be bent about its major axis



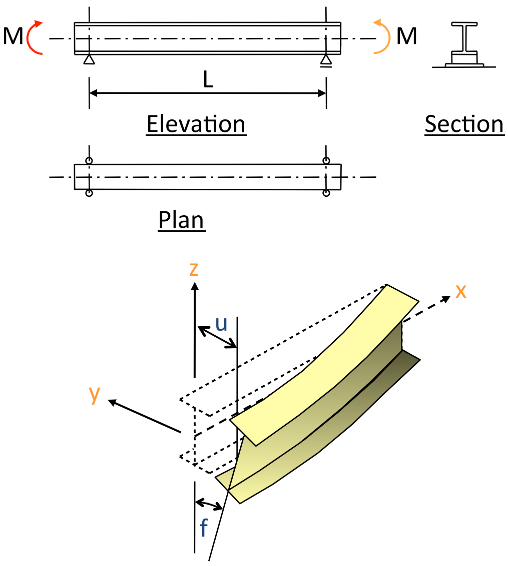






- Slender structural elements loaded in a stiff plane tend to fail by buckling in a more flexible plane.
- In the case of a beam bent about its major axis, failure may occur by a form of buckling which involves both lateral deflection and twisting.

Lateral-torsional buckling



- Perfectly elastic, initially straight, loaded by equal and opposite end moments about its major axis.

- Unrestrained along its length.
- End Supports
  - Twisting and lateral deflection prevented.
  - Free to rotate both in the plane of the web and on plan.

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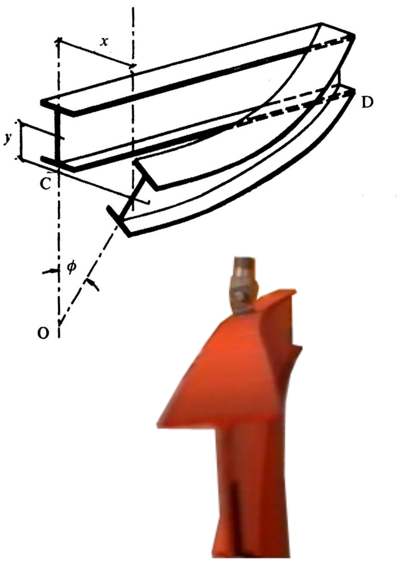
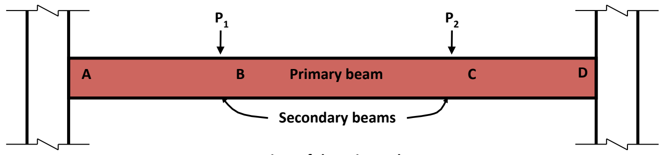
Unrestrained beam



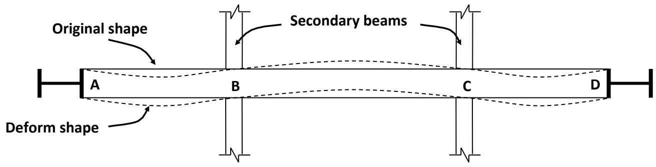
The compression flange is not restrained from deflect laterally and rotate about the plan of the section, which is called lateral torsional buckling

Three components of displacement i.e. vertical, horizontal and torsional displacement

- Lateral restraint may be of along the span or at some points along the span.



Front view of the primary beam



Plan view

Points A, B, C and D are restrained from deform laterally by the secondary beams and the connection at column

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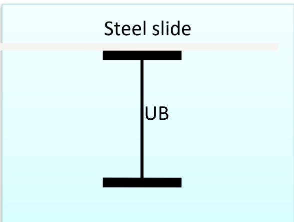
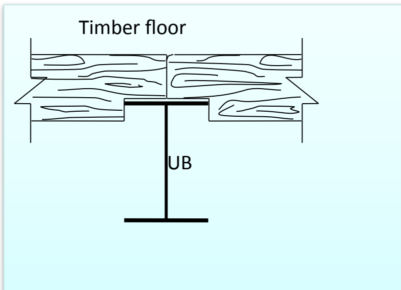
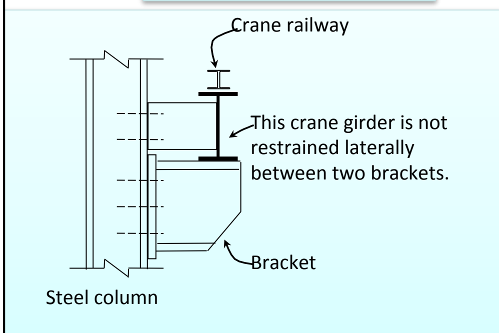
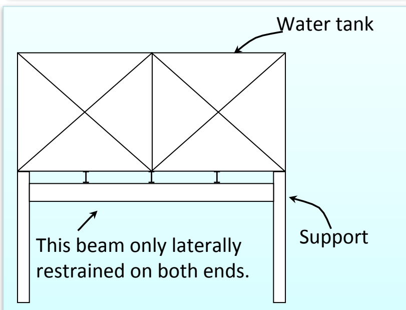
Unrestrained Beam

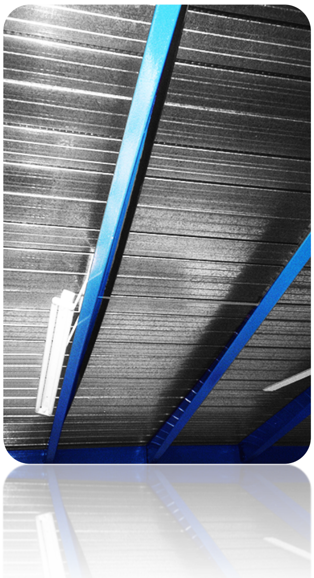
Examples :











Design factors which will influence the lateral stability can be summarized as:

- The slenderness of the member between adequate lateral restraints;
- the shape of cross-section;
- the variation of moment along the beam;
- the form of end restraint provided,
- the manner in which the load is applied, i.e. to tension or compression flange.

Elastic buckling of beams

Critical Buckling Moment for uniform bending moment diagram is

$$M_{cr} = \frac{\pi^2 EI_z}{L^2}\sqrt{\left[\frac{I_w}{I_z} + \frac{L^2 GI_t}{\pi^2 EI_z}\right]}$$

Includes:

- Lateral flexural stiffness EI_z
- Torsional and Warping stiffnesses GI_t and Ei_w

Their relative importance depends on the type of cross-section used.

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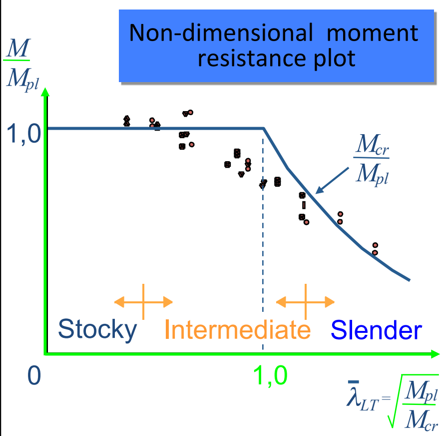
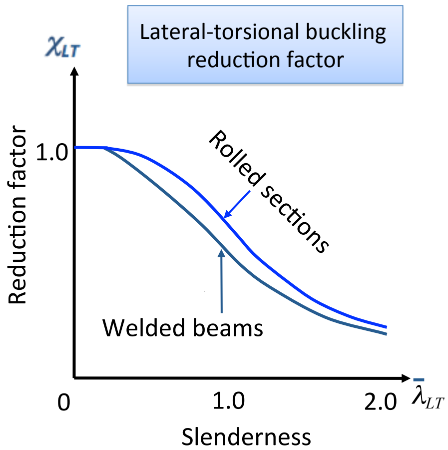
Effect of Slenderness



Non-dimensional plot permits results from different test series to be compared

- **Stocky beams** ($\bar{\lambda}_{LT} < 0,4$) unaffected by lateral torsional buckling
- **Slender beams** ($\bar{\lambda}_{LT} > 1,2$) resistance close to theoretical elastic critical moment M_{cr}
- Intermediate slenderness - adversely affected by inelasticity and geometric imperfections
- EC3 uses a reduction factor χ_{LT} on plastic resistance moment to cover the whole slenderness range

Design buckling resistance

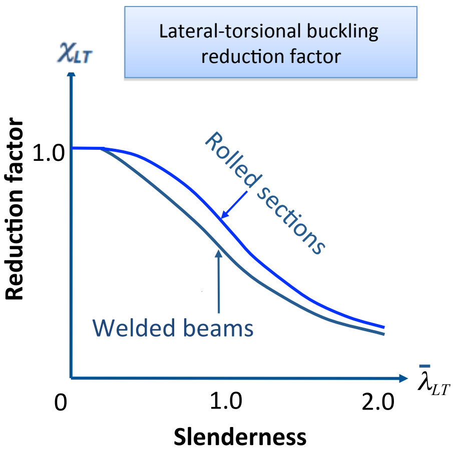


The design buckling resistance moment $M_{b.Rd}$ of a laterally unrestrained beam is calculated as

$$M_{b.Rd} = \chi_{LT}\beta_w W_{pl.y} f_y / \gamma_{M1}$$

which is effectively the plastic resistance of the section multiplied by the reduction factor χ_{LT}

Reduction factor for LTB



$$\chi_{LT} = \frac{1}{\varphi_{LT} + \left[\varphi_{LT}^2 - \bar{\lambda}_{LT}^2\right]^{0.5}}$$

where

$$\varphi_{LT} = 0.5\left[1 + \alpha_{LT}(\bar{\lambda}_{LT} - 0.2) + \bar{\lambda}_{LT}^2\right]$$

and

$\alpha_{LT} = 0.21$ for rolled sections

$\alpha_{LT} = 0.49$ for welded sections

Determining $\bar{\lambda}_{LT}$

The non-dimensional slenderness $\bar{\lambda}_{LT} = \sqrt{M_{pl.Rd} / M_{cr}}$

calculated by calculating the plastic resistance moment $M_{pl.Rd}$ and elastic critical moment M_{cr} from first principles

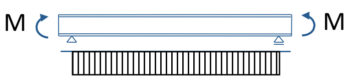
or using $\bar{\lambda}_{LT} = \left[\frac{\lambda_{LT}}{\lambda_1}\right] \beta_w^{0.5}$ where $\lambda_1 = \pi \left[\frac{E}{fy}\right]^{0.5}$

For any plain **I** or **H** section with equal flanges, under uniform moment with simple end restraints

$$\lambda_{LT} = \frac{L / i_z}{\left[1 + \frac{1}{20}\left[\frac{L / i_z}{h / t_f}\right]^2\right]^{0.25}}$$

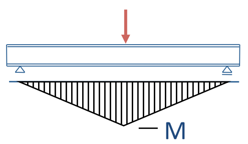
Effect of load pattern on LTB

The elastic critical moment for a beam under uniform bending moment is



$$M_{cr} = \frac{\pi}{L}\sqrt{EI_z GI_t}\sqrt{1 + \frac{\pi^2 EI_w}{L^2 GI_t}}$$

The elastic critical moment (mid-span moment) for a beam with a central point load is



$$M_{cr} = \frac{4,24}{L}\sqrt{EI_z GI_t}\sqrt{1 + \frac{\pi^2 EI_w}{L^2 GI_t}}$$

… which is increased from the basic (uniform moment) case by a factor $C_1 = 4.24/\pi = 1.365$

18

C_1 factor

Loads	Bending moment	M_{max}	C_1
M M		M	1.00
M		M	1.879
M -M		M	2.752
F		FL/4	1.365
F		FL/8	1.132
F F		FL/4	1.046

EC3 expresses the elastic critical moment ***M_{cr}*** for a particular loading case as

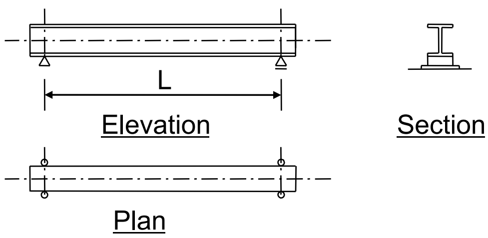
$$M_{cr} = C_1 \frac{\pi}{L}\sqrt{EI_z GI_t}\sqrt{1+\frac{\pi^2 EI_w}{L^2 GI_t}}$$

C_1 appears:

- as a simple multiplier in expressions for Mcr
- as $1/C_1^{0.5}$ in expressions for λLT.

End support conditions

- Basic case assumes end conditions which prevent lateral movement and twist but permit rotation on plan.



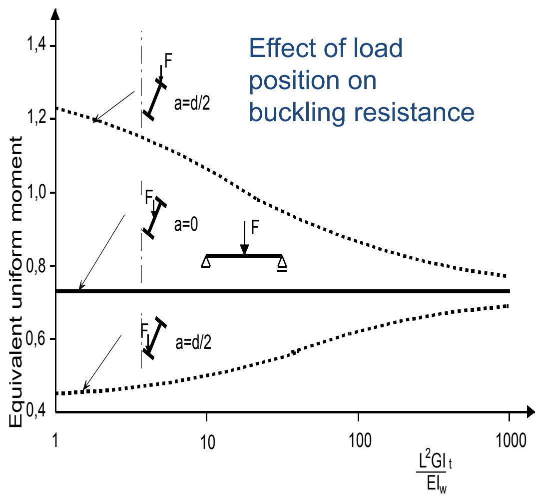
End support conditions

- End conditions which prevent rotation on plan enhance the elastic buckling resistance
- Can include the effect of different support conditions by redefining the unrestrained length as an effective length
- Two effective length factors, k and k_w.
- Reflect the two possible types of end fixity, lateral bending restraint and warping restraint.
- Note: it is recommended that k_w be taken as 1.0 unless special provision for warping fixing is made.
- EC3 recommends k values of 0,5 for fully fixed ends, 0,7 for one free and one fixed end and of course 1,0 for two free ends.

Choice of k is at the designer's discretion

21

Level of application of load



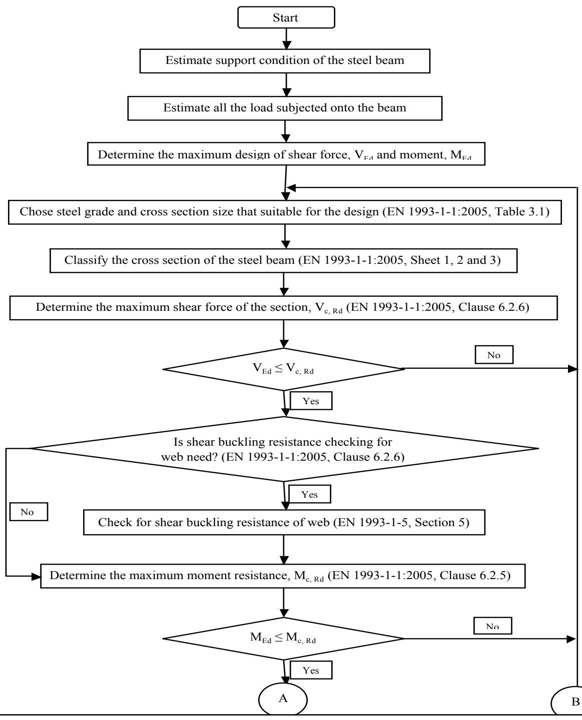
- Loads applied to top flange are destabilising
- Problem increases with depth of section and/or as span reduces
- EC3 introduces C_2 factor into expressions for λ_{LT}

22

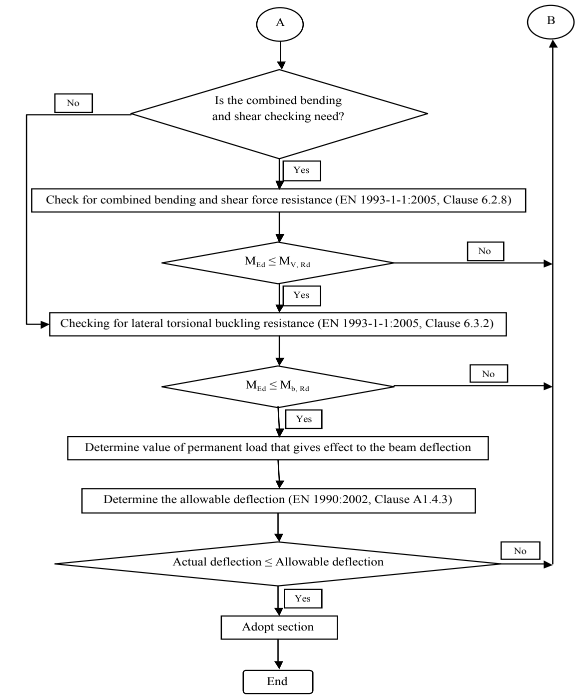
Beams with intermediate lateral support

- If beams have lateral restraints at intervals along the span the segments of the beam between restraints must be treated in isolation
- beam design is based on the most critical segment
- Lengths of beams between restraints should use an effective length factor k of 1.0

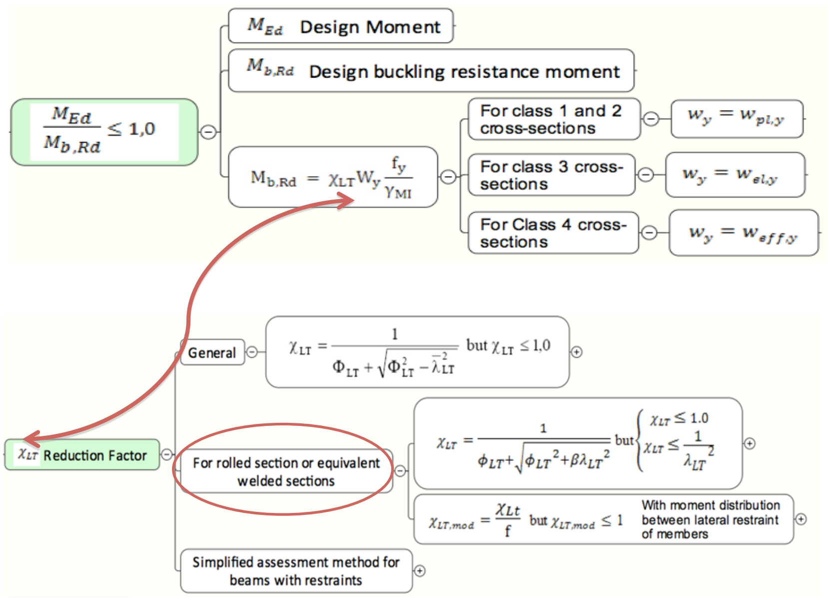
Design Procedure :



Design Procedure :



Design Procedure :



For rolled section or equivalent welded sections

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 + \beta\lambda_{LT}^2}} \text{ but } \begin{cases} \chi_{LT} \leq 1.0 \\ \chi_{LT} \leq \frac{1}{\lambda_{LT}^2} \end{cases}$$

- $\Phi_{LT} = 0{,}5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}\right) + \beta\bar{\lambda}_{LT}^2\right]$
- α_{LT} Imperfection factor
  - Table 6.3
  - Table 6.5
- $\bar{\lambda}_{LT,0} = 0{,}4$ (maximum value)
- $\beta = 0.75$

$$\chi_{LT,mod} = \frac{\chi_{Lt}}{f} \text{ but } \chi_{LT,mod} \leq 1$$

With moment distribution between lateral restraint of members

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}}$$

$$M_{cr} = C_1 \frac{\pi^2 EI_z}{L_{cr}^2}\left(\frac{I_W}{I_z} + \frac{L_{cr}^2 GI_T}{\pi^2 EI_z}\right)^{0.5}$$

- $E = 210\,000 N/mm^2$
- L_{cr} is the buckling length
- $G = 81\,000 N/mm^2$
- C_1
  - $C_1 = 1.88 - 1.40\Psi + 0.52\Psi^2$ — but C1 £ 2.70, where Ψ is the ratio of the end moments for restrained ends
  - Alternatively

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Table 6.3

Table 6.3: Recommended values for imperfection factors for lateral torsional buckling curves

Buckling curve	a	b	c	d
Imperfection factor α_{LT}	0,21	0,34	0,49	0,76

Table 6.5

Table 6.5: Recommendation for the selection of lateral torsional buckling curve for cross sections using equation (6.57)

Cross-section	Limits	Buckling curve
Rolled I-sections	$h/b \leq 2$	b
	$h/b > 2$	c
Welded I-sections	$h/b \leq 2$	c
	$h/b > 2$	d

Example 1 : Design of an unrestrained beam

Thank You