SSCE1693 ENGINEERING MATHEMATICS

CHAPTER 8:
POLAR COORDINATES

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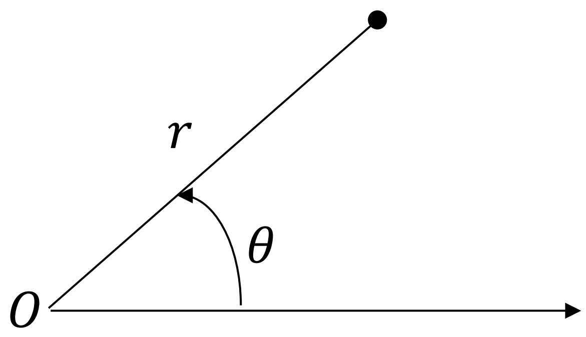
8.1 POLAR COORDINATES SYSTEM

Definition:

The polar coordinates of point P is written as an ordered pair (r, θ), that is $P(r, \theta)$ where

r - distance from origin to P

θ - angle from polar axis to the line OP



Note:

1. θ is ***positive in anticlockwise*** direction, and it is ***negative in clockwise*** direction.
2. Polar coordinate of a point is not unique.
3. A point $(-r, \theta)$ is in the opposite direction of point (r, θ).

Example 8.1:

Plot the following set of points in the same diagram:

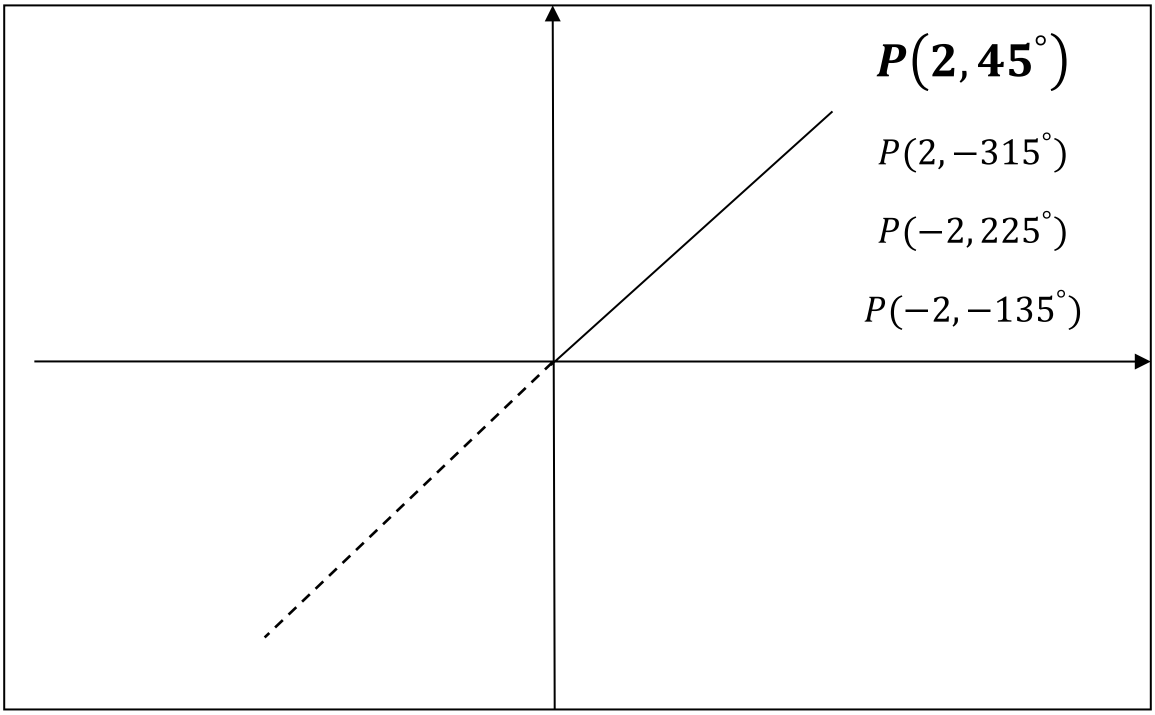
a) $(3, 225^{\circ}), (1, 225^{\circ}), (-3, 225^{\circ})$

b) $\left(2, \frac{\pi}{3}\right), \left(2, -\frac{\pi}{3}\right), \left(-2, \frac{\pi}{3}\right)$

a)

b)

For every point $P(r,\theta)$ in $0 \leq \theta \leq 2\pi$, there exist 3 more coordinates that represent the point P.



Example 8.2:

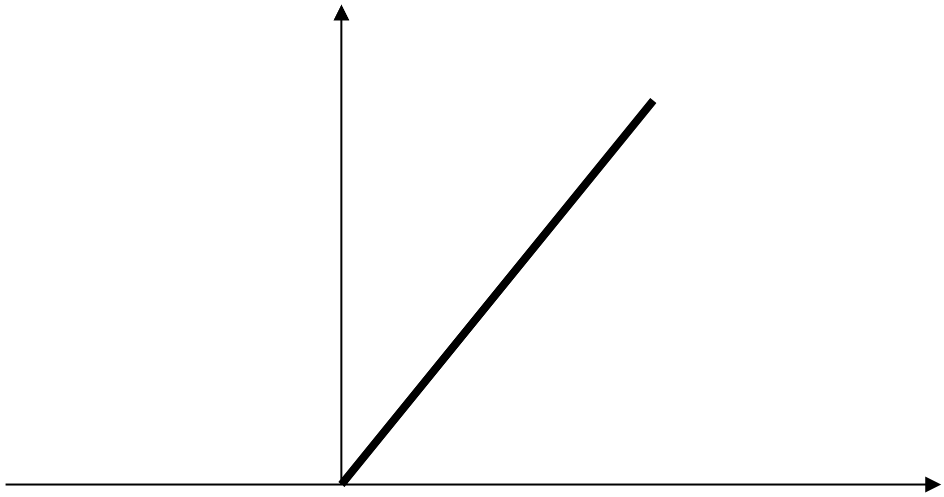
Find all possible polar coordinates of the points whose polar coordinates are given as the following:

a) $P(1, 45^\circ)$

b) $Q(2, -60^\circ)$

c) $R(-1, 225^\circ)$

8.2 Relationship between Cartesian and Polar Coordinates



1) Polar ⇨ Cartesian

$$x = r\cos\theta \qquad y = r\sin\theta$$

2) Cartesian ⇨ Polar

$$r = \sqrt{x^2 + y^2} \qquad \tan\theta = \frac{y}{x}$$

Example 8.3:

Find the Cartesian coordinates of the points whose polar coordinates are given as

a) $\left(1, \frac{7\pi}{4}\right)$

b) $\left(-4, \frac{2\pi}{3}\right)$

c) $\left(2, -30^{\circ}\right)$

Example 8.4:

Find the Cartesian coordinates of the points whose polar coordinates are given as

a) $(11,5\ )$

b) $(0,2)$

c) $(-4,-4)$

8.3 Forming polar equations from Cartesian equations and vice-versa.

To change the equation in Cartesian coordinates to polar coordinates, and conversely, use equation

$$x = r\cos\theta \qquad y = r\sin\theta \qquad r = \sqrt{x^2 + y^2}$$

Example 8.5:

Express the following rectangular equations in polar equations.

(a) $y = x^2$ (b) $x^2 + y^2 = 16$ (c) $xy = 1$

Example 8.6:

Express the following polar equations in rectangular equations.

(a) $r = 2\sin\theta$ (b) $r = \dfrac{3}{4\cos\theta + 5\sin\theta}$

(c) $r = 4\cos\theta + 4\sin\theta$ (d) $r = \tan\theta\sec\theta$

8.4 Graph Sketching of Polar Equations

There are two methods to sketch a graph of $r = f(\theta)$:

Method 1:

Form a table for r and θ, where $0 \leq \theta \leq 2\pi$. From the table, plot the (r, θ) points.

Method 2:

Symmetry test of the polar equation. The polar equations is symmetrical

(a) about the *x*-axis if $f(-\theta) = f(\theta)$

- consider θ in range [0, 180^0] only

(b) about the *y*-axis if $f(\pi - \theta) = f(\theta)$

- consider θ in range [0, 90^0] **and** [270^0, 360^0]

(c) at the origin if $f(\pi + \theta) = f(\theta)$

- consider θ in range [0, 180^0] **or** [180^0, 360^0]

* if symmetry at all, consider θ in range [0, 90^0] **only.**

Example 8.7:

Sketch the graph of $r = 2\sin\theta$

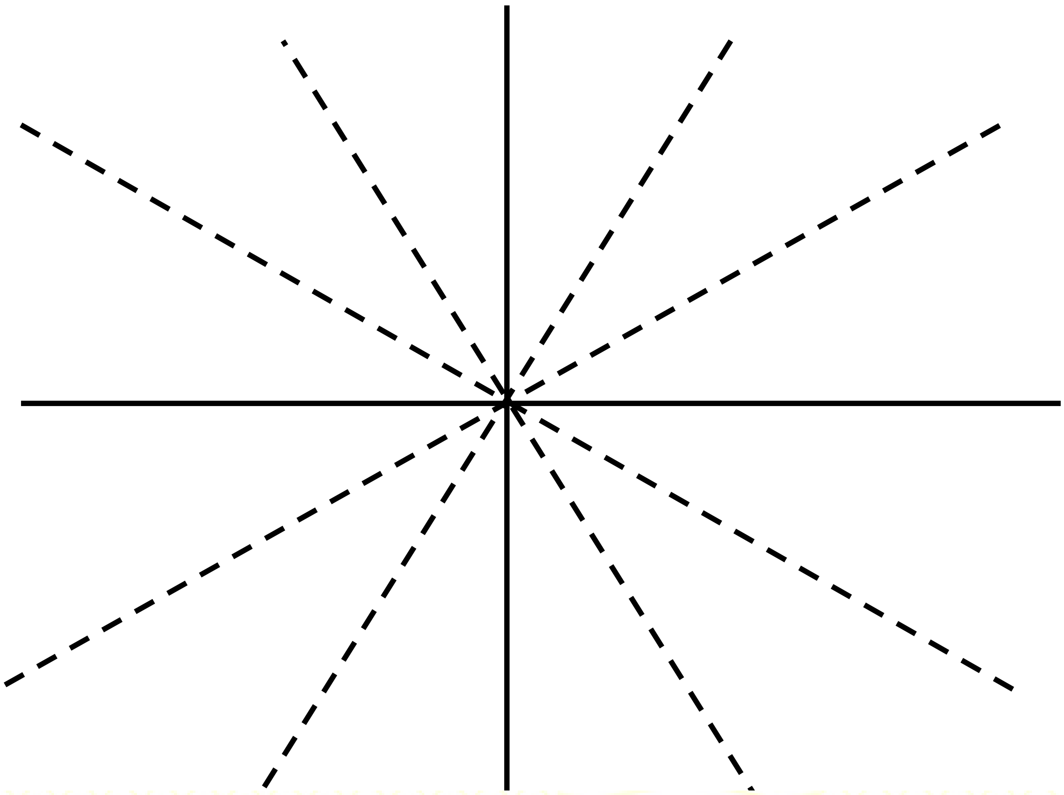
Solution:

Method 1

Here is the complete table

θ	0	30	60	90	120	150	180	210
$r = 2\sin\theta$	0	1.0	1.732	2	1.732	1	0	-1.0

θ	240	270	300	330	360
$r = 2\sin\theta$	-1.732	-2	-1.732	-1	0



Method 2

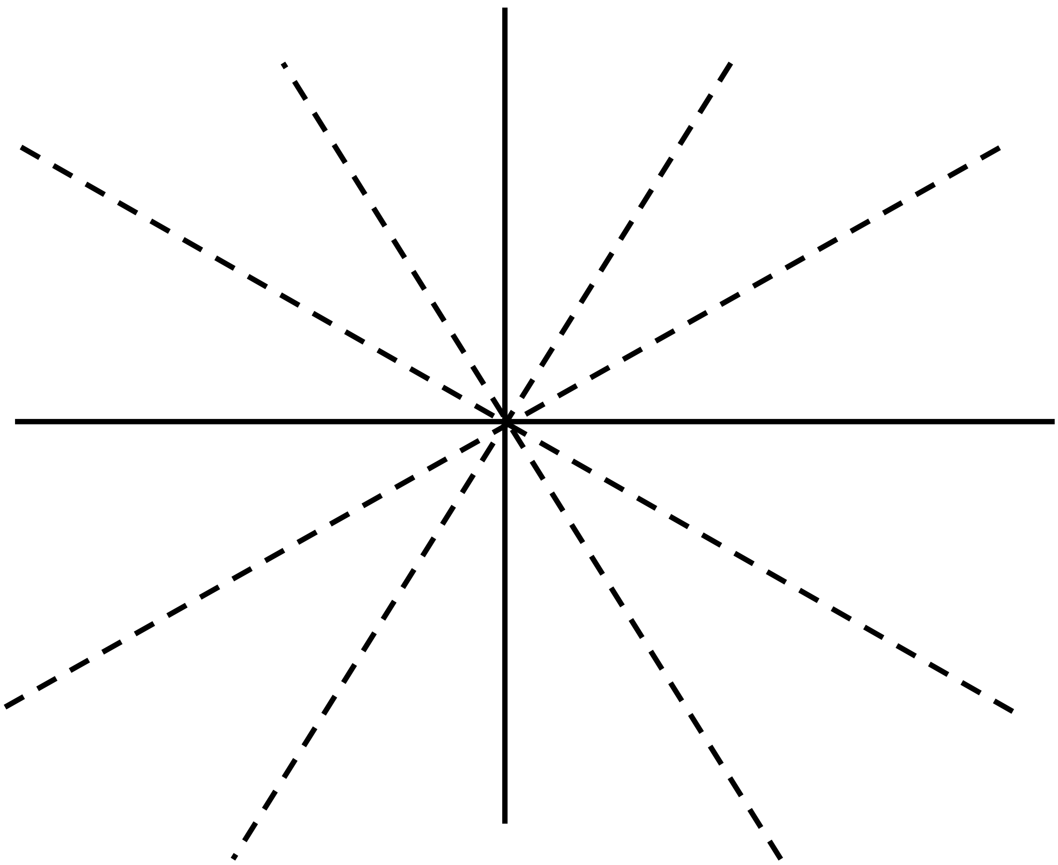
Symmetrical test for $f(\theta) = 2\sin\theta$.

Symmetry	Symmetrical test
About x-axis	$f(-\theta) = f(\theta)$?
About y-axis	$f(\pi-\theta) = f(\theta)$?
About origin	$f(\pi+\theta) = f(\theta)$?

Since r symmetry at y-axis, consider θ in range

$[0, 90^0]$ **and** $[270^0, 360^0]$

θ	0	30	60	90	270	300	330	360
$r = 2\sin\theta$	0	1.0	1.732	2	-2	-1.732	-1	0



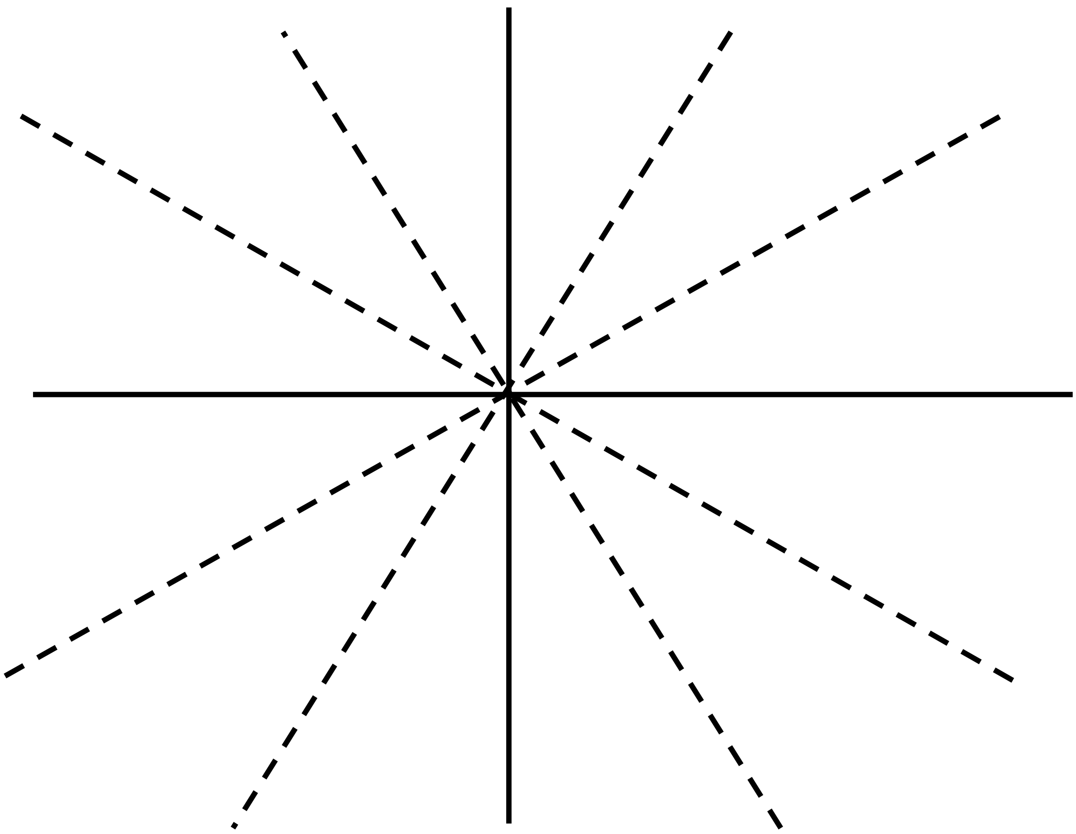
Example 8.8:

Sketch the graph of $r = \frac{3}{2} - \cos\theta$

Symmetry	Symmetrical test
About x-axis	$f(-\theta) = f(\theta)$?
About y-axis	$f(\pi - \theta) = f(\theta)$?
About origin	$f(\pi + \theta) = f(\theta)$?

Since r symmetry at x-axis, consider θ in range $[0, 180^0]$ only.

θ	0	30	60	90	120	150	180
$r = \frac{3}{2} - \cos\theta$							



Example 8.9:

Sketch the graph of $r = 2\sin^2\theta$

Symmetry	Symmetrical test
About x-axis	$f(-\theta) = f(\theta)$?
About y-axis	$f(\pi - \theta) = f(\theta)$?
About origin	$f(\pi + \theta) = f(\theta)$?

Since r symmetry at ____________, consider θ in range ____________ only.

θ							
$r = 2\sin^2\theta$