CHAPTER 2
Mathematical Modeling in Transfer Function Form

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Chapter Outline

2.1
- Introduction to Laplace Transform and Transfer Function
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  - 2.1.2 Transfer function

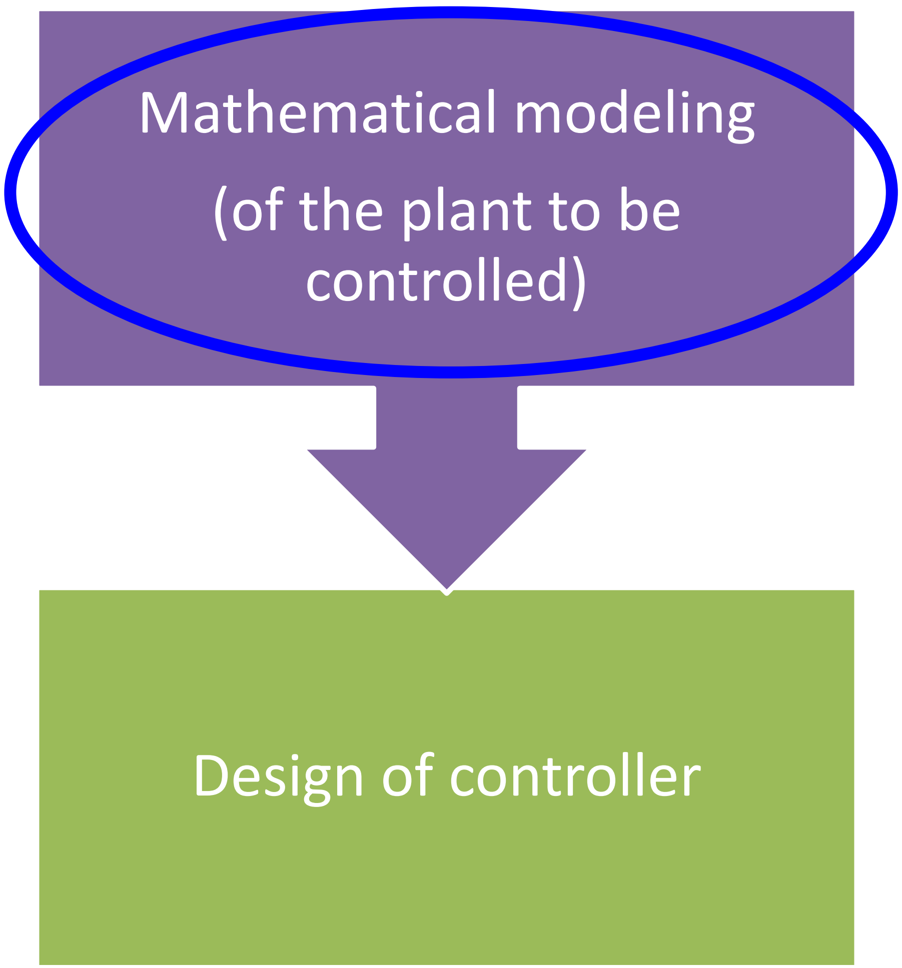
2.2
- Modeling of Electrical Systems

2.3
- Modeling of Mechanical Systems
  - 2.3.1 Translational system
  - 2.3.2 Rotational system
  - 2.3.3 Rotational system with gears

2.4
- Modeling of Electromechanical Systems

2.1 Introduction to Laplace Transform and Transfer Function

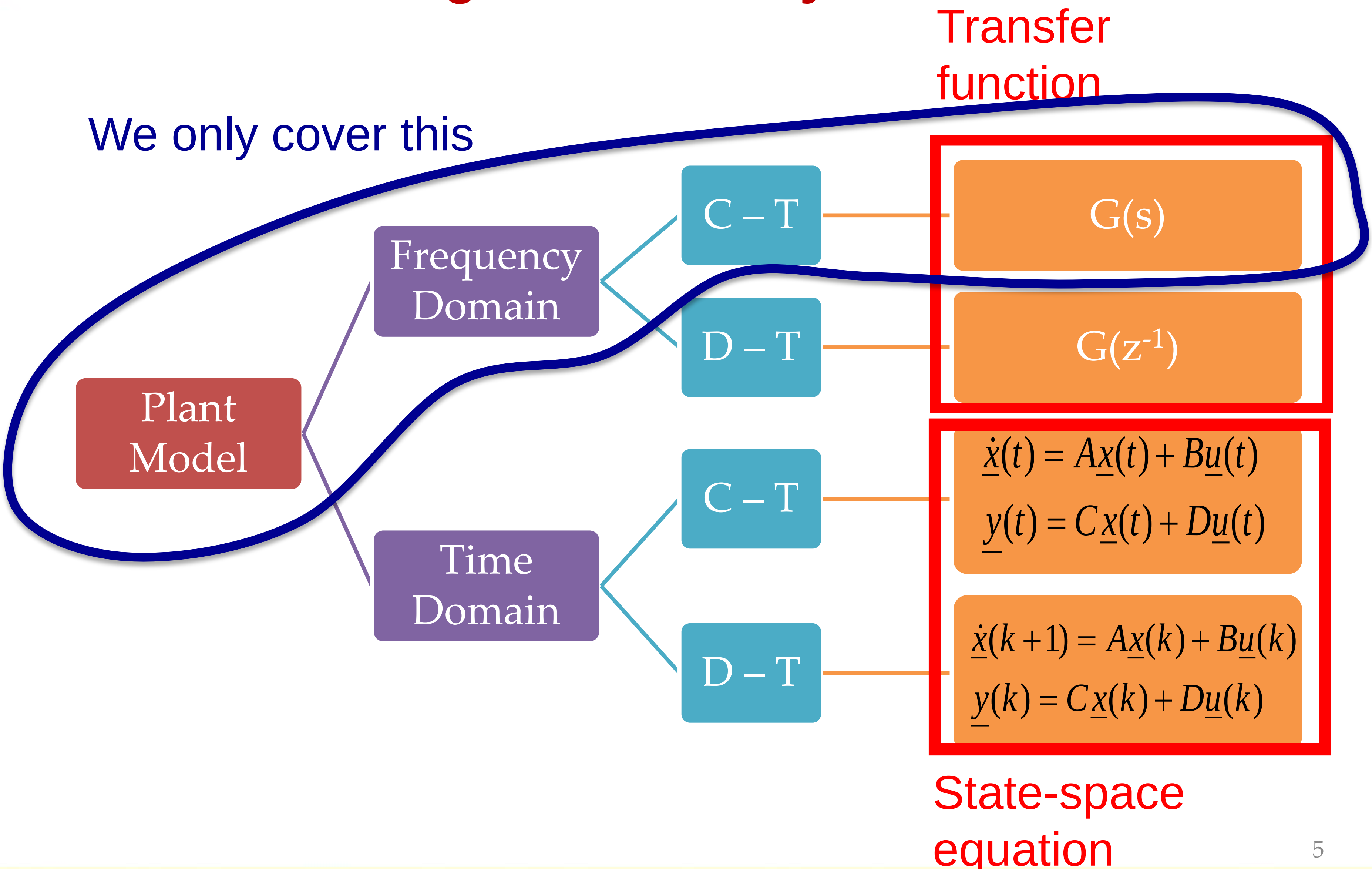
The Need for a Mathematical Model



Mathematical model of a dynamical system:

- May be obtained from the schematics of the physical systems,
- Based on physical laws of engineering
- Newton's Laws of motion
- Kirchoff's Laws of electrical network
- Ohm's Law

Modeling of Control System Plants



2.1.1 Laplace Transform

Time-domain signals ⟺ **Frequency-domain** signals

Equations:

Laplace Transform: $$\boldsymbol{L}\left[f(t)\right] = F(s) = \int_0^\infty f(t)e^{-st}dt$$

Inverse Laplace Transform: $$\boldsymbol{L}^{-1}\left[F(s)\right] = f(t)u(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} F(s)e^{st}ds$$

$$f(t) \underset{\boldsymbol{L}^{-1}}{\overset{\boldsymbol{L}}{\rightleftarrows}} F(s)$$

$$\begin{aligned} u(t) &= 1, \quad t > 0 \\ &= 0, \quad t < 0 \end{aligned}$$

Laplace Transform Table

No.	$f(t)$	$F(s)$
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t \, u(t)$	$\frac{\omega}{s^2+\omega^2}$
7.	$\cos \omega t \, u(t)$	$\frac{s}{s^2+\omega^2}$

Given ***f (t)***, what is ***F(s)***?

Laplace Transform Theorem

No.	*Theorem*	*Description*
1.	$L[f(t)] = \int_0^{\infty} e^{-st} f(t)dt = F(s)$	Laplace definition
2.	$L[kf(t)] = kF(s)$	Linearity theorem
3.	$L[f_1(t) + f_2(t)] = F_1(s) + F_2(s)$	Linearity theorem
4.	$L[e^{-at} f(t)] = F(s+a)$	Frequency shift theorem
5.	$L[f(at)] = \frac{1}{a} F(\frac{s}{a})$	Scaling theorem
6.	$L\left[\frac{df}{dt}\right] = sF(s) - f(0)$	Differentiation theorem
7.	$L\left[\frac{d^2 f}{dt^2}\right] = s^2 F(s) - sf(0) - f'(0)$	Differentiation theorem
8.	$L\left[\frac{d^n f}{dt^n}\right] = s^n F(s) - \sum_{k=1}^{n} s^{n-k} f^{k-1}(0)$	Differentiation theorem (in general)
9.	$f(\infty) = \lim_{s \to 0} sF(s)$	Final value theorem

Example 1:

- Find the Laplace Transform of *y*(*t*), assuming zero initial condition

$$\frac{d^2y(t)}{dt^2}+12\frac{dy(t)}{dt}+32y(t)=32u(t)$$

where *u*(*t*) is a unit step.

- Solution:

- [Answer: $s^2Y(s)+12sY(s)+32Y(s)=32U(s)$]

Inverse Laplace Transform

- Recall:

$$f(t) \underset{L^{-1}}{\overset{L}{\rightleftarrows}} F(s)$$

- Therefore, for Inverse Laplace Transform,

Given *F*(*s*), what is *f* (*t*)?

- Refer to Laplace Transform Table on page 8.

Inverse Laplace Transform

$$F(s) = \frac{N(s)}{D(s)}$$

numerator

denominator

- 3 situations:

  i. Roots of ***D*(*s*)** are real & distinct, e.g. $F(s) = \frac{2}{(s+1)(s+2)}$

  ii. Roots of ***D*(*s*)** are real & repeated, e.g. $F(s) = \frac{2}{(s+1)(s+2)^2}$

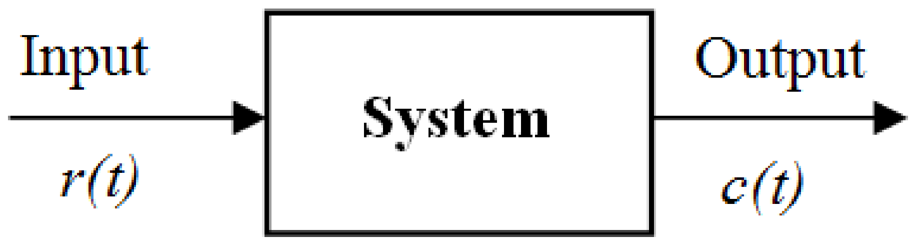
  iii. Roots of ***D*(*s*)** are complex, e.g. $F(s) = \frac{2}{s(s^2+2s+5)}$

- Hint: Use '**Partial Fraction Expansion**'

2.1.2 Transfer Function

2.1.2 Transfer Function, *G*(*s*)

- Definition:



$$G(s) = \frac{\text{Laplace transform of output signal, c}(t)}{\text{Laplace transform of input signal, r}(t)}$$

$$= \frac{C(s)}{R(s)}$$

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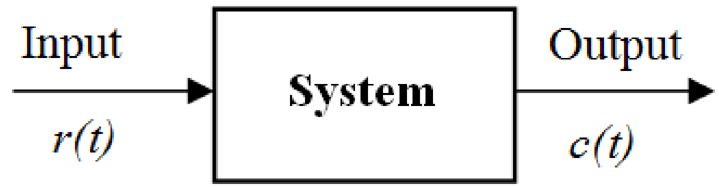


- Differential equation model:

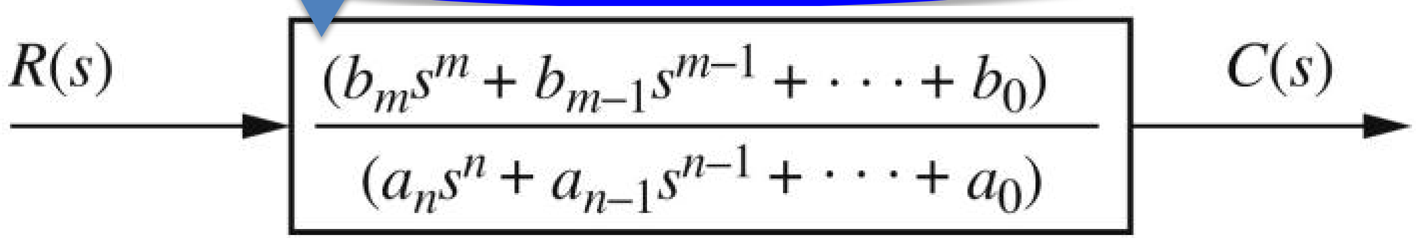
$$a_n \frac{d^n c(t)}{dt^n} + a_{n-1} \frac{d^{n-1} c(t)}{dt^{n-1}} + \ldots + a_0 c(t) = b_m \frac{d^m r(t)}{dt^m} + b_{m-1} \frac{d^{m-1} r(t)}{dt^{m-1}} + \ldots + b_0 r(t)$$

- Laplace transform both sides ('**Differentiation Theorem**' from p11) – assume zero initial condition:

$$a_n s^n C(s) + a_{n-1} s^{n-1} C(s) + \ldots + a_0 C(s) = b_m s^m R(s) + b_{m-1} s^{m-1} R(s) + \ldots + b_0 R(s)$$

$$\frac{C(s)}{R(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_0}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_0} = G(s)$$

Transfer function



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Example 3:

- Find the transfer function represented by:

$$\frac{dc(t)}{dt} + 2c(t) = r(t)$$

- Use MATLAB to create the above transfer function.

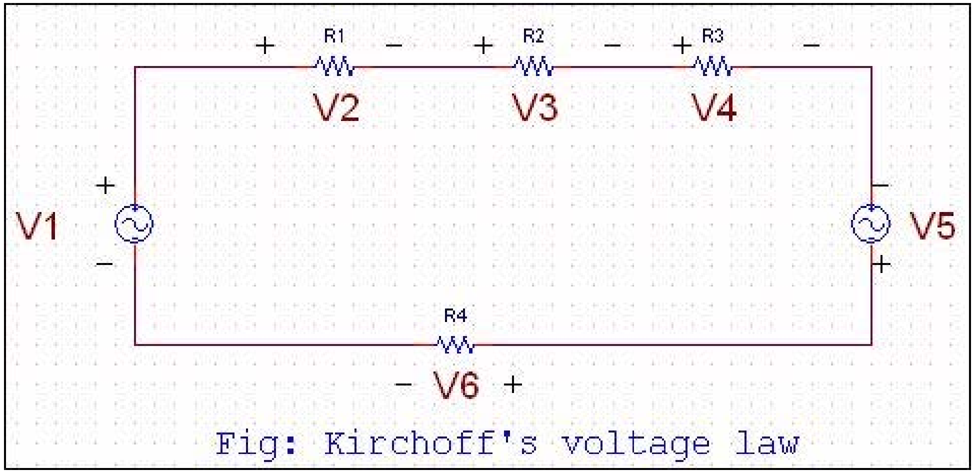
- Find the response, $c(t)$, to an input $r(t) = u(t)$, a unit step input, assuming zero initial condition.

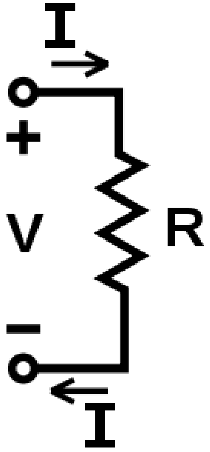
- [Answer: $\frac{C(s)}{R(s)} = \frac{1}{s+2}$]

2.2
Modeling of Electrical System

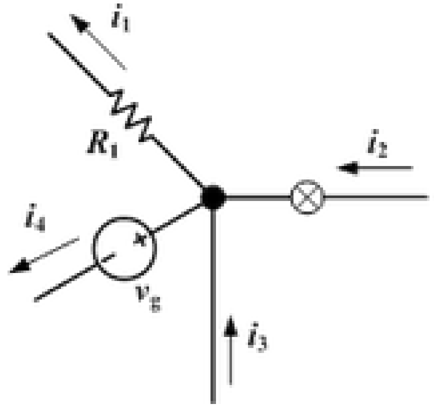
Review on Electrical Circuit Analysis

- Ohm's Law
- Kirchoff's Voltage Law
- Kirchoff's Current Law
- Mesh & Nodal Analysis





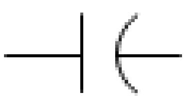
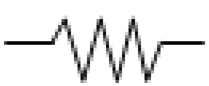
Ohm's Law



Kirchoff's Current Law

Electrical Components

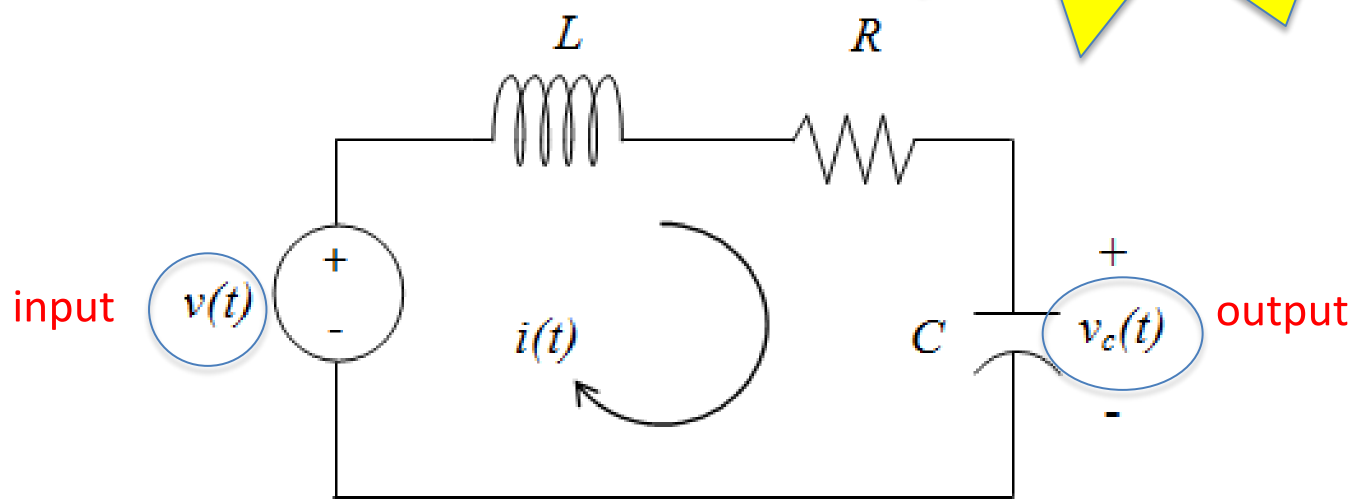
- Passive linear components
  - i. Capacitor (C) – **store** energy
  - ii. Resistor (R) – **dissipate** energy
  - iii. Inductor (L) – **store** energy
- Relationships:

Component	Voltage-current	Voltage-charge	Impedance $Z(s) = V(s)/I(s)$
 Capacitor	$v(t) = \frac{1}{C}\int i(\tau)d\tau$	$v(t) = \frac{1}{C}q(t)$	$\frac{1}{Cs}$
 Resistor	$v(t) = Ri(t)$	$v(t) = R\frac{dq(t)}{dt}$	R
 Inductor	$v(t) = L\frac{di(t)}{dt}$	$v(t) = L\frac{d^2q(t)}{dt^2}$	Ls

Example 4: Single-loop network

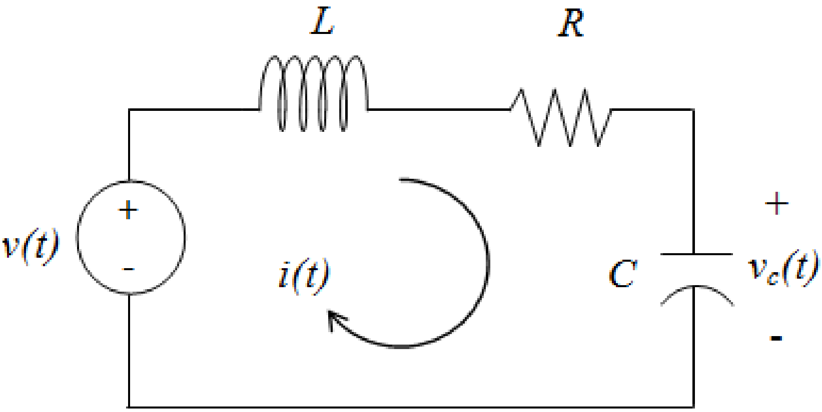
- Find the transfer function of the circuit using
  - Differential Equation Method
  - Mesh Analysis (Laplace)
  - Nodal Analysis (Laplace)

Kirchoff's Law



Solution for example 4

1. Using differential equation



KVL $\Rightarrow$ $$L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int_0^t i(t)dt = v(t)$$

Current to charge $\quad i(t) = \dfrac{dq(t)}{dt}$

$$L\frac{d^2q(t)}{dt^2} + R\frac{dq(t)}{dt} + \frac{1}{c}q(t) = v(t)$$

From table $\quad q(t) = Cv_c(t)$

$$LC\frac{d^2v_c(t)}{dt^2} + RC\frac{dv_c(t)}{dt} + v_c(t) = v(t)$$

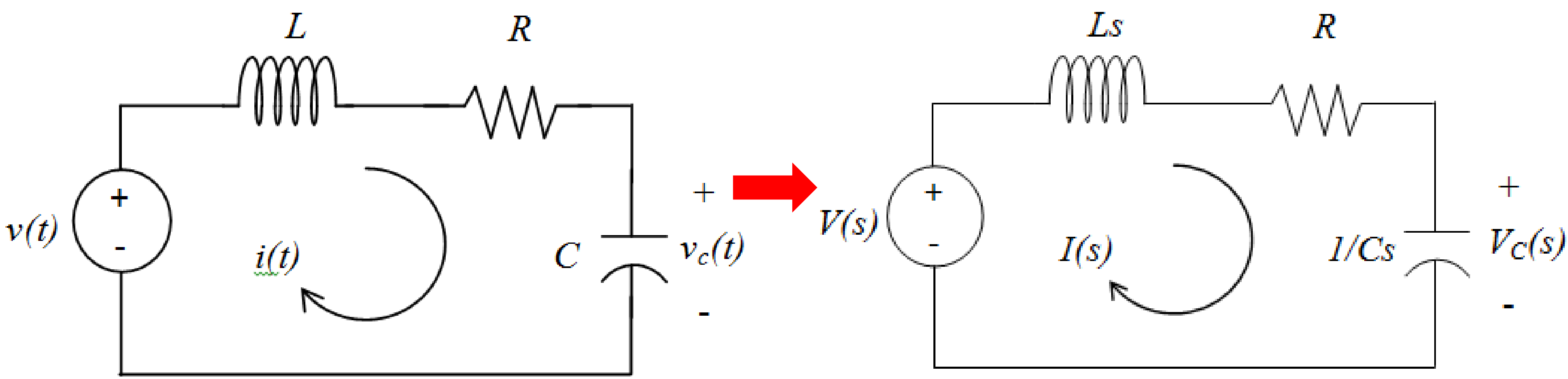
Laplace transform

$$\left(LCs^2 + RCs + 1\right)V_c(s) = V(s)$$

TRANSFER FUNCTION

$$\boxed{\frac{V_C(s)}{V(s)} = \frac{1}{LCs^2 + RCs + 1}}$$

2. Solving in Laplace domain



Time domain *Laplace domain*

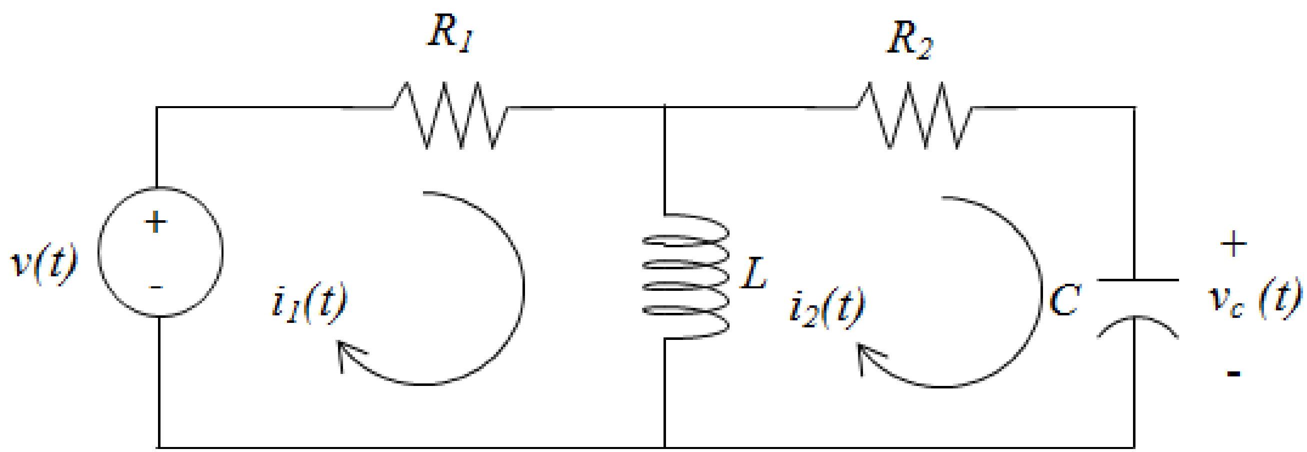
$$V_R(s), V_L(s), V_C(s) = ?$$

KVL: $$RI(s) + LsI(s) + \frac{1}{Cs}I(s) = V(s)$$

$$\frac{I(s)}{V(s)} = ? \qquad \frac{V_C(s)}{V(s)} = ?$$

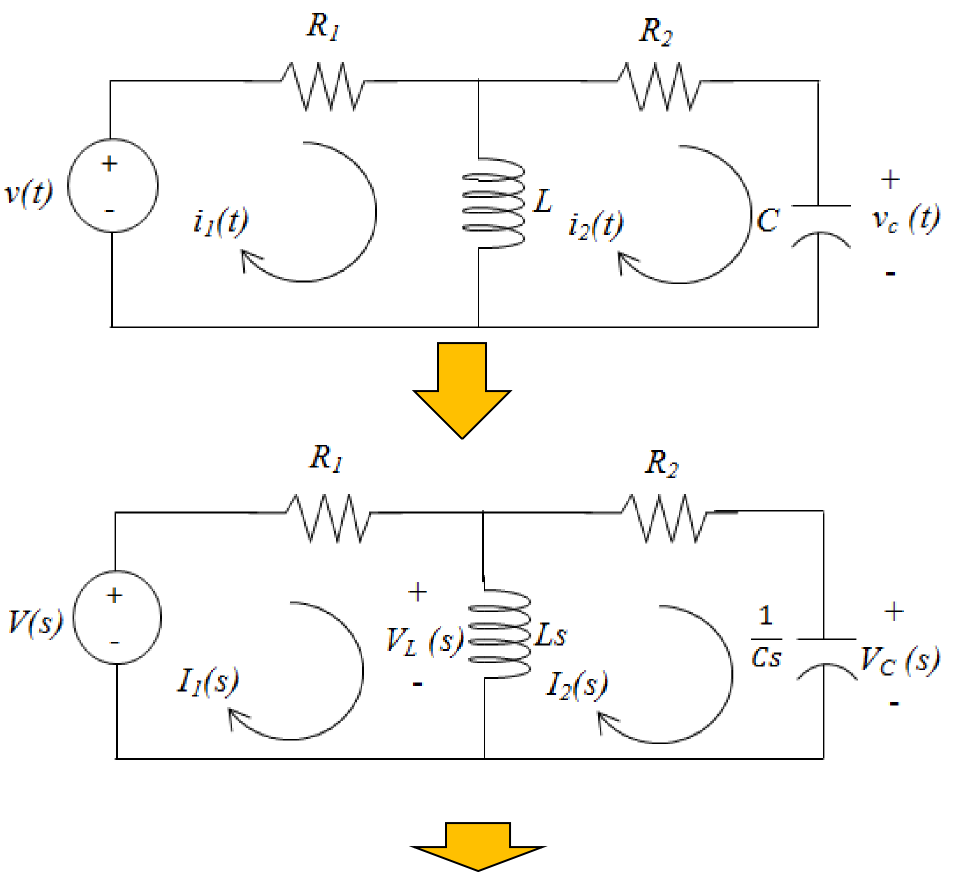
Example 5: Multiple-loop network

- Find the transfer function $\dfrac{I_2(s)}{V(s)}$ of the circuit using
  - Mesh Analysis
  - Nodal Analysis



Solution for Example 5

Find the transfer function, $\dfrac{I_2(s)}{V(s)}$



$$\frac{LCs^2}{(R_1+R_2)LCs^2+(R_1R_2C+L)s+R_1}$$

Next, find $\dfrac{V_C(s)}{V(s)} = ?$

Generic equations – Two-loop Electrical System

- Depending on number of loops in the circuit, use the following rule to obtain simultaneous equations →

	(Sum of impedances around Mesh 1)	$I_1(s)$	-	(Sum of impedances common to Mesh 1 and Mesh 2)	$I_2(s)$	=	(Sum of applied voltage around Mesh 1)
-	(Sum of impedances common to Mesh 1 and mesh 2)	$I_1(s)$	+	(Sum of impedances around Mesh 2)	$I_2(s)$	=	(Sum of applied voltage around Mesh 2)

2.3 Modeling of Mechanical System

- Translational
- Rotational
- Rotational with Gears

2.3.1 Translational

- Newton's Laws of Motion:

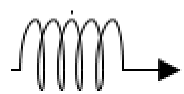
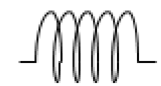
i. **First law**: The velocity of a body remains constant unless the body is acted upon by an external force.

ii. **Second law**: The acceleration **a** of a body is parallel and directly proportional to the net force **F** and inversely proportional to the mass *m*, i.e., $\mathbf{F} = m\mathbf{a}$.

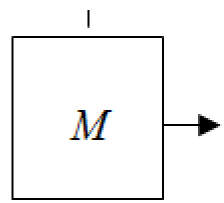
iii. **Third law**: The mutual forces of action and reaction between two bodies are equal, opposite and collinear.

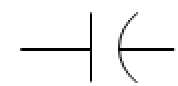
Translational Mechanical System

- 3 <u>passive</u> and <u>linear</u> components in mechanical system:
  - **Spring** - energy storage element ⟷ inductor

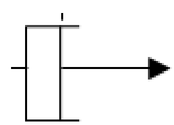



  - **Mass** - energy storage element ⟷ capacitor

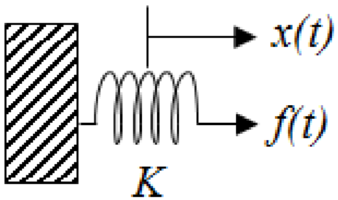
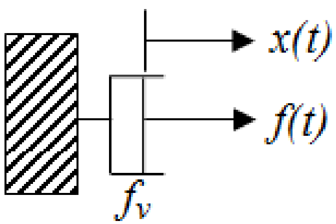
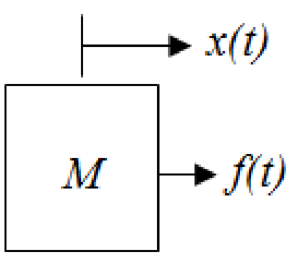




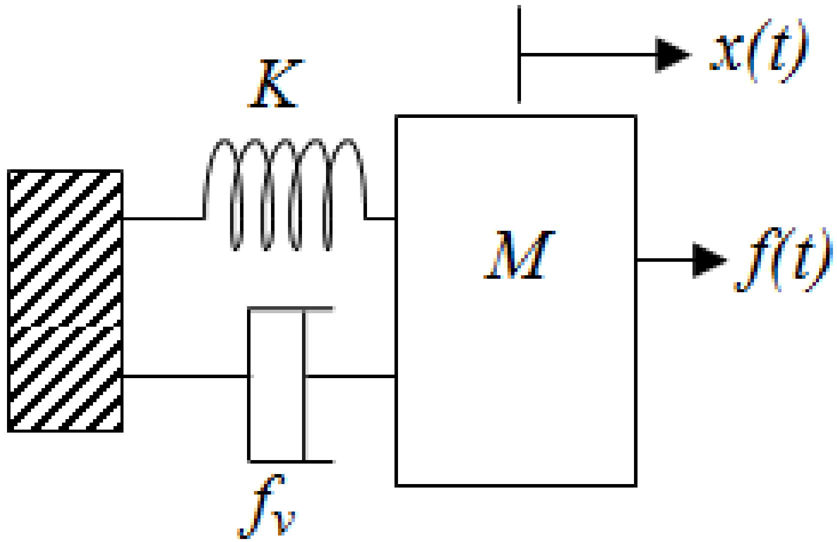
  - **Viscous damper** - energy-dissipative element ⟷ resistor




Translational Mechanical Components

Component	Force-velocity	Force-displacement	Impedance $Z_M(s) = F(s)/X(s)$
 Spring	$f(t) = K\int_0^{t} v(\tau)d\tau$	$f(t) = Kx(t)$	K
 Viscous damper	$f(t) = f_v v(t)$	$f(t) = f_v \frac{dx(t)}{dt}$	$f_v s$
 Mass	$f(t) = M\frac{dv(t)}{dt}$	$f(t) = M\frac{d^2x(t)}{dt^2}$	Ms^2

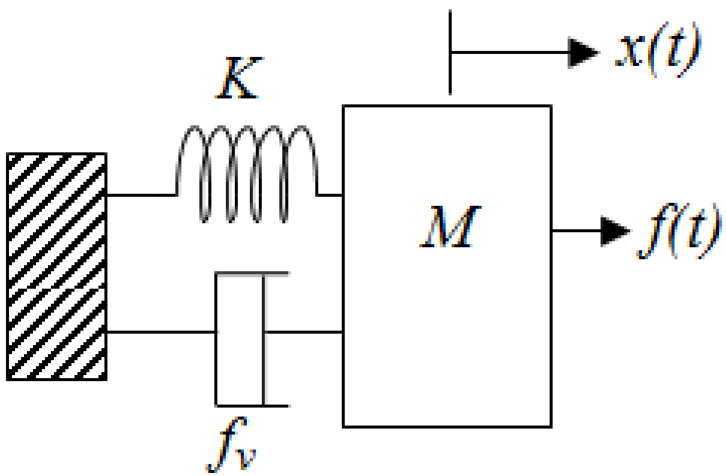
Spring, Mass & Damper in action



- Applied force *f*(*t*) points to the right
- Mass is traveling toward the right
- All other forces impede the motion and act to opposite direction
- Single input single output (SISO) system

Example 6:

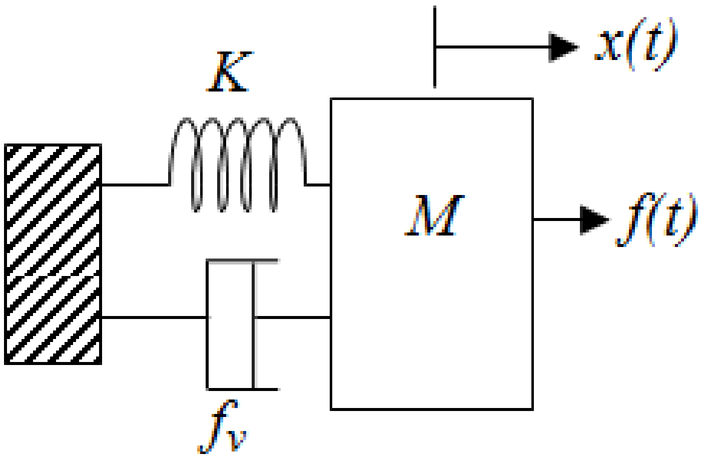
- Find the transfer function $X(s)/F(s)$, for the following mechanical system.

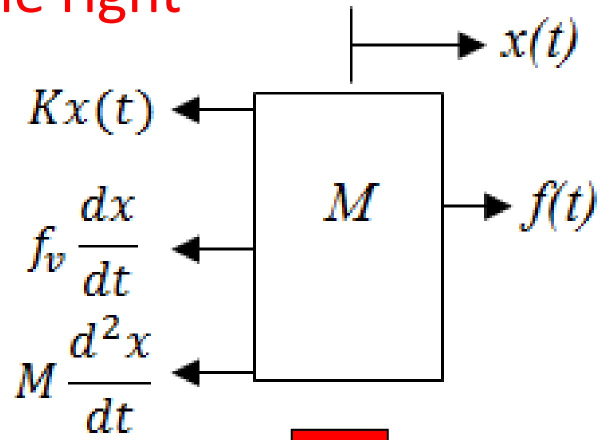


Find the transfer function *X*(*s*)/*F*(*s*)

Free body diagram

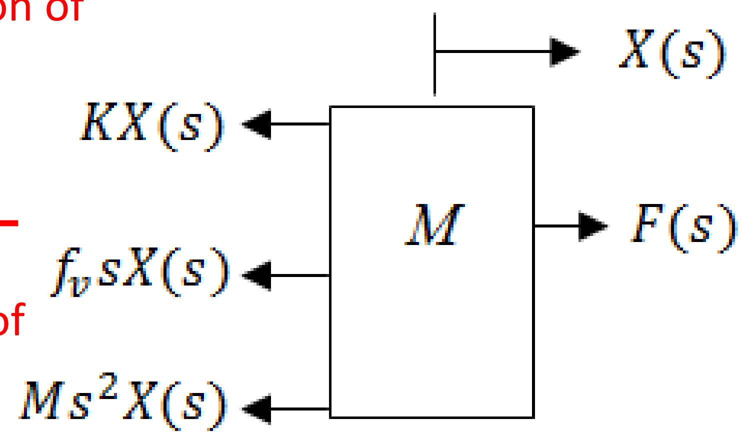
Equation of motion: → positive direction to the right





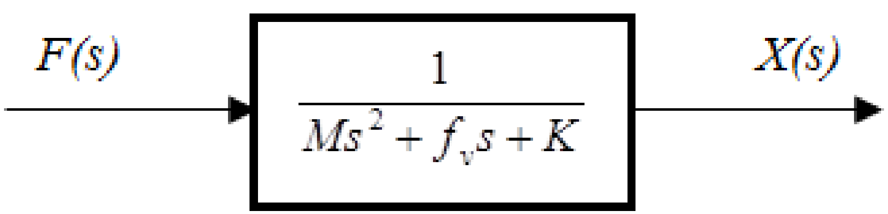
$$M\frac{d^2x(t)}{dt^2} + f_v\frac{dx(t)}{d(t)} + Kx(t) = f(t)$$

Differential equation of motion



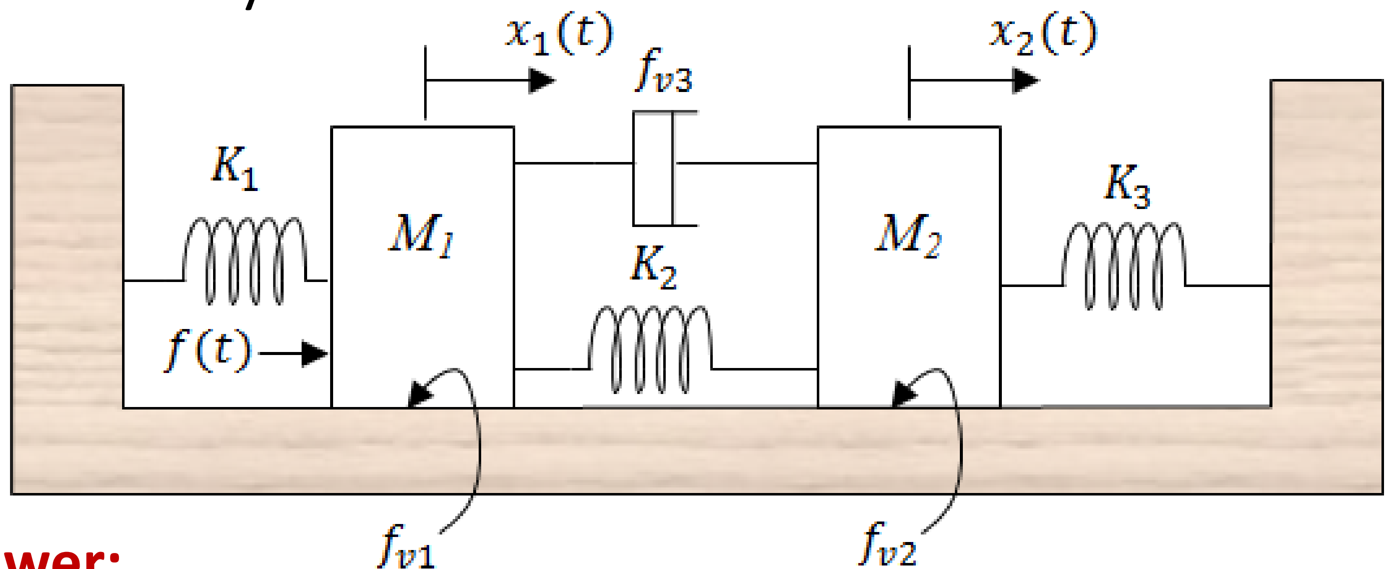
$$Ms^2X(s) + f_v sX(s) + KX(s) = F(s)$$

Laplace equation of motion



Example 7:

- Find the transfer function $X_2(s)/F(s)$, for the following mechanical system.



- **Answer:**

$$G(s) = \frac{X_2(s)}{F(s)} = \frac{(f_{V3}s + K_2)}{\Delta}$$

where

$$\Delta = \begin{vmatrix} [M_1 s^2 + (f_{V1} + f_{V3})s + (K_1 + K_2)] & -(f_{V3}s + K_2) \\ -(f_{V3}s + K_2) & [M_2 s^2 + (f_{V2} + f_{V3})s + (K_2 + K_3)] \end{vmatrix}$$

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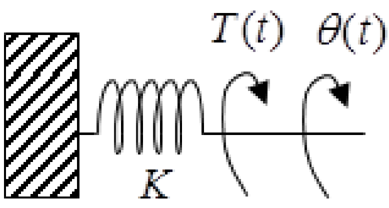
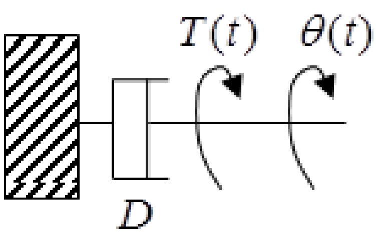
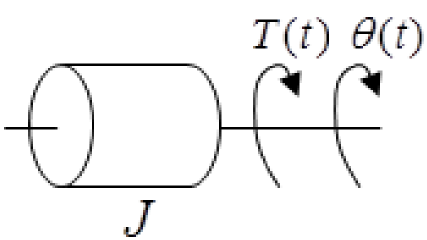
Generic equations – Two Translational Body System

$$\left[\begin{array}{c}\text{Sum of}\\ \text{impedances}\\ \text{connected to}\\ \text{the motion at}\\ x_1\end{array}\right] X_1(s) - \left[\begin{array}{c}\text{Sum of}\\ \text{impedances}\\ \text{between } x_1\\ \text{and } x_2\end{array}\right] X_2(s) = \left[\begin{array}{c}\text{Sum of}\\ \text{applied}\\ \text{forces at}\\ x_1\end{array}\right]$$

$$-\left[\begin{array}{c}\text{Sum of}\\ \text{impedances}\\ \text{between } x_1\\ \text{and } x_2\end{array}\right] X_1(s) + \left[\begin{array}{c}\text{Sum of}\\ \text{impedances}\\ \text{connected to}\\ \text{the motion at}\\ x_2\end{array}\right] X_2(s) = \left[\begin{array}{c}\text{Sum of}\\ \text{applied}\\ \text{forces at}\\ x_2\end{array}\right]$$

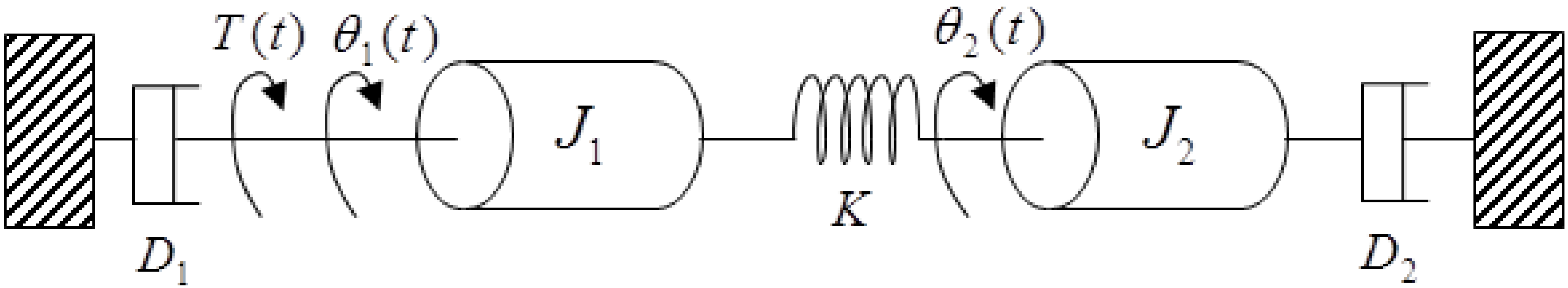
2.3.2 Rotational Mechanical System

Component	Torque-angular velocity	Torque-angular displacement	Impedance $Z_M(s) = T(s)/\theta(s)$
 Spring	$T(t) = K\int_0^t \omega(\tau)d\tau$	$T(t) = K\theta(t)$	K
 Viscous damper	$f(t) = f_v v(t)$	$T(t) = D\frac{d\theta(t)}{dt}$	Ds
 Inertia	$T(t) = D\frac{d\omega(t)}{dt}$	$T(t) = J\frac{d^2\theta(t)}{dt^2}$	Js^2

Example 8:

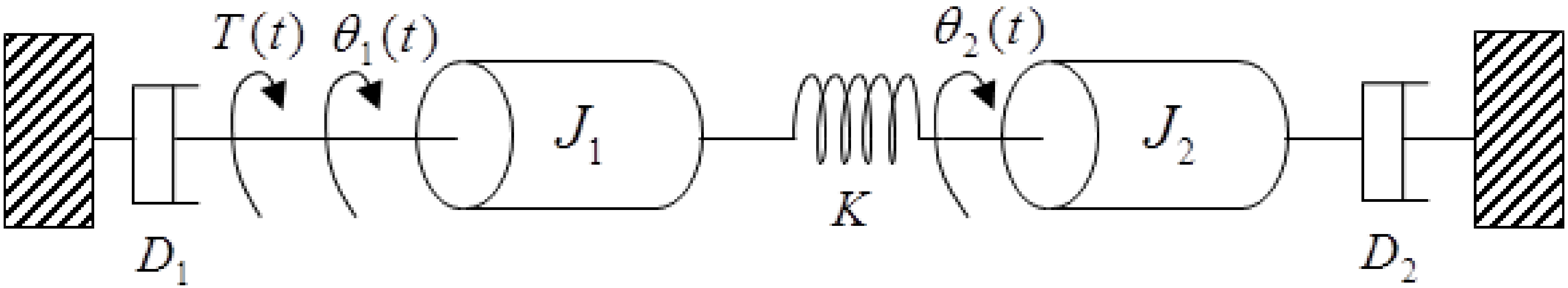
Find the transfer function $\theta_2(s) / T(s)$.

The rod is supported by bearing and at either end is undergoing torsion. A torque is applied at the left and the displacement is measured at the right.



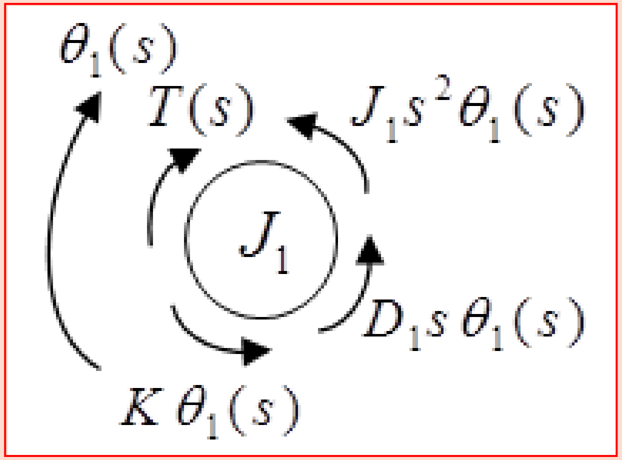
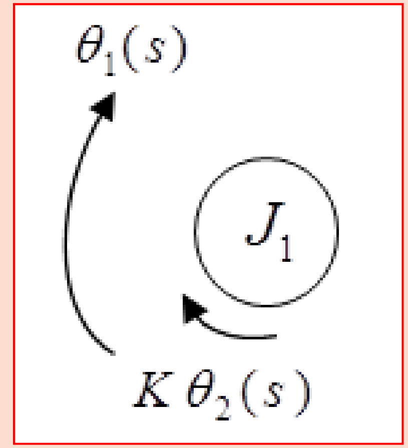
Solution 8:

- Obtain the schematic from the physical system
- Assume:
  - The torsion acts like a spring, concentrated at one particular point in the rod
  - Inertia J_1 to the left and J_2 to the right
  - The damping inside the flexible shaft is negligible

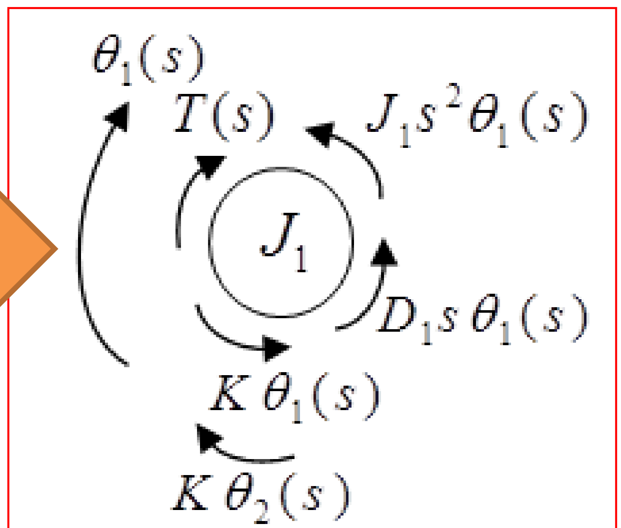


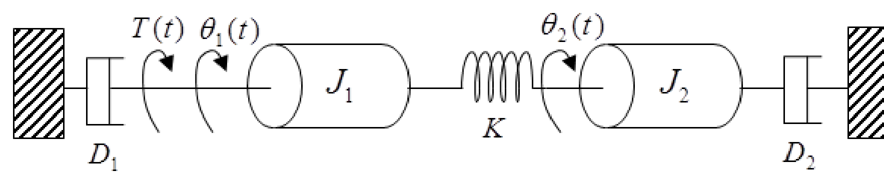
$$J_1 \frac{d^2\theta_1}{dt^2} + D_1 \frac{d\theta_1}{dt} + K(\theta_1 - \theta_2) = T(t) \qquad (1)$$

$$J_2 \frac{d^2\theta_2}{dt^2} + D_2 \frac{d\theta_2}{dt} + K(\theta_2 - \theta_1) = 0 \qquad (2)$$

Torques on J1 if J2 held still	Torques on J1 when J2 is in motion
	

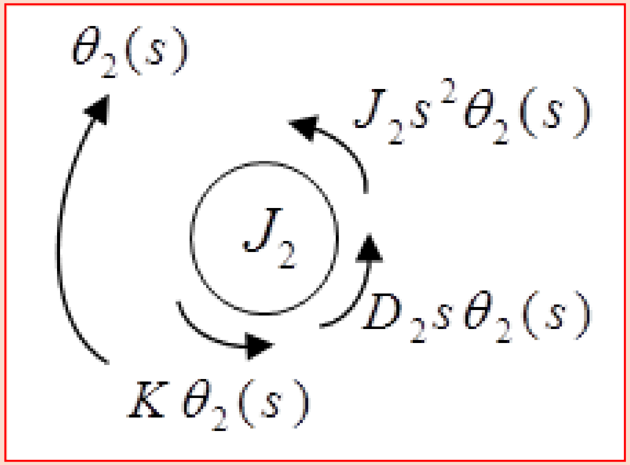
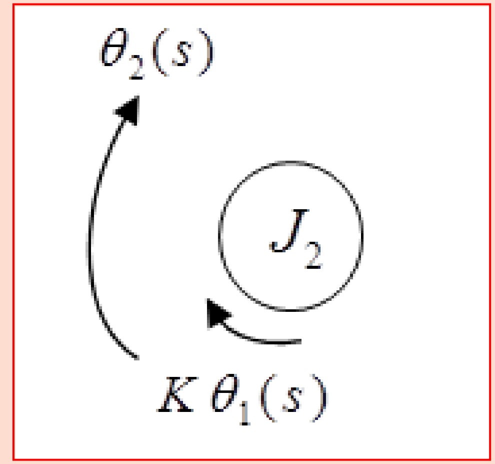
Final free body diagram for J_1



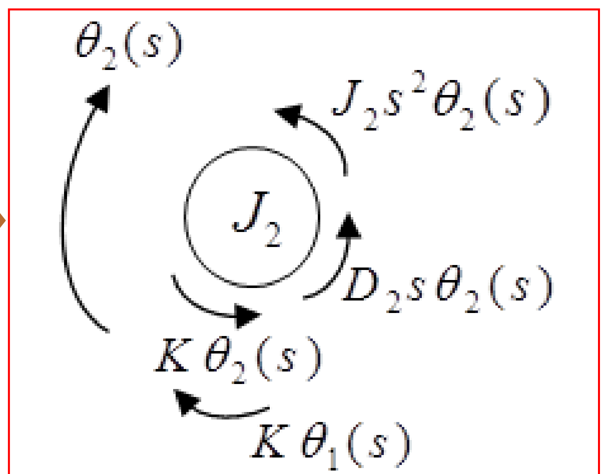


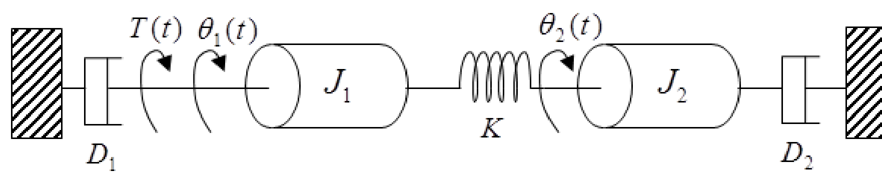
Equation of motion

$$(J_1 s^2 + D_1 s + K)\theta_1(s) - K\theta_2(s) = T(s)$$

Torques on J2 if J2 in motion	Torques on J2 if J1 in motion
	

Final free body diagram for J_2





Equation of motion

$$-K\theta_1(s) + (J_2 s^2 + D_2 s + K)\,\theta_2(s) = 0$$

- Equations of motions:

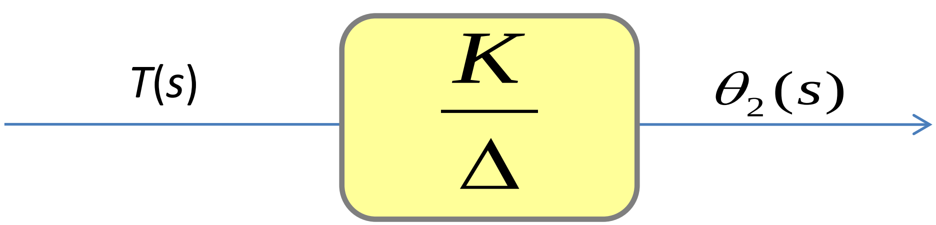
$$(J_1 s^2 + D_1 s + K)\theta_1(s) - K\theta_2(s) = T(s)$$
$$-K\theta_1(s) + (J_2 s^2 + D_2 s + K) = 0$$

- Hence giving the transfer function

$$\frac{\theta_2(s)}{T(s)} = \frac{K}{\Delta}$$

$$where \quad \Delta = \begin{vmatrix} (J_1 s^2 + D_1 s + K) & -K \\ -K & (J_2 s^2 + D_2 s + K) \end{vmatrix}$$

- giving the block diagram



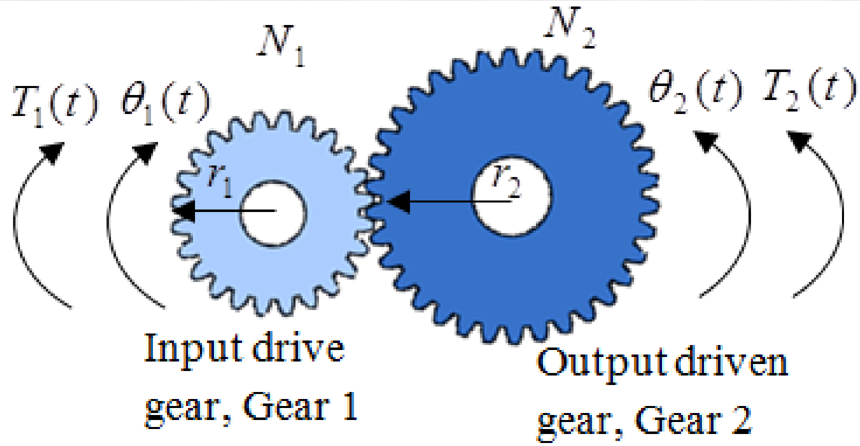
Generic equations – Two Rotational Body System

$\begin{bmatrix}\text{Sum of impedances connected to the motion at } \theta_1\end{bmatrix}\theta_1(s)$	$-\begin{bmatrix}\text{Sum of impedances between } \theta_1 \text{ and } \theta_2\end{bmatrix}\theta_2(s)$	$=$	$\begin{bmatrix}\text{Sum of applied torques at } \theta_1\end{bmatrix}$
$-\begin{bmatrix}\text{Sum of impedances between } \theta_1 \text{ and } \theta_2\end{bmatrix}\theta_1(s)$	$+\begin{bmatrix}\text{Sum of impedances connected to the motion at } \theta_2\end{bmatrix}\theta_2(s)$	$=$	$\begin{bmatrix}\text{Sum of applied torques at } \theta_2\end{bmatrix}$

2.3.3 Rotational with gears

- Gears
  - Used with rotational systems (esp. those driven by motors).
  - Match driving systems with loads.
  - E.g. Bicycles with gearing systems
    - **Uphill**: shift gear for more torque & less speed
    - **Level road**: shift gear for more speed & less torque

Input gear with radius r_1 and N_1 teeth rotated through angle $\theta_1(t)$ due to torque $T_1(t)$

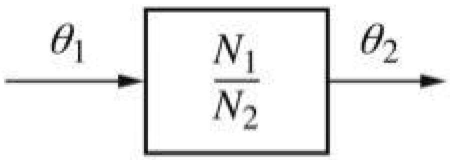


Output gear with radius r_2 and N_2 teeth responds through angle $\theta_2(t)$ and delivering a torque $T_2(t)$

$$\text{ratio number of teeth} \propto \text{ratio of radius} \qquad \frac{N_1}{N_2} = \frac{r_1}{r_2}$$

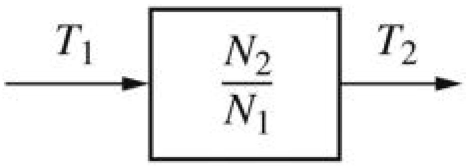
$$\text{ratio angular disp} \propto \frac{1}{\text{ratio of number of teeth}} \qquad \frac{\theta_1}{\theta_2} = \frac{N_2}{N_1}$$

$$\Rightarrow \quad \frac{\theta_2}{\theta_1} = \frac{r_1}{r_2} = \frac{N_1}{N_2}$$

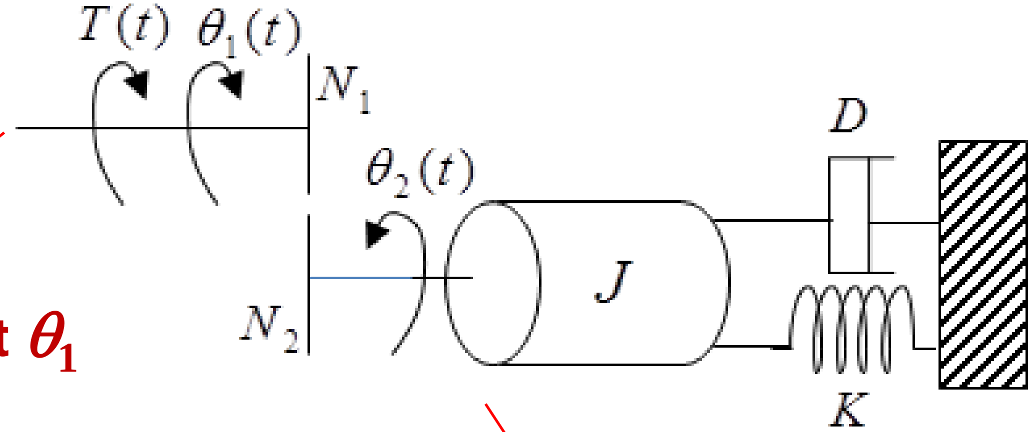


Just like translational motion, energy = force x disp.

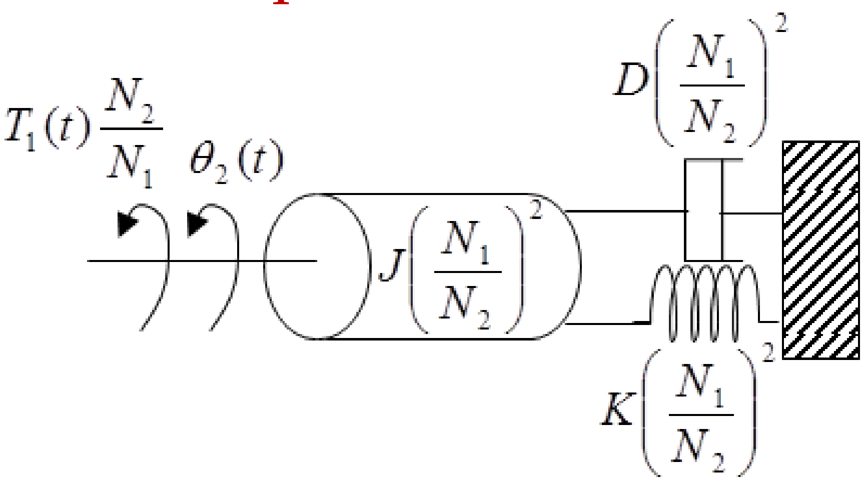
$$T_1\theta_1 = T_2\theta_2 \quad \Rightarrow \quad \frac{T_2}{T_1} = \frac{\theta_1}{\theta_2} = \frac{N_2}{N_1}$$



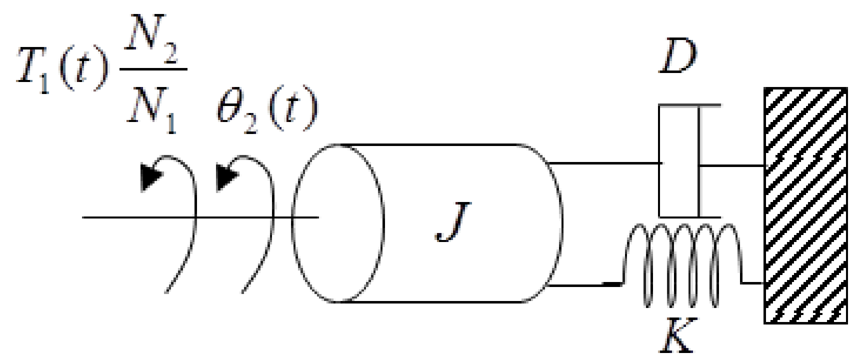
$$\frac{\theta_2}{\theta_1} = \frac{N_1}{N_2}$$



Equivalent system at θ_1 at input N_1



Equivalent system at θ_2 at output N_2



$$T_1 = J\left(\frac{N_1}{N_2}\right)^2 s^2\theta_1(s) + D\left(\frac{N_1}{N_2}\right)^2 s\theta_1(s) + K\left(\frac{N_1}{N_2}\right)^2 \theta_1(s)$$

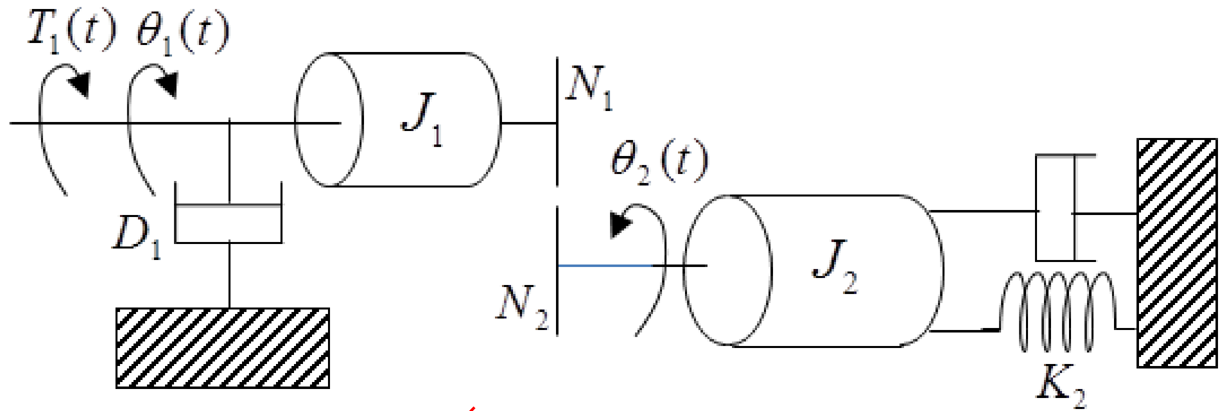
$$T_1\frac{N_2}{N_1} = Js^2\theta_2(s) + Ds\theta_2(s) + K\theta_2(s)$$

- Generalizing the result →
  - We can say that the rotational mechanical impedances can be reflected through gear trains by multiplying the mechanical impedance by the ratio

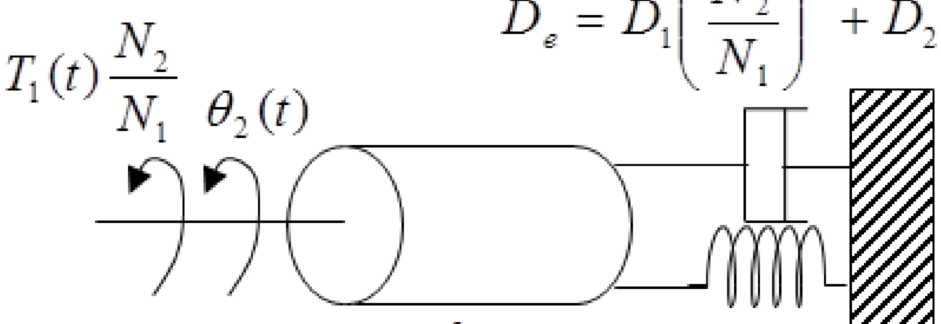
$$\left(\frac{Number\ of\ teeth\ of\ gear\ on\ destination\ shaft}{Number\ of\ teeth\ of\ gear\ on\ source\ shaft} \right)^2$$

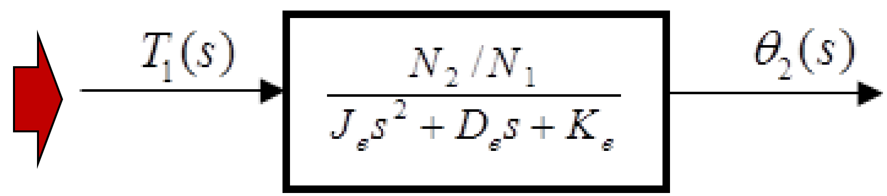
Example 9 and solution:

Find the transfer function $\theta_2(s)/T_1(s)$



$$D_e = D_1\left(\frac{N_2}{N_1}\right)^2 + D_2$$



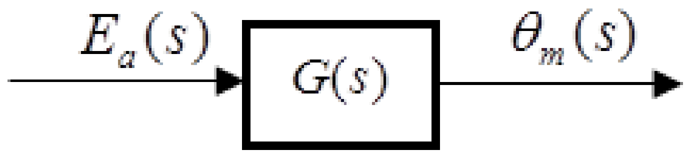


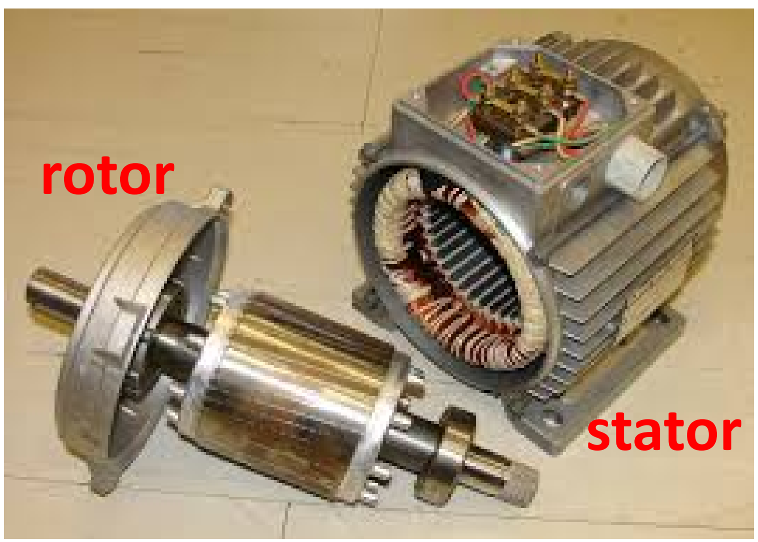
$$J_e = J_1\left(\frac{N_2}{N_1}\right)^2 + J_2$$

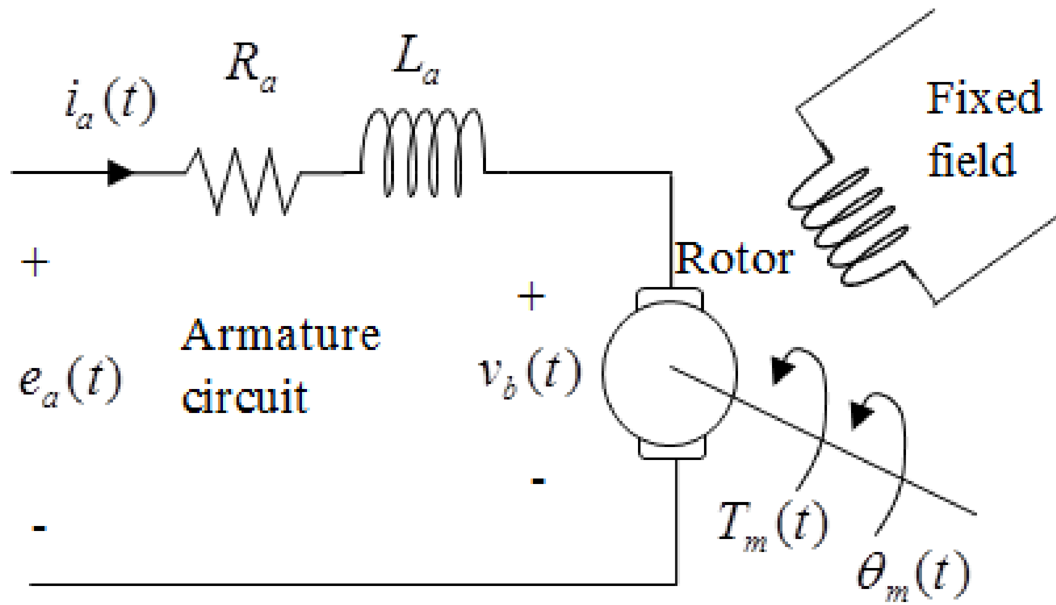
2.4 Modeling of Electromechanical System

Electromechanical System

- Electromechanical system: **electrical + mechanical** components that generates a mechanical output by an electrical input (motor)



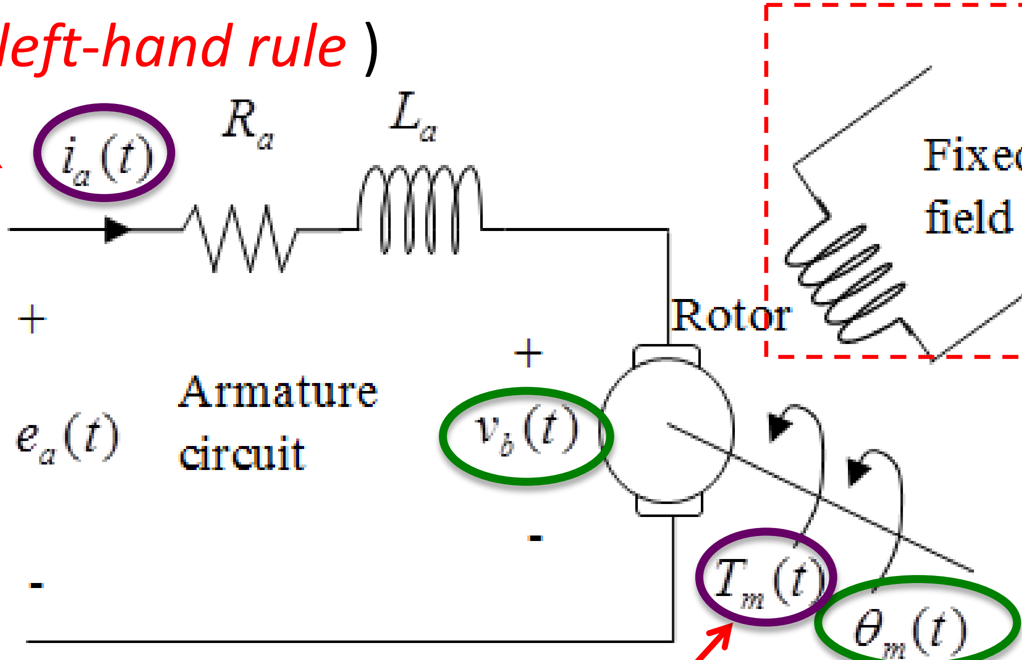


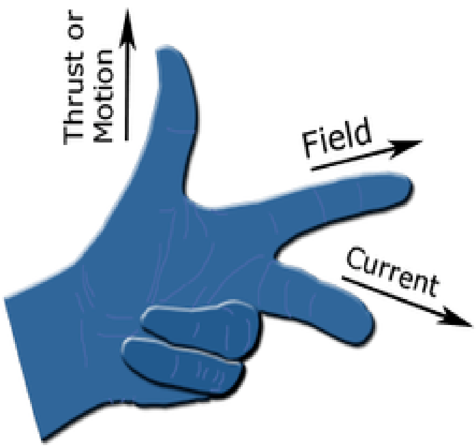


Schematic diagram

Develop magnetic field *B* by stationary permanent magnet

Armature circuit with $i_a(t)$, passes through magnetic field and produces a force *F*- (*Fleming's left-hand rule*)





The resulting torque turns the rotor (rotating member of motor)

Since the current-carrying armature is rotating in a magnetic field, its voltage v_b (back electromagnetic force emf) is proportional to angular valocity.

$$v_b(t) = K_b \frac{d\theta_m(t)}{dt} \qquad (1)$$

Taking the *L.T*:

$$\boxed{V_b(s) = K_b s\theta_m(s)} \qquad (2)$$

KVL around the armature circuit

$$\boxed{R_a I_a(s) + L_a s I_a(s) + V_b(s) = E_a(s)} \qquad (3)$$

Torque developed by the motor (T_m) is proportional to the armature current (i_a).

$$\boxed{T_m(s) = K_t I_a(s)} \quad \textbf{or} \quad I_a(s) = \frac{1}{K_t} T_m(s) \qquad (4)$$

Substitute (2) and (4) into (3)

$$\frac{(R_a + L_a s)T_m(s)}{K_t} + K_b s\theta_m(s) = E_a(s) \qquad (5)$$

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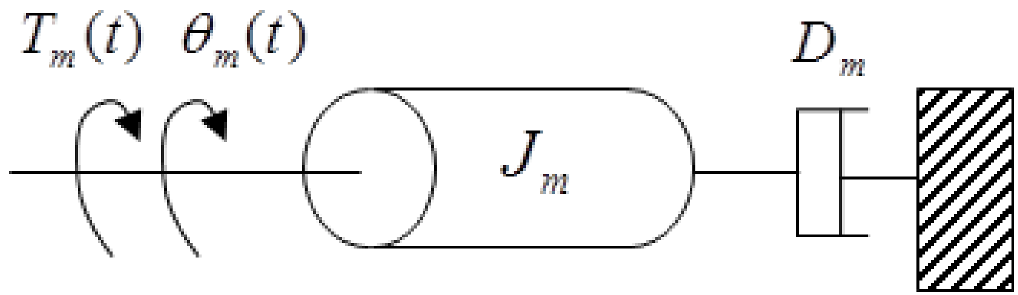
Equivalent mechanical loading on motor



$$T_m(s) = (J_m s^2 + D_m s)\theta_m(s) \quad (6)$$

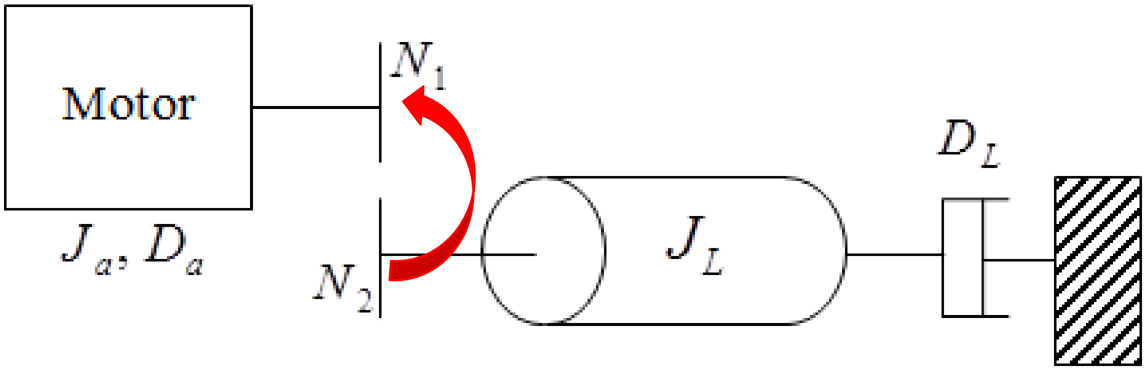
Substitute (6) into (5) yields

$$\frac{(R_a + L_a s)(J_m s^2 + D_m s)\theta_m(s)}{K_t} + K_b s\theta_m(s) = E_a(s) \quad (7)$$

Assume La <<<Ra , then (7) becomes

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_t / R_a J_m}{s\left(s + \frac{1}{J_m}\left(D_m + \frac{K_b K_t}{R_a}\right)\right)} \Rightarrow \frac{K}{s(s+\alpha)} \quad (8)$$

DC motor driving a rotational mechanical load



$$J_m(s) = J_a + J_L\left(\frac{N_1}{N_2}\right)^2 \quad ; \quad D_m(s) = D_a + D_L\left(\frac{N_1}{N_2}\right)^2$$

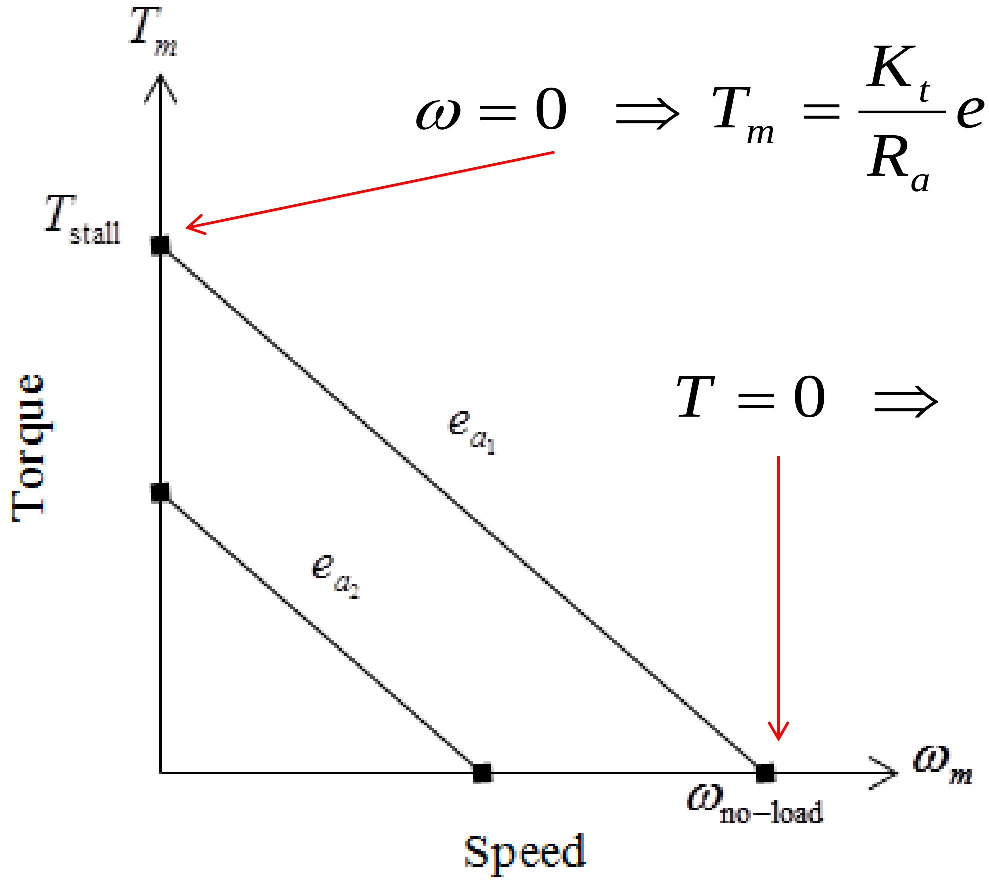
From (5), with $L_a = 0$

$$\frac{R_a T_m(s)}{K_t} + K_b s\theta_m(s) = E_a(s)$$ taking inverse Laplace transform

$$\frac{R_a T_m(t)}{K_t} + K_b \frac{d\theta_m(t)}{dt} = e_a(t)$$

$$T_m(t) = -\frac{K_b K_t}{R_a}\frac{d\theta_m(t)}{dt} + \frac{K_t}{R_a}e_a(t) = -\frac{K_b K_t}{R_a}\omega_m + \frac{K_t}{R_a}e_a(t)$$

At steady state: $T_m = -\frac{K_b K_t}{R_a}\omega_m + \frac{K_t}{R_a}e_a$



$$\omega = 0 \Rightarrow T_m = \frac{K_t}{R_a}e_a$$

$$T = 0 \Rightarrow \omega = \frac{e_a}{K_b}$$

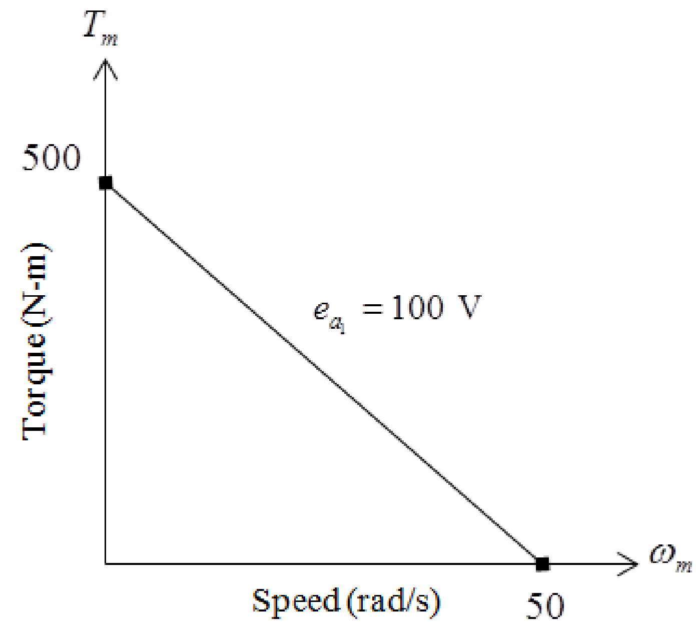
Electrical constants

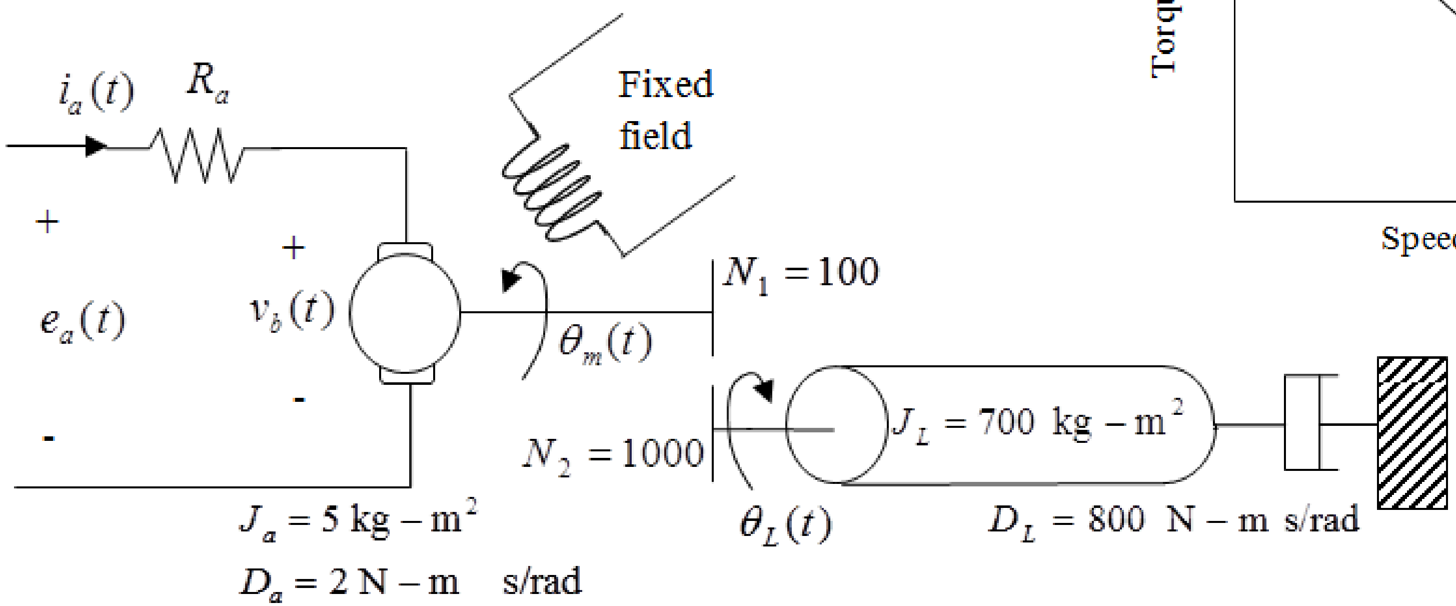
$$\frac{K_t}{R_a} = \frac{T_{stall}}{e_a}$$

$$K_b = \frac{e_a}{\omega}$$

Example 10:

Find $\frac{\theta_L(s)}{E_a(s)}$





Answer: $\frac{\theta_L(s)}{E_a(s)} = \frac{0.0417}{s(s+1.667)}$

Solution 10:

1) Find Mechanical Constants J_m and D_m

a. **Total Inertia** at the armature

$$J_m = J_a + J_L\left(\frac{N_1}{N_2}\right)^2 = 5 + 700\left(\frac{1}{10}\right)^2 = 12 \qquad (1)$$

b. **Total damping** at the armature

$$D_m = D_a + D_L\left(\frac{N_1}{N_2}\right)^2 = 2 + 800\left(\frac{1}{10}\right)^2 = 10 \qquad (2)$$

BY NC SA

2) Find the electrical constants $\frac{K_t}{R_a}$ and K_b

From the Torque - speed curve

$$T_{stall} = 500$$

$$\omega_{no-load} = 50$$

$$e_a = 100$$

$\therefore$ Electrical constants

$$\frac{K_t}{R_a} = \frac{T_{stall}}{e_a} = \frac{500}{100} = 5 \qquad (3)$$

$$K_b = \frac{e_a}{\omega_{no-load}} = \frac{100}{50} = 2 \qquad (4)$$

3) Substitute (1), (2), (3) and (4) into :

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_\tau/(R_a J_m)}{s\left[s + \frac{1}{J_m}(D_m + \frac{K_\tau K_b}{R_a})\right]} = \frac{0.417}{s(s+1.667)}$$

4) Find $\frac{\theta_L(s)}{E_a(s)}$ by establishing the ratio of $\frac{\theta_m(s)}{\theta_L(s)}$

$$\frac{\theta_m(s)}{\theta_L(s)} = \frac{N_2}{N_1} = \frac{1000}{100} = 10$$

$$i.e. \quad \theta_m(s) = 10\ \theta_L(s)$$

$$\therefore \frac{\theta_L(s)}{E_a(s)} = \frac{0.417/10}{s(s+1.667)} = \frac{0.0417}{s(s+1.667)}$$

END OF CHAPTER 2

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