SCR 1013 : Digital Logic

Module 2:

NUMBER SYSTEM & CODES

Module 2 Content

- Number system
  - Decimal Number
  - Binary Number
  - Hexadecimal Numbers
  - Octal Number
- Conversion between Number system
- Digital Codes (BCD, Parity, Gray, ASCII)
- Number representations in Digital System
- Arithmetic operations in Digital System

Numbering System

- A number system is the set of symbols used to express quantities as the basis for counting, determining order, comparing amounts, performing calculations, and representing value.
- Examples include the Arabic, Babylonian, Chinese, Egyptian, Greek, Mayan, and Roman number systems.
- Any positive integer B ($B > 1$) can be chosen as the base or radix of a numbering system.
- If base is B, then B digits ($0, 1, 2, \ldots, B - 1$) are used.

Decimal Number

- The **decimal numbering** system has 10 digits **0** through **9**.
- The decimal numbering system has a base of 10 with each position **weighted** by a **factor** of **10**

$$\ldots 10^5\ 10^4\ 10^3\ 10^2\ 10^1\ 10^0.\ 10^{-1} 10^{-2}\ 10^{-3}\ 10^{-4}\ 10^{-5} \ldots$$

- Example:

  Express decimal 47 as a sum of the values of each digit.

  $47_{10} = (4 \text{ x } 10^1) + (7 \text{ x } 10^0) = 40 + 7$

  $= 47$

Binary Number

- The binary numbering system has 2 digits 0 and 1.
- The binary numbering system has a base of 2 with each position weighted by a factor of 2.
- Example: 0, 1, 10, 11, 100, 101, 110, 111, 1000, …

1	0	1	1 .	0	1	binary number
2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	place values

Octal Number

- Uses base 8.
- Includes only the digits 0 through 7.
- Based on the binary system with a 3-bit boundary.
- The binary numbering system has a base of 8 with each position **weighted** by a **factor** of **8**.
- Example: 0, 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 20, …

4	5	3	6	0 .	7	2	octal number
8^4	8^3	8^2	8^1	8^0	8^{-1}	8^{-2}	place values

Hexadecimal Number System

- Allows for convenient handling of long binary strings.
- Base 16
  - 16 possible symbols
  - 0-9 and A-F
  - $(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F)_{16}$
    $(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)_{10}$
  - 0, 1, …, F, 10, 11, 12, …, 1F, 20, 21, …

9	C	F	3	.	A	1	octal number
16^3	16^2	16^1	16^0		16^{-1}	16^{-2}	place values

Hex, Octal, Binary and Decimal Numbering System

Hexadecimal	Octal	Binary	Decimal
0	0	0000	0
1	1	0001	1
2	2	0010	2
3	3	0011	3
4	4	0100	4
5	5	0101	5
6	6	0110	6
7	7	0111	7
8	10	1000	8
9	11	1001	9
A	12	1010	10
B	13	1011	11
C	14	1100	12
D	15	1101	13
E	16	1110	14
F	17	1111	15

CODES IN DIGITAL SYSTEMS

What are codes?

- Code is a rule for converting a piece of information into another form or representation, not necessarily of the same type.
- One reason for coding is to enable communication in places where ordinary spoken or written language is difficult or impossible.

Binary Coded Decimal (BCD)

- BCD is a way to express each of the decimal digits with a binary code.
- There are only 10 code groups in the BCD system, one for every digit (0000 – 1001)
- Invalid codes are 1010, 1011, 1100, 1101, 1110 and 1111
- Coded values for decimal digit:

Decimal	BCD	Decimal	BCD
0	0000	5	0101
1	0001	6	0110
2	0010	7	0111
3	0011	8	1000
4	0100	9	1001

Gray Codes

- Designed to prevent false output from electromechanical switches.
- Are widely used to facilitate error correction in digital communications such as digital terrestrial television and some cable TV systems.
- In modern digital communications, Gray codes play an important role in error correction.
- It is arranged so that every transition from one value to the next value involves only one bit change.
- Sometimes referred to as reflected binary, because the first eight values compare with those of the last 8 values, but in reverse order.

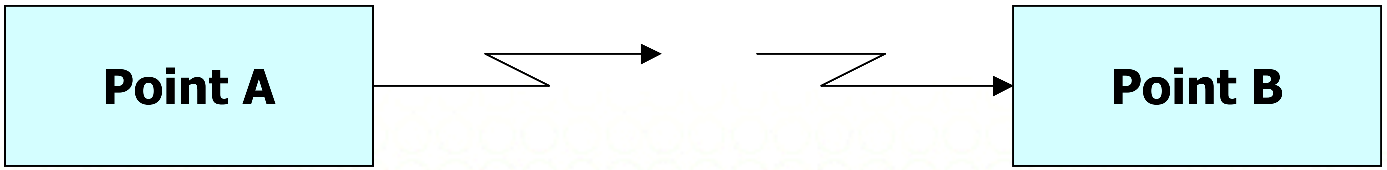
Parity Code

- Parity bit used for bit error detection
  - Even parity – total number of 1s even
  - Odd parity – total number of 1s odd
- Parity bit is append to the code at the leftmost position (MSB).
- Example:

Code	Number of 1s	Even/Odd	Even Parity	Odd Parity
110010	3	Odd	**1** 110010	**0** 110010
101110	4	Even	**0** 101110	**1** 101110
101000	2	Even	**0** 101000	**1** 101000
110111	5	Odd	**1** 110111	**0** 110111

Error Detection by Parity Checking

- Assume that data = 0101
- It uses even parity.
- Therefore the appended parity bit is 0.
- The data with parity bit: 0 0101
- The data is transmitted.
- The data is received as 00001 → odd no. of 1, not even!!



Data: 0 0101
(even parity)

Data: 0 0001
(even parity)
but does not conform
to even parity

American Standard Code for Information Interchange (ASCII)

- It has 128 characters and symbols represented in 7-bit binary code
- Example :
- $A = 1000001_2$
- $a = 1100001_2$
- A parity bit is added so that the total number of bits is 8 → a byte.

NUMBER REPRESENTATION IN DIGITAL SYSTEM

- Unsigned Number representation
- Signed Number representations

Integer Representation

- Numbers can be represented as a combination of a value, or magnitude and sign, plus or minus
- Unsigned integer
- Signed integer

Unsigned Integer Representation

- By unsigned integer, it is mean no negative values.
  - E.g. 0, 1, 2, ..., 254, 255, 256, 257, ... 65535, 65536, 65537, ..., 2000000000, 2000000001, ...
- A **bit** can store unsigned integers from 0 to 1 .
- A **byte** of 8 bits can store unsigned integers from 0 to 255 = $2^8 - 1$.
- In binary arithmetic, if the length of the number is restricted to 8 digits (0s and 1s), the largest value is $1111\ 1111_2$ = 255, and the smallest is 0.
- A **word** of 16 bits can store unsigned integers from 0 to 65535 = $2^{16} - 1$.
- In binary arithmetic, if the length of the number is restricted to 16 digits (0s and 1s), the largest value is $1111\ 1111\ 1111\ 1111_2$ = 65535, and the smallest is 0.

Signed Numbers

- Integers that do not have a sign indication are considered as positive numbers and they are referred to as unsigned numbers.
  - 01000, 11101
- However, integers can be positive and negative.
  - +01000, +11101, -10001, -0111001
  - Need for a code to represent '-' and '+'.
- Positive and negative integers use a code system to indicate the sign.
  - Signed bit: **0 (+ve)** or **1 (-ve)** positioned at MSB
  - Positive numbers → 0 01000, 0 11101
  - Negative numbers → 1 10101, 1 0101001
  - This is referred as signed numbers.

Signed Numbers Representation

- Three representation:
    - Sign and magnitude (simple representation)
    - 1's complement
    - 2's complement

Arithmetic Operations

- Integer Numbers
  - Unsigned Numbers
  - Signed Numbers
- Addition
- Subtraction

Arithmetic Operation: Addition

A	B	A + B
0	0	0
0	1	1
1	0	1
1	1	10

- Example:

	1	0	0	1	0
+	0	1	1	0	0
	1	1	1	1	0

	1	1			
	1	0	1	1	0
+	0	1	1	0	0
1	0	0	0	1	0

Arithmetic Operation: Subtraction

- In digital system, subtraction is performed by using 2's complement and addition.
- Carry from the MSB (signed bit) is deleted.
- Example:

010011 – 001111 = 010011 + (– 001111)
= 010011 + (110001)
= 000100

	1			1	1	
	0	1	0	0	1	1
+	1	1	0	0	0	1
~~1~~	0	0	0	1	0	0