

Chap 6: Identification of ARIMA models

OPENCOURSEWARE

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Chap 6: Identification of ARIMA models

Outline:

- Theoretically of ACF and PACF for MA(1)
- Sample ACF and PACF for MA(1)
- Theoretically of ACF and PACF for MA(2)
- Sample ACF and PACF for MA(2)
- Theoretically of ACF and PACF for AR(1)
- Sample ACF and PACF for AR(1)
- Theoretically of ACF and PACF for AR(2)
- Sample ACF and PACF for AR(2)
- Theoretically of ACF and PACF for ARMA(1,1)
- Sample ACF and PACF for ARMA(1,1)







Theoretically of ACF and PACF for MA(1)

The first Moving Average Model or MA(1) model $Z_t = \varepsilon_t + \theta \varepsilon_{t-1}$ Invertibility condition: $-1 < \theta < 1$

Theoretical of ACF

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Theoretical of PACF

$$\rho_k = \begin{cases} \frac{-\theta}{1+\theta^2} & \text{for } k=1\\ 0 & \text{others} \end{cases} \qquad \rho_{kk} = \frac{-\theta^k (1-\theta^2)}{1+\theta^{2(k+1)}} \text{ for } k=1,2,3, \end{cases}$$





Theoretical of ACF and PACF for MA(1)





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Sample ACF and PACF for MA(1)









Theoretical of ACF and PACF for MA(2)

The second order Moving Average or MA(2) model

$$Z_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$$

Invertibility condition:

$$\theta_1 + \theta_2 < 1; \theta_2 - \theta_1 < 1; |\theta_2| < 1$$

Theoretical of ACF

$$\rho_{k} = \begin{cases} \frac{-\theta_{1}(1-\theta_{2})}{1+\theta_{1}^{2}+\theta_{2}^{2}} &, k = 1 \\ \frac{-\theta_{2}}{1+\theta_{1}^{2}+\theta_{2}^{2}} &, k = 2 \\ 0 &, k > 2 \end{cases}$$

Theoretical of PACF

Dies down (according to a mixture of damped Exponentials and/or damped sine waves





Theoretical of ACF and PACF for MA(2)





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Sample ACF and PACF for MA(2)



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Sample ACF and PACF for MA(2)







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Theoretically of ACF and PACF for AR(1)

The first Autoregressive Model or AR(1) model

$$Z_t = \phi_1 Z_{t-1} + \varepsilon_t$$

Stationarity condition:

$$-1 < \phi_1 < 1$$

Theoretical of ACF

Theoretical of PACF

$$\rho_k = \phi_1^k$$
 for k = 0, 1, 2, ...

$$\rho_{kk} = \begin{cases} \rho_1 \text{ for } k = 1\\ 0 \text{ others} \end{cases}$$





Theoretical of ACF and PACF for AR(1)



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Sample ACF and PACF for AR(1)









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Theoretical of ACF and PACF for AR(2)

The second order Autoregressive or AR(2) model

$$Z_t = \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + \varepsilon_t$$

Invertibility condition:

$$\phi_1 + \phi_2 < 1; \phi_2 - \phi_1 < 1; |\phi_2| < 1$$

Theoretical of ACF

Theoretical of PACF

$$\rho_{k} = \begin{cases} \frac{\phi_{1}}{1 - \phi_{2}} & , k = 1 \\ \phi_{1} \rho_{k-1} + \phi_{2} \rho_{k-2} & , k = 2, 3, \dots \end{cases} \qquad \rho_{kk} = \begin{cases} \rho_{1} & , k = 1 \\ \phi_{2} & , k = 2 \\ 0 & , k = 3, 4, 5, \dots \end{cases}$$





Theoretical of ACF and PACF for AR(2)





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Theoretical of ACF and PACF for AR(2)



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Sample ACF and PACF for AR(2)



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ACF [-0.5,0.3]



Sample ACF and PACF for AR(2)



PACF [-0.5,0.3]



ACF [0.5,-0.3]







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Theoretically of ACF and PACF for ARMA(1,1)

The Mixed Autoregressive Moving Average Model or ARMA(1,1) model

$$Z_t = \phi_1 Z_{t-1} + \theta_1 \varepsilon_{t-1} + \varepsilon_t$$

Stationarity and invertibility condition:

$$-1 < \phi_1 < 1$$
; $-1 < \theta_1 < 1$

Theoretical of ACF

$$\rho_{k} = \begin{cases} \frac{(\phi_{1} - \theta_{1})(1 - \phi_{1}\theta_{1})}{1 + \theta_{1}^{2} - 2\phi_{1}\theta_{1}}, & k = 1\\ \phi_{1}\rho_{k-1}, & k \ge 2 \end{cases}$$

Theoretical of PACF

Dies down (according to a mixture of damped Exponentials and/or damped sine waves



Theoretical of ACF and PACF for ARMA(1,1)



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Theoretical of ACF and PACF for ARMA(1,1)



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Sample ACF and PACF for ARMA(1,1)





PACF [0.5,0.3]





PACF [-0.5,-0.3]

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Sample ACF and PACF for ARMA(1,1)



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