Chap 6: Identification of ARIMA models

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Chap 6: Identification of ARIMA models

Outline:

- Theoretically of ACF and PACF for MA(1)
- Sample ACF and PACF for MA(1)
- Theoretically of ACF and PACF for MA(2)
- Sample ACF and PACF for MA(2)
- Theoretically of ACF and PACF for AR(1)
- Sample ACF and PACF for AR(1)
- Theoretically of ACF and PACF for AR(2)
- Sample ACF and PACF for AR(2)
- Theoretically of ACF and PACF for ARMA(1,1)
- Sample ACF and PACF for ARMA(1,1)

Theoretically of ACF and PACF for MA(1)

The first Moving Average Model or MA(1) model

$$Z_t = \varepsilon_t + \theta\varepsilon_{t-1}$$

Invertibility condition:

$$-1 < \theta < 1$$

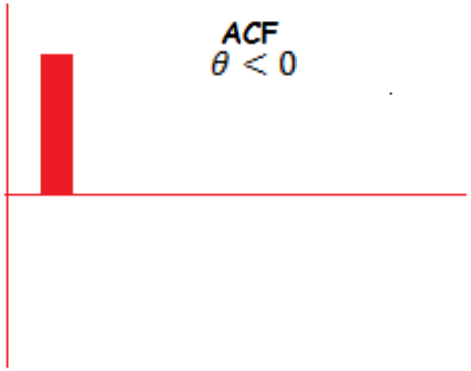
Theoretical of ACF

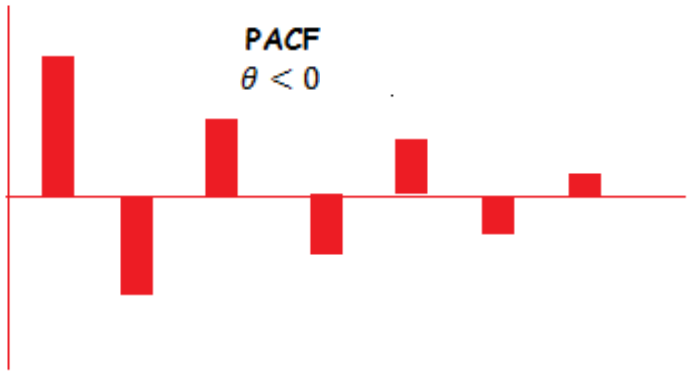
$$\rho_k = \begin{cases} \dfrac{-\theta}{1+\theta^2} & \text{for k} = 1 \\ 0 & \text{others} \end{cases}$$

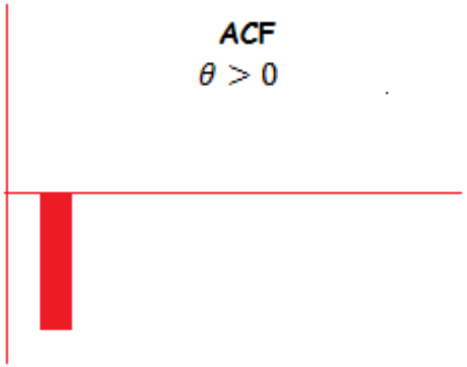
Theoretical of PACF

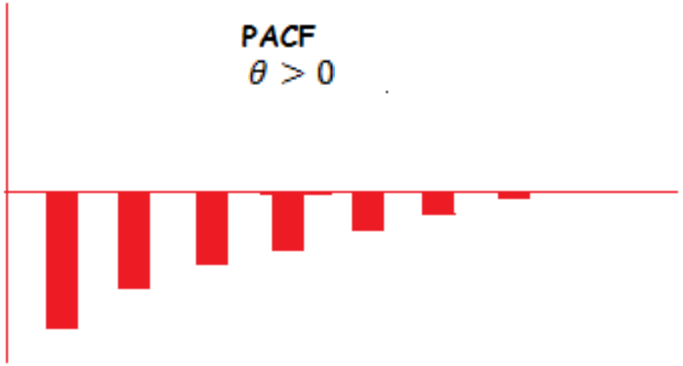
$$\rho_{kk} = \frac{-\theta^k(1-\theta^2)}{1+\theta^{2(k+1)}} \text{ for k} = 1, 2, 3, \ldots$$

Theoretical of ACF and PACF for MA(1)

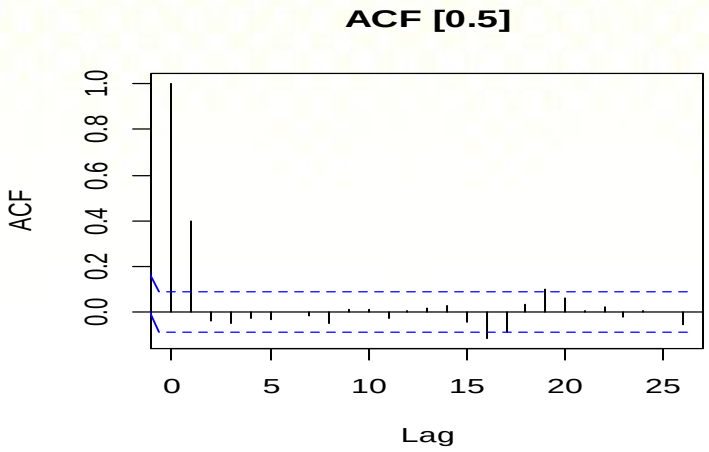


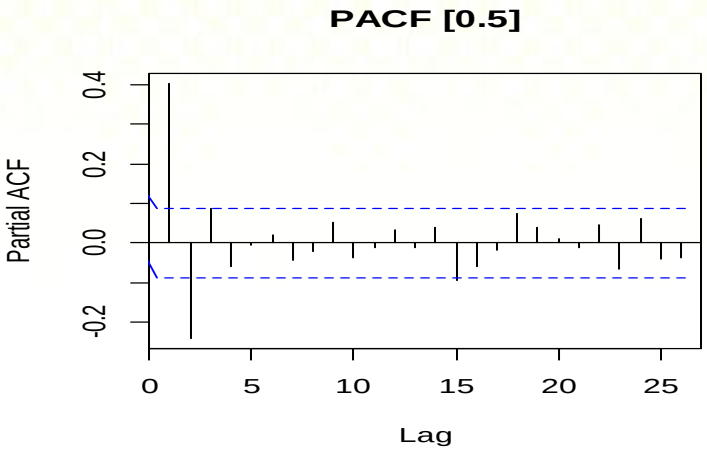


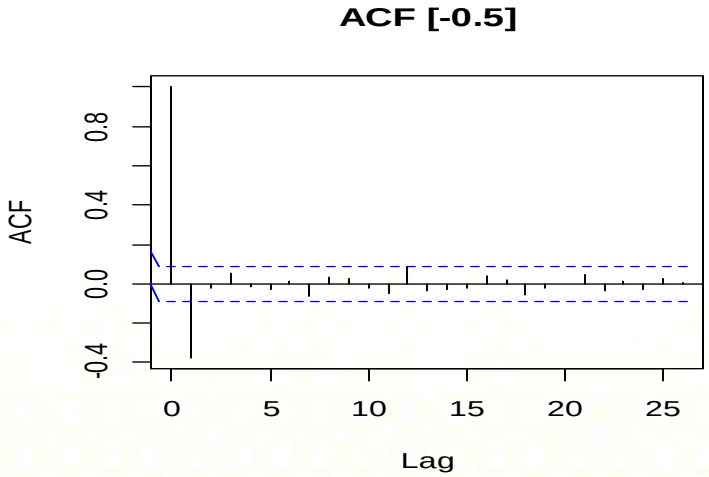


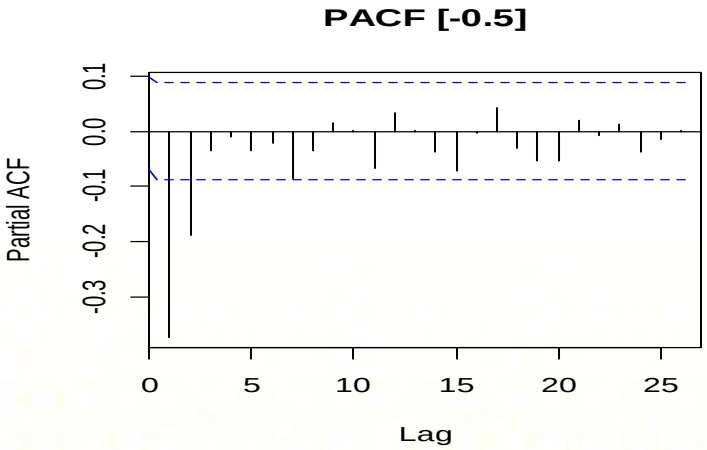


Sample ACF and PACF for MA(1)









Theoretical of ACF and PACF for MA(2)

The second order Moving Average or MA(2) model

$$Z_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$$

Invertibility condition:

$$\theta_1 + \theta_2 < 1; \theta_2 - \theta_1 < 1; |\theta_2| < 1$$

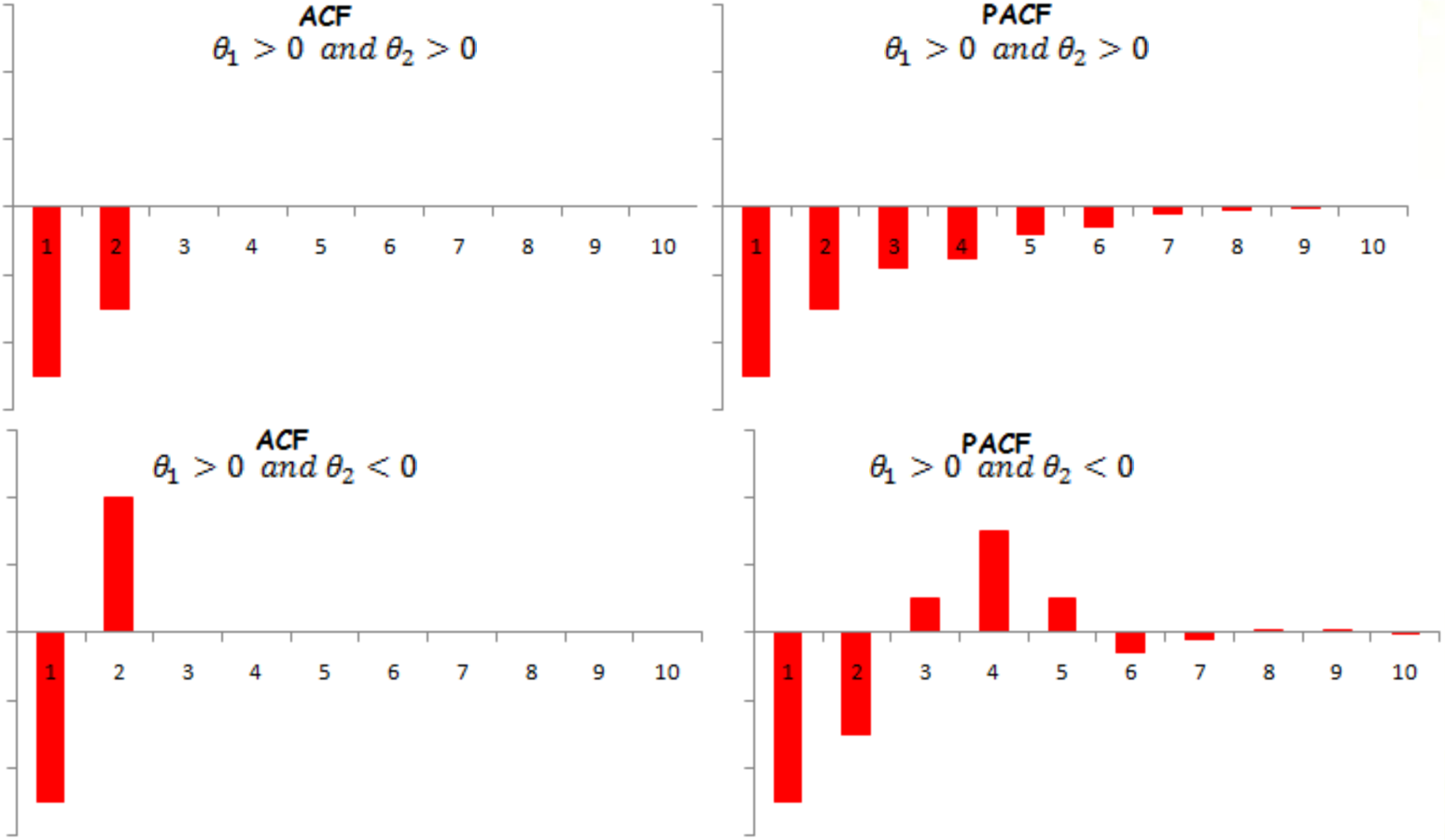
Theoretical of ACF

$$\rho_k = \begin{cases} \dfrac{-\theta_1(1-\theta_2)}{1+\theta_1^2+\theta_2^2} & , k = 1 \\ \dfrac{-\theta_2}{1+\theta_1^2+\theta_2^2} & , k = 2 \\ 0 & , k > 2 \end{cases}$$

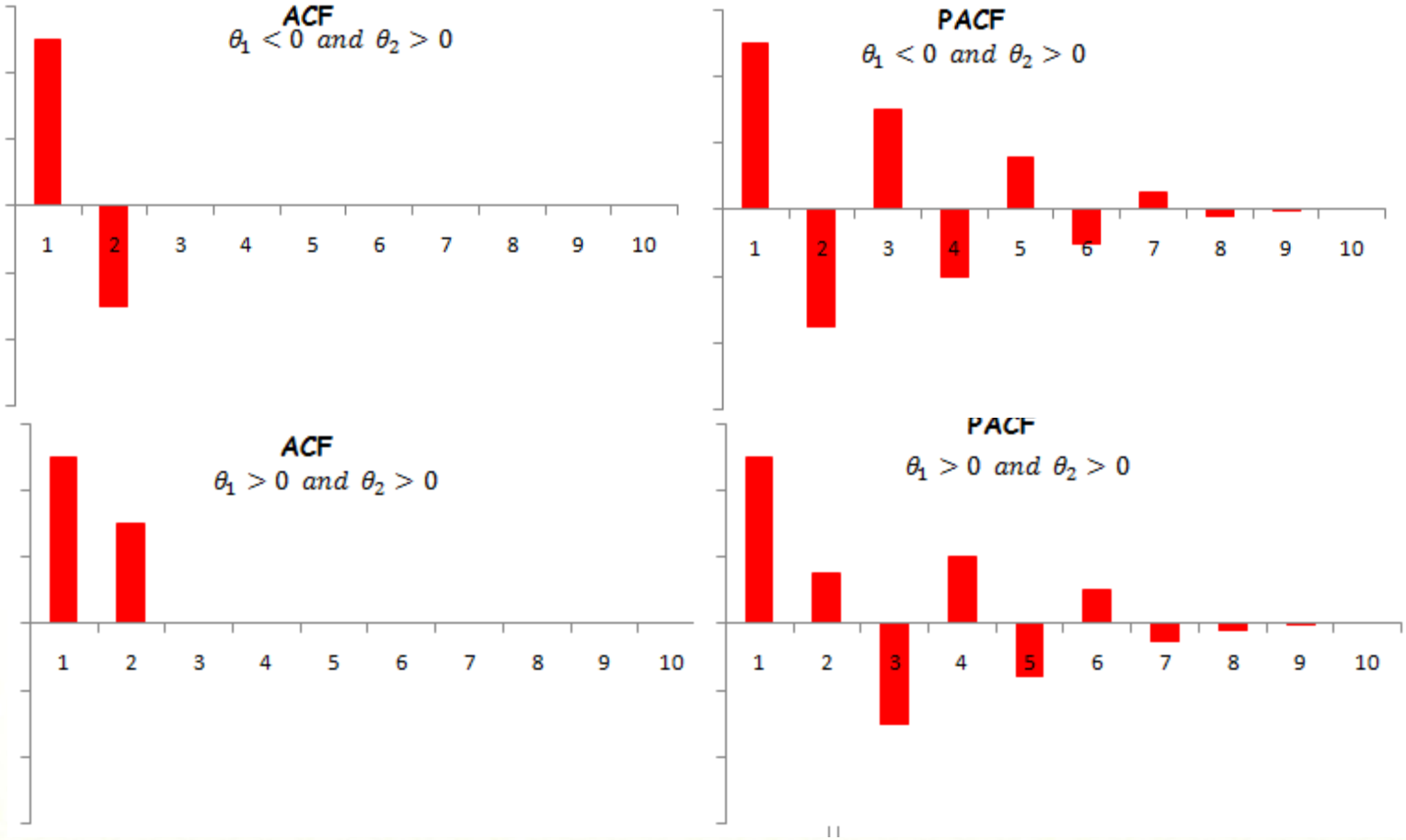
Theoretical of PACF

Dies down (according to a mixture of damped Exponentials and/or damped sine waves

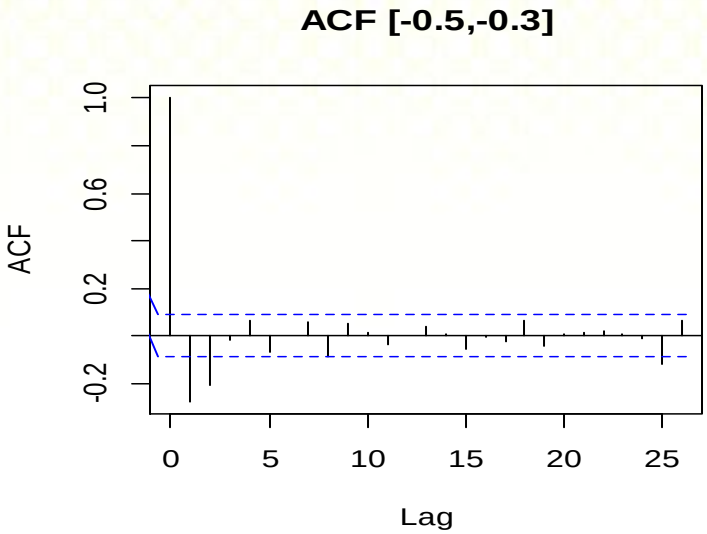
Theoretical of ACF and PACF for MA(2)

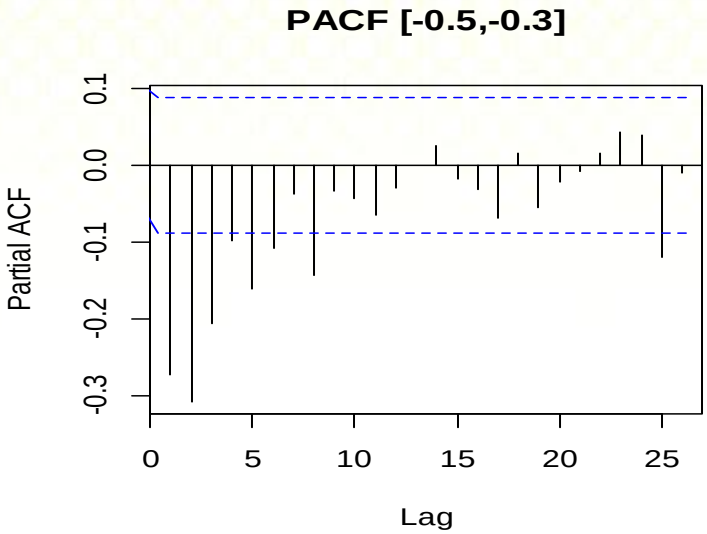


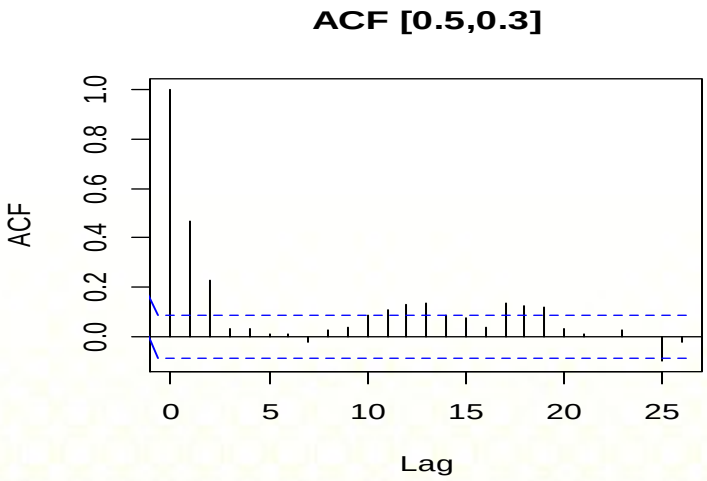
Theoretical of ACF and PACF for MA(2)

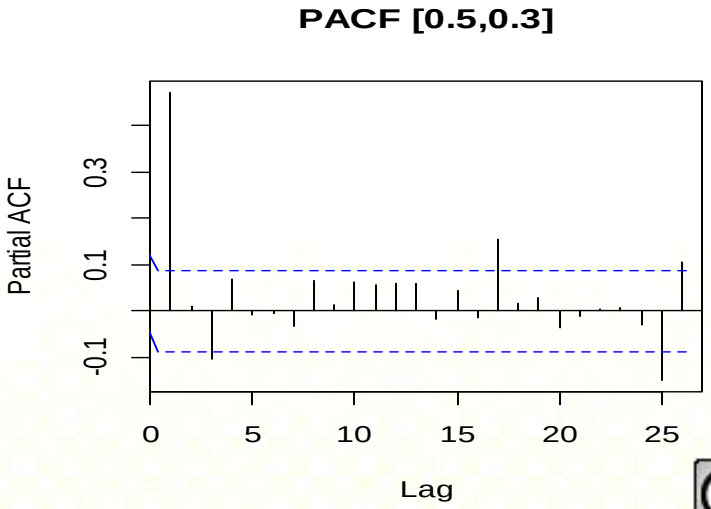


Sample ACF and PACF for MA(2)

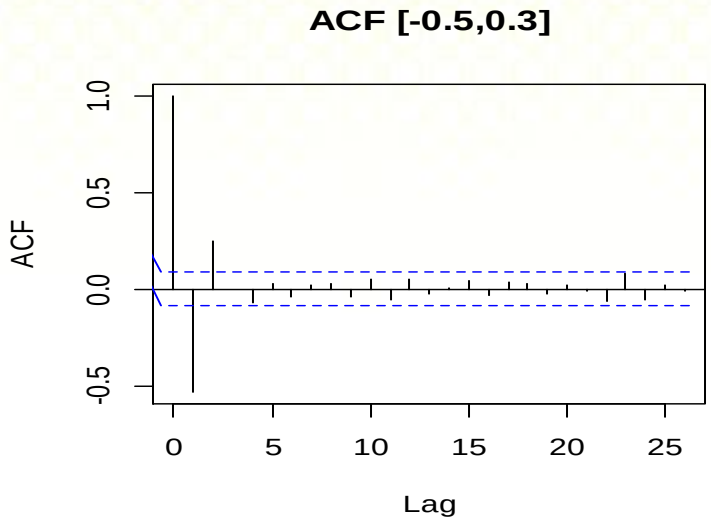


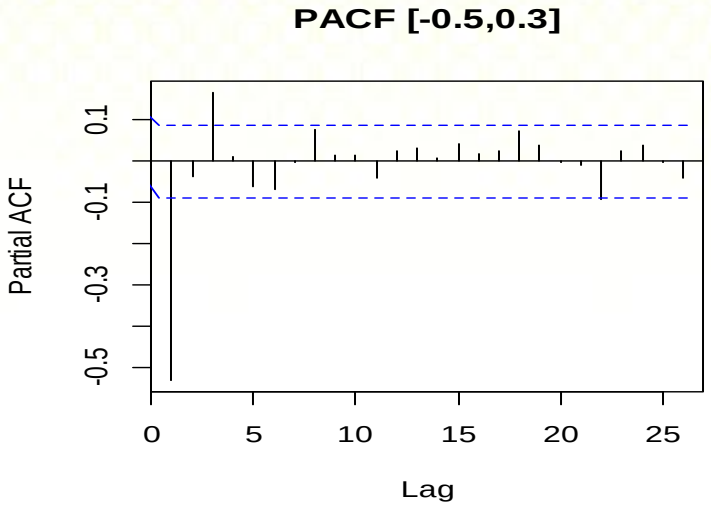


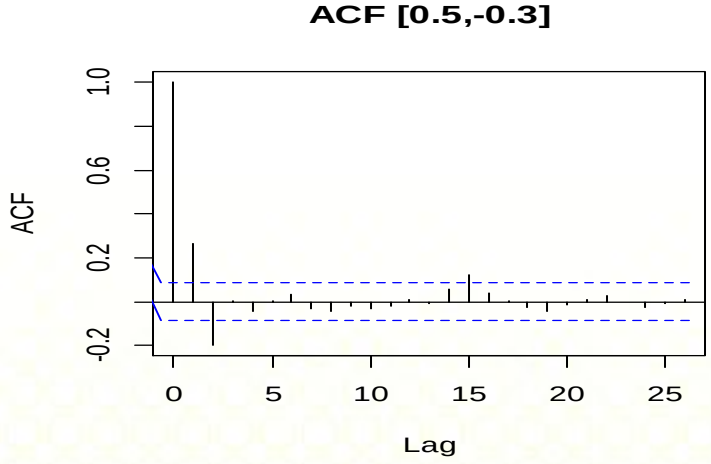


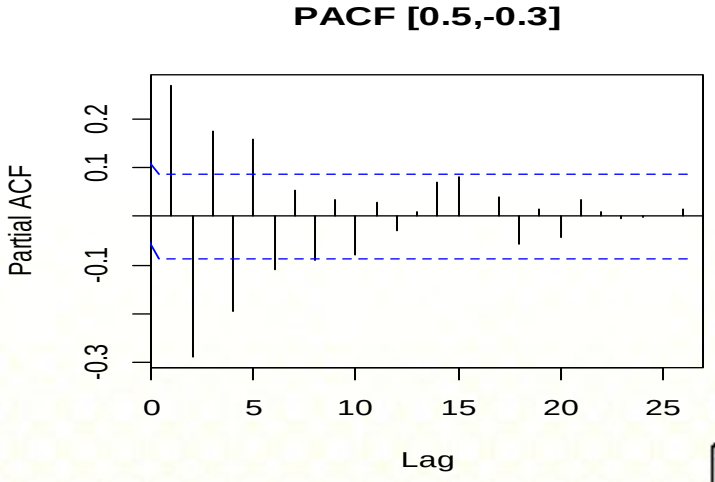


Sample ACF and PACF for MA(2)









Theoretically of ACF and PACF for AR(1)

The **first Autoregressive Model or** AR(1) model

$$Z_t = \phi_1 Z_{t-1} + \varepsilon_t$$

Stationarity condition:

$$-1 < \phi_1 < 1$$

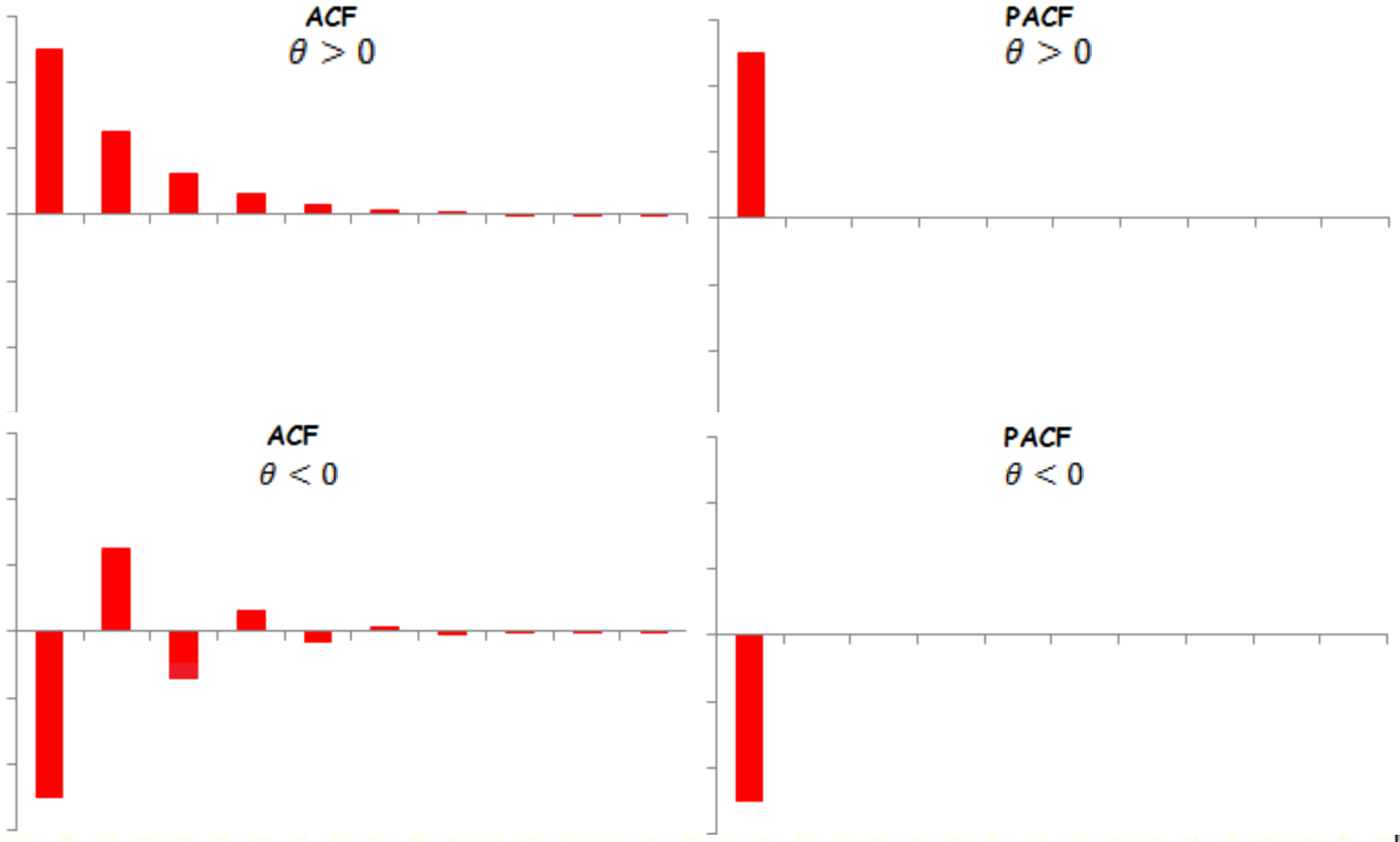
Theoretical of ACF

$$\rho_k = \phi_1^k \text{ for k} = 0, 1, 2, \ldots$$

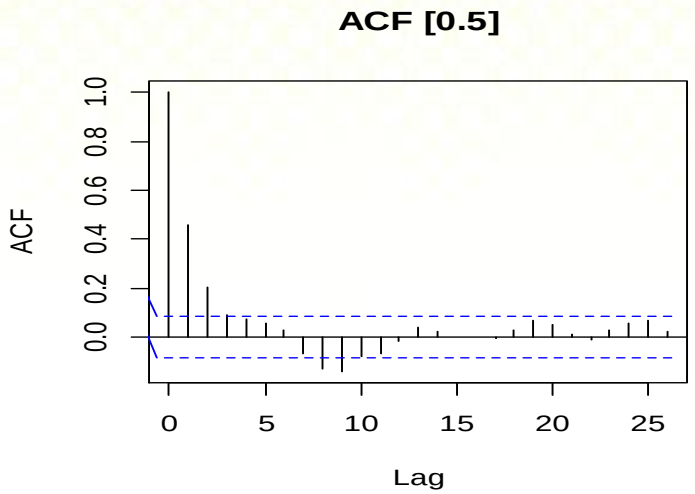
Theoretical of PACF

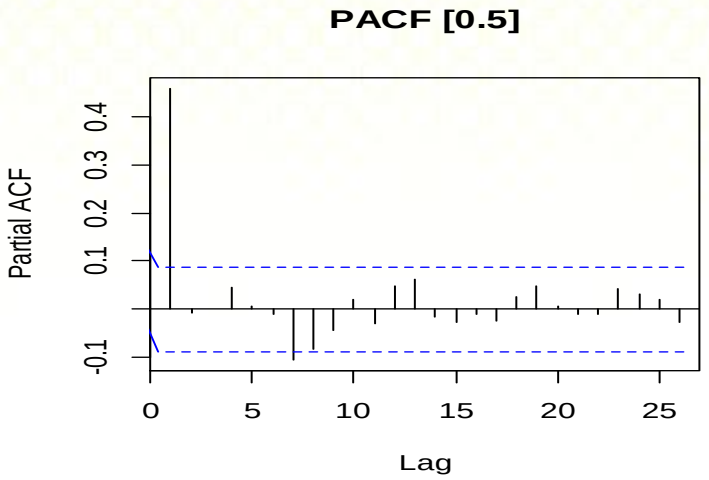
$$\rho_{kk} = \begin{cases} \rho_1 \text{ for k} = 1 \\ 0 \quad \text{others} \end{cases}$$

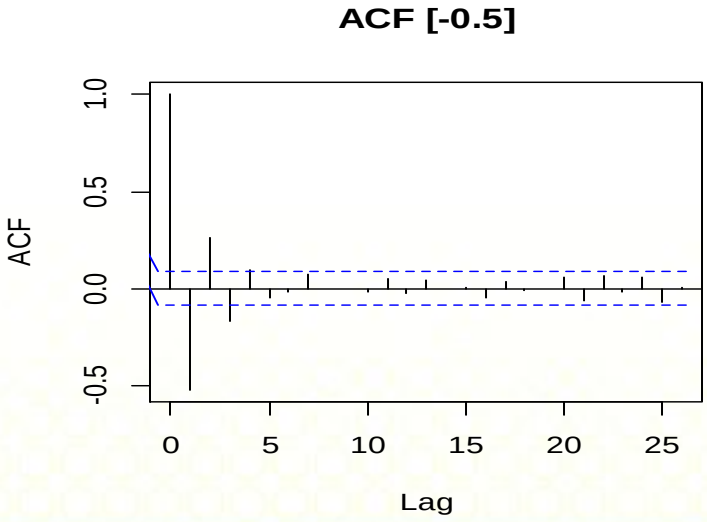
Theoretical of ACF and PACF for AR(1)

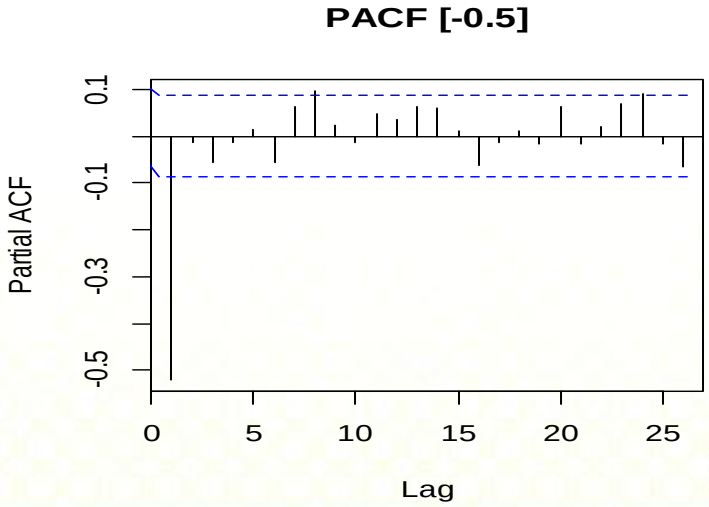


Sample ACF and PACF for AR(1)









Theoretical of ACF and PACF for AR(2)

The second order Autoregressive or AR(2) model

$$Z_t = \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + \varepsilon_t$$

Invertibility condition:

$$\phi_1 + \phi_2 < 1; \phi_2 - \phi_1 < 1; |\phi_2| < 1$$

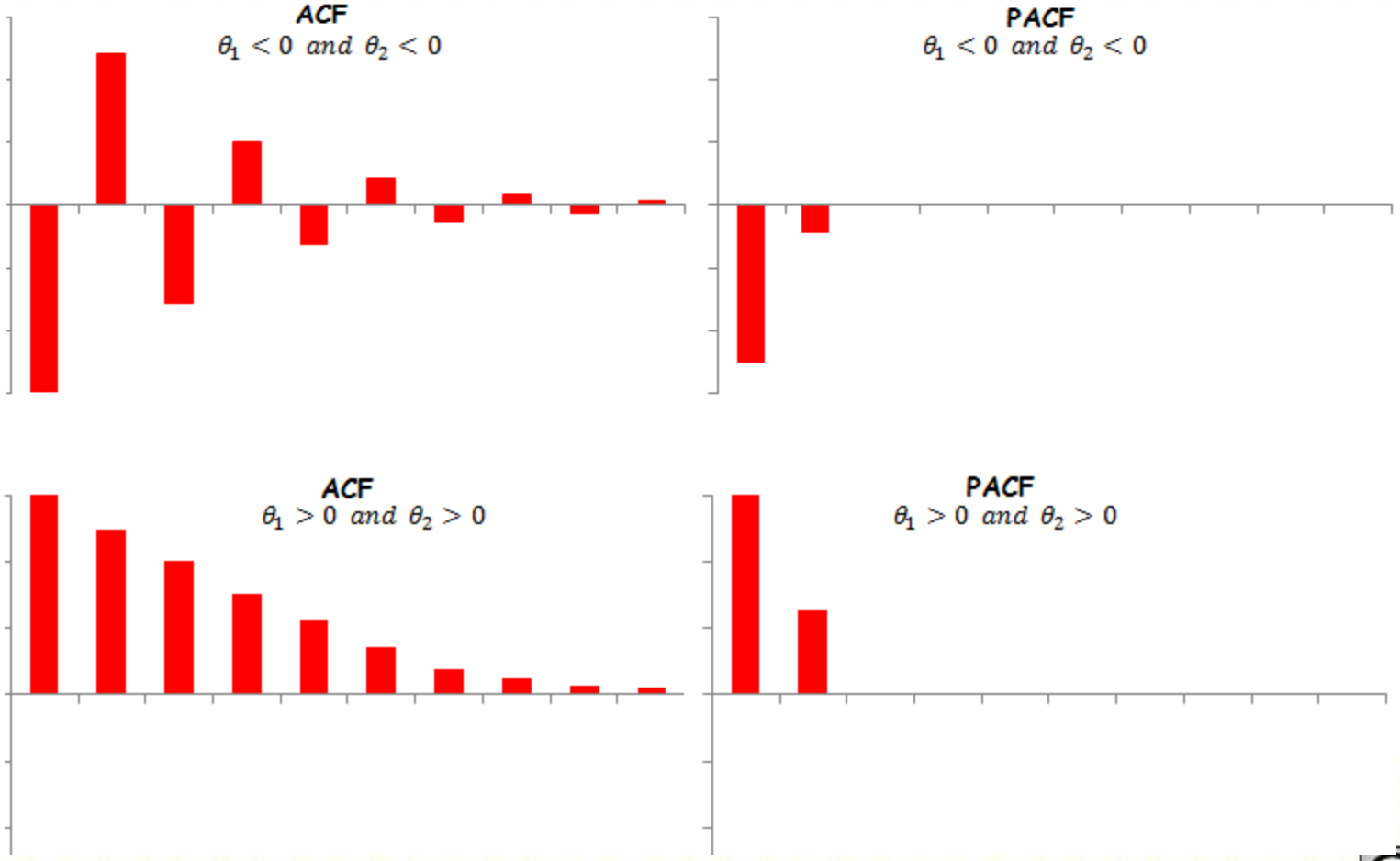
Theoretical of ACF

$$\rho_k = \begin{cases} \dfrac{\phi_1}{1-\phi_2} & , k = 1 \\ \phi_1 \rho_{k-1} + \phi_2 \rho_{k-2} & , k = 2,3,... \end{cases}$$

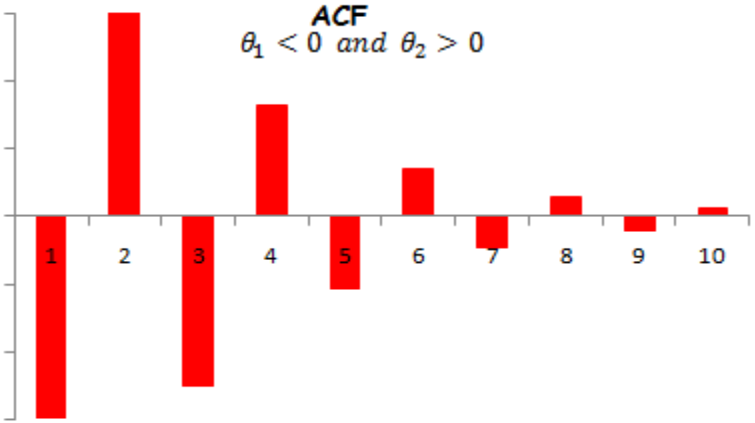
Theoretical of PACF

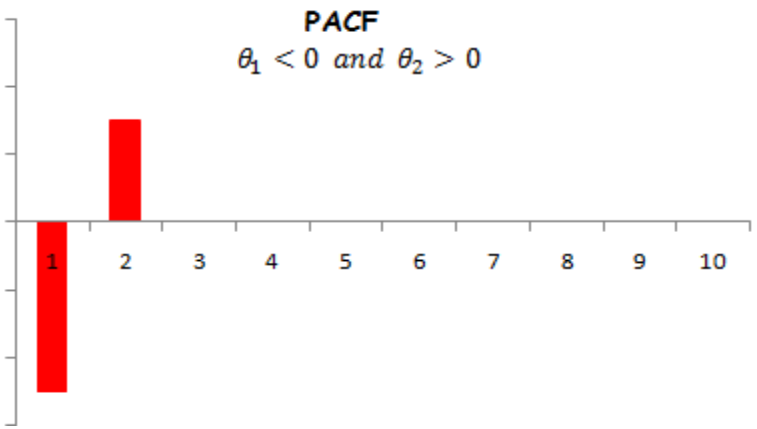
$$\rho_{kk} = \begin{cases} \rho_1 & , k = 1 \\ \phi_2 & , k = 2 \\ 0 & , k = 3,4,5,... \end{cases}$$

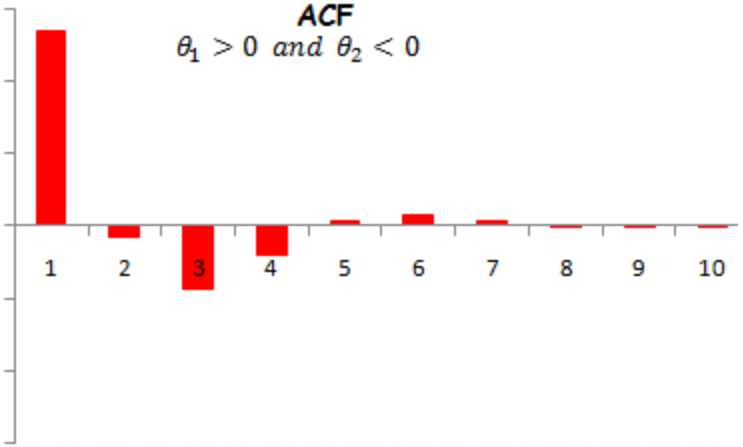
Theoretical of ACF and PACF for AR(2)

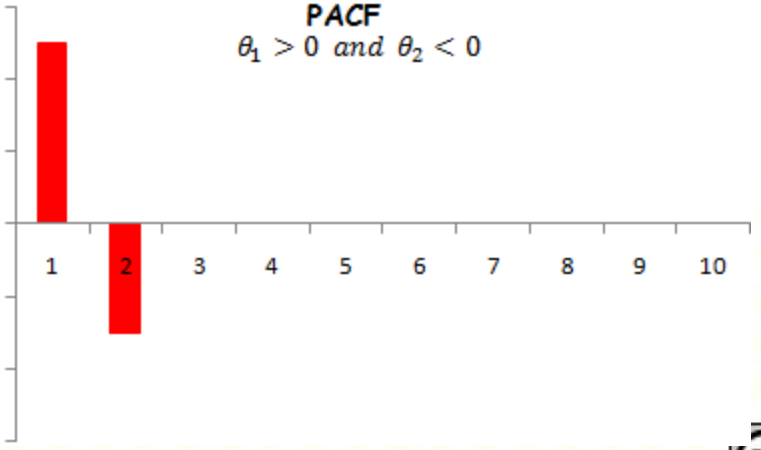


Theoretical of ACF and PACF for AR(2)

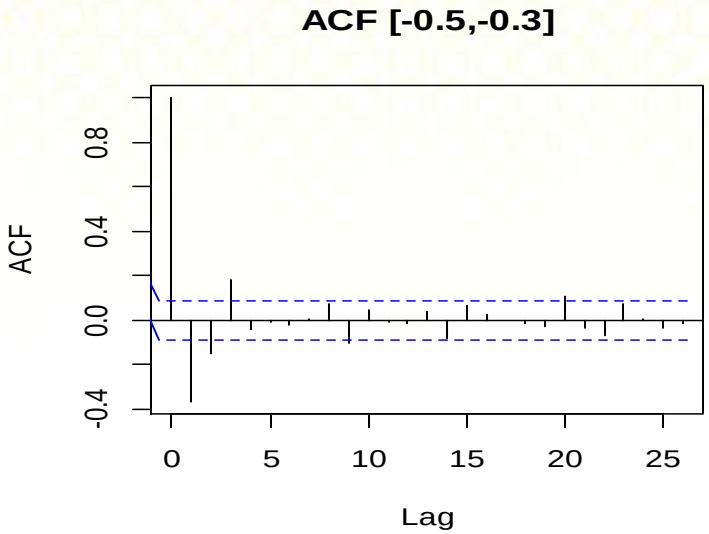


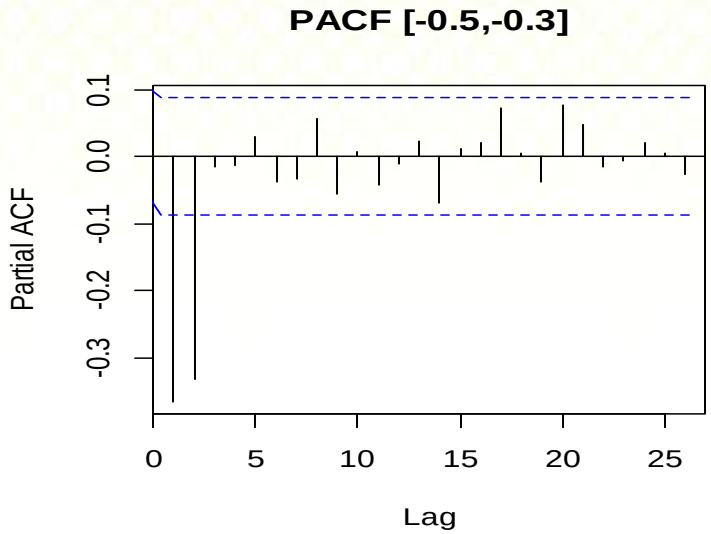


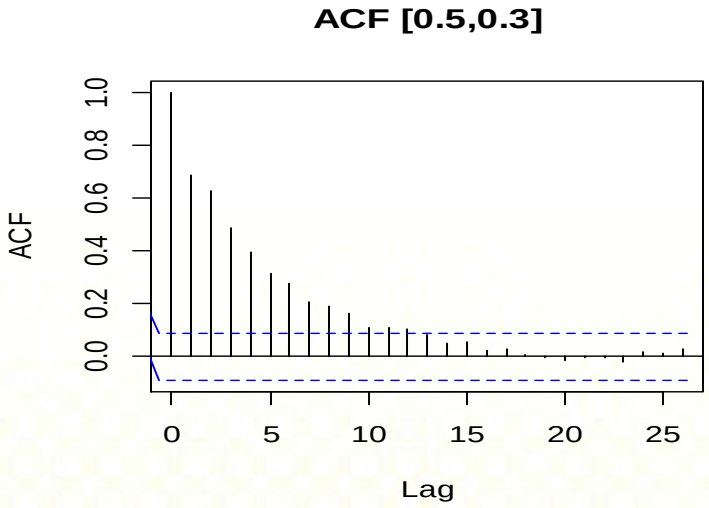


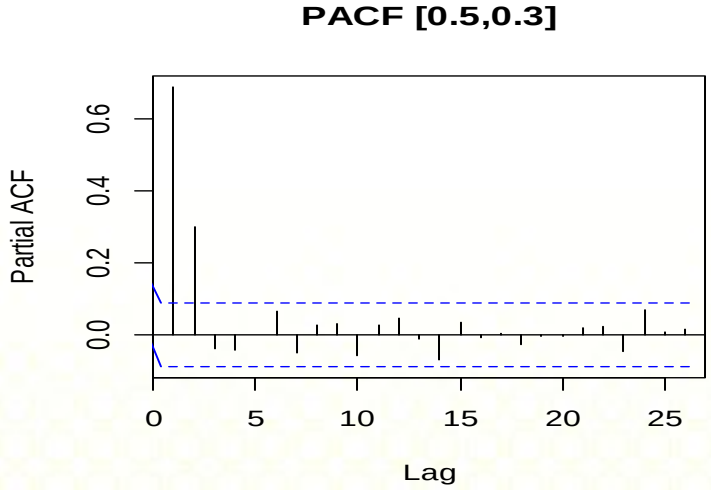


Sample ACF and PACF for AR(2)

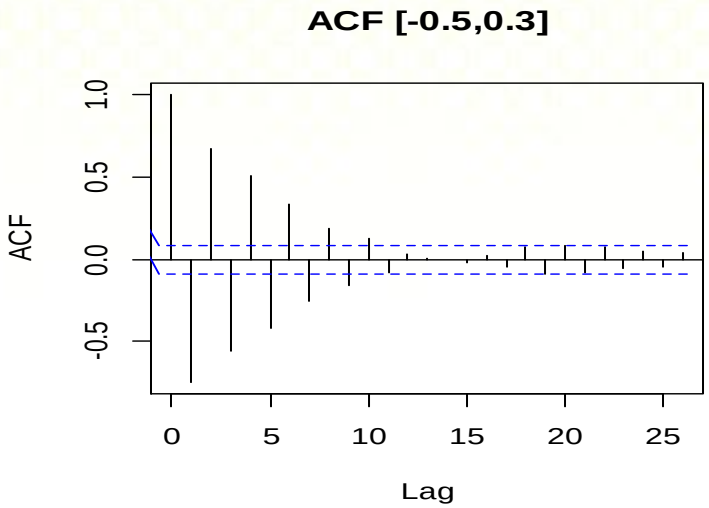


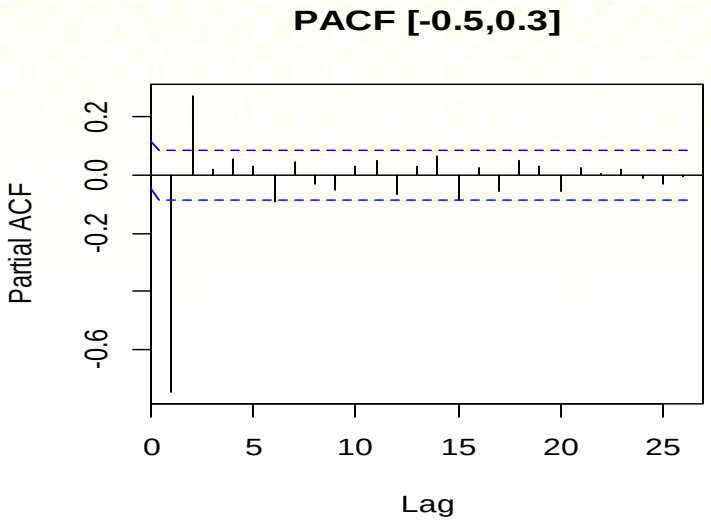


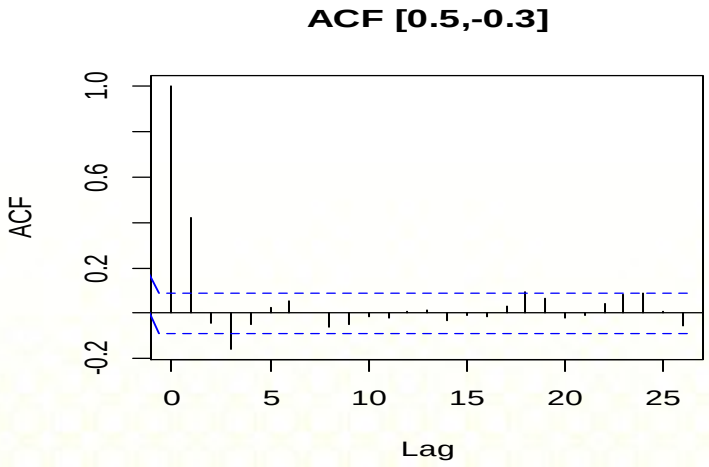


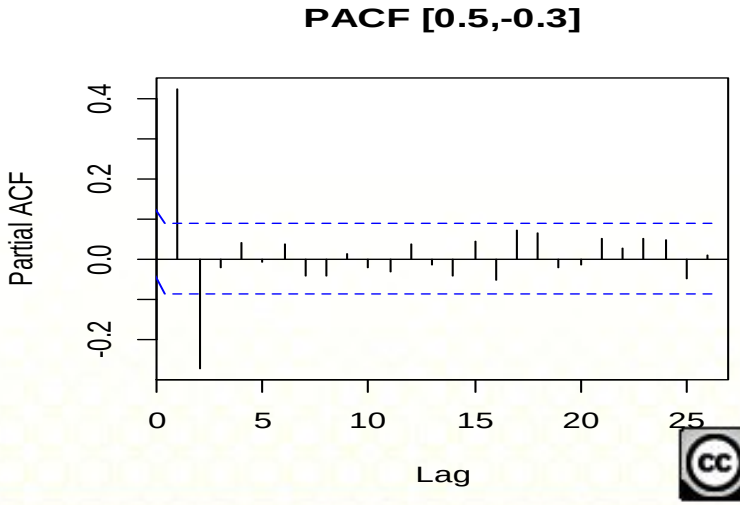


Sample ACF and PACF for AR(2)









Theoretically of ACF and PACF for ARMA(1,1)

The Mixed Autoregressive Moving Average Model or ARMA(1,1) model

$$Z_t = \phi_1 Z_{t-1} + \theta_1 \varepsilon_{t-1} + \varepsilon_t$$

Stationarity and invertibility condition:

$$-1 < \phi_1 < 1 \quad ; -1 < \theta_1 < 1$$

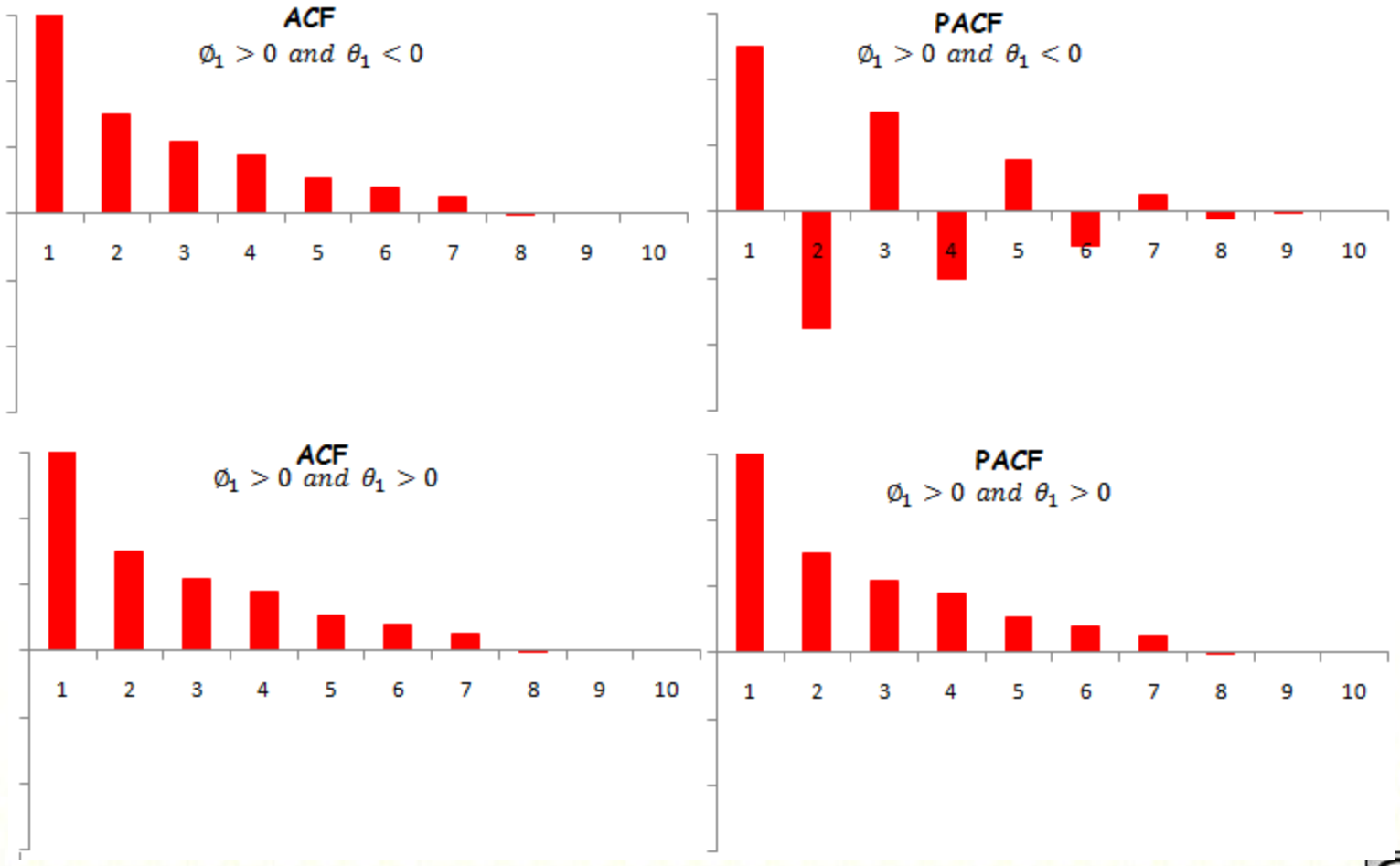
Theoretical of ACF

$$\rho_k = \begin{cases} \dfrac{(\phi_1 - \theta_1)(1 - \phi_1\theta_1)}{1 + \theta_1^2 - 2\phi_1\theta_1}, & k = 1 \\ \phi_1 \rho_{k-1}, & k \geq 2 \end{cases}$$

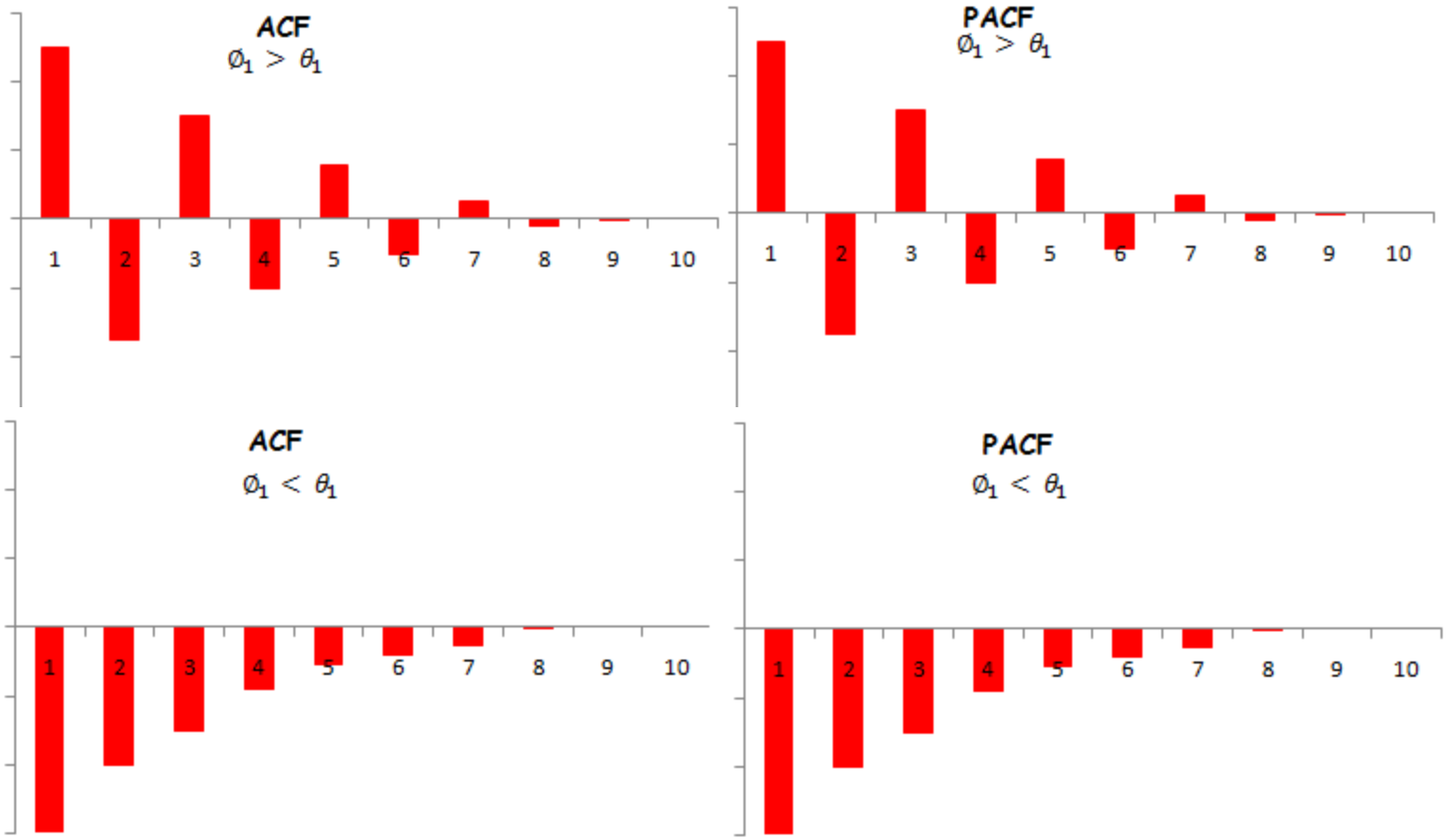
Theoretical of PACF

Dies down (according to a mixture of damped Exponentials and/or damped sine waves

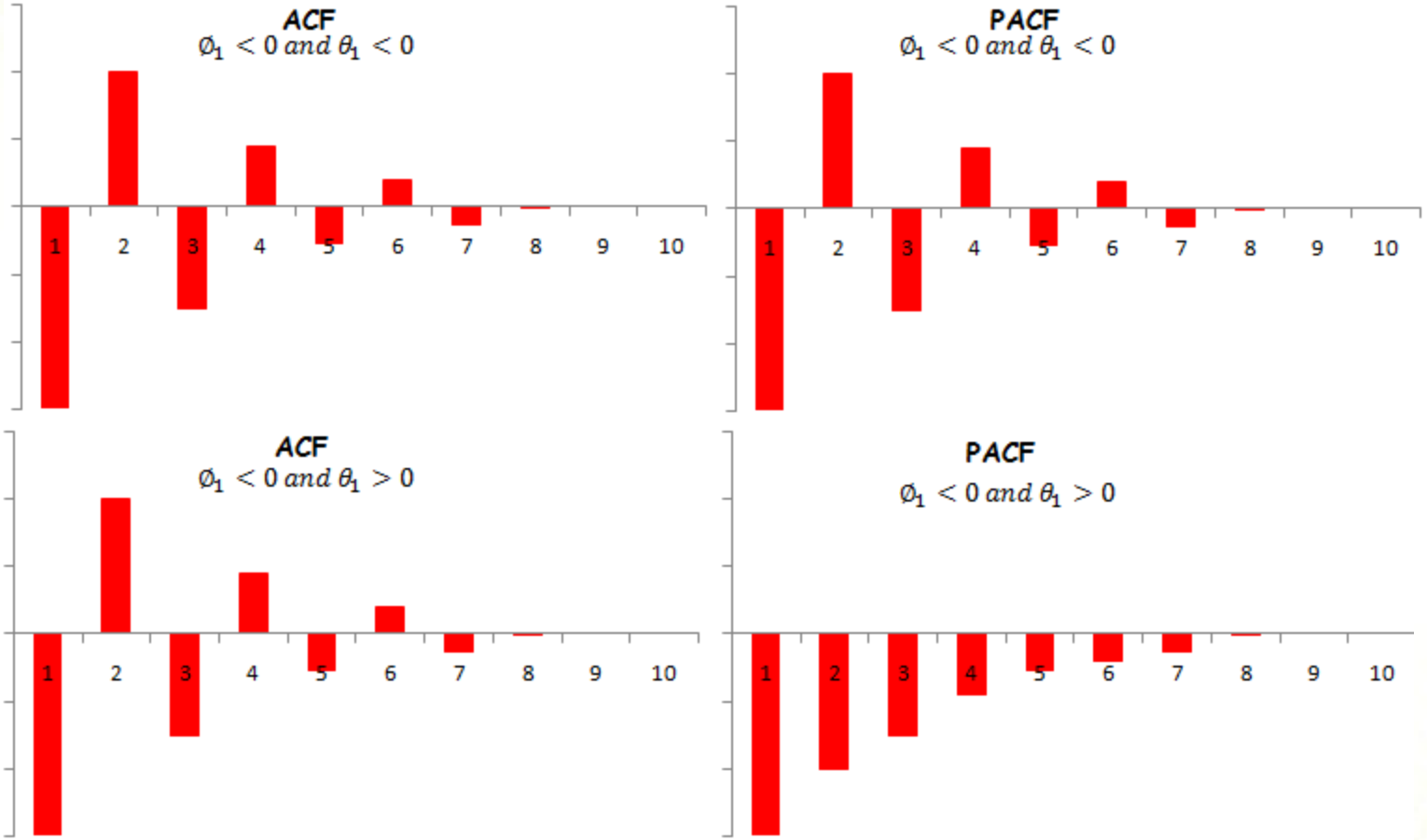
Theoretical of ACF and PACF for ARMA(1,1)



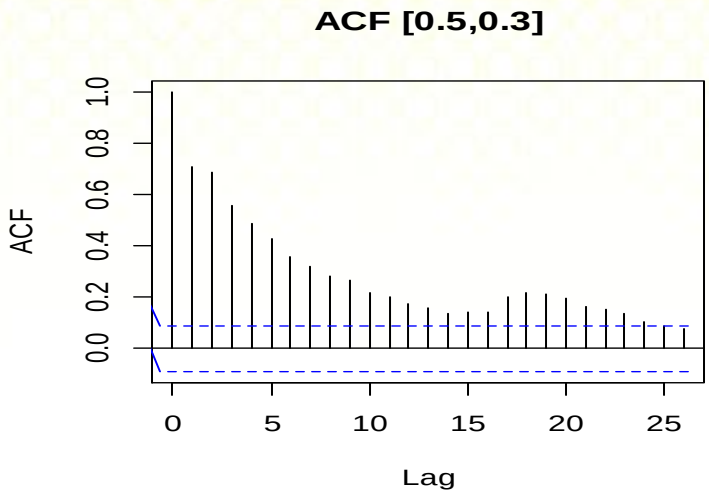
Theoretical of ACF and PACF for ARMA(1,1)

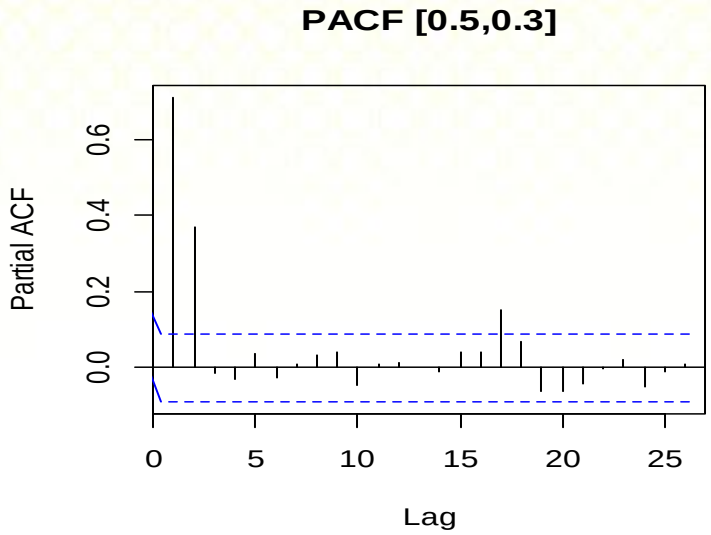


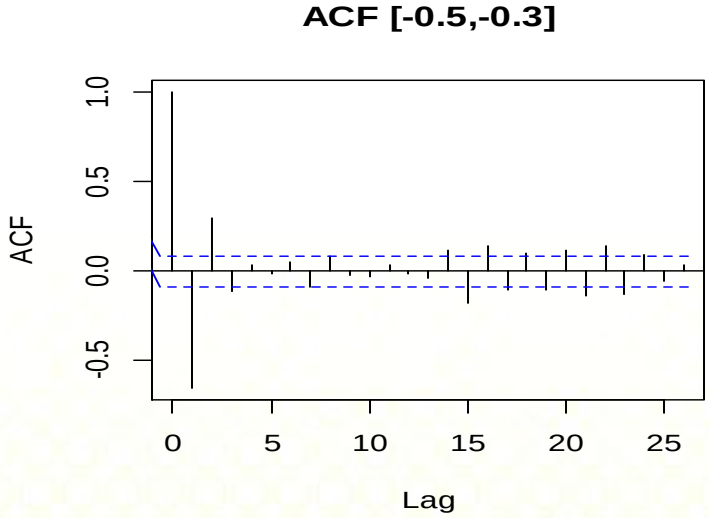
Theoretical of ACF and PACF for ARMA(1,1)

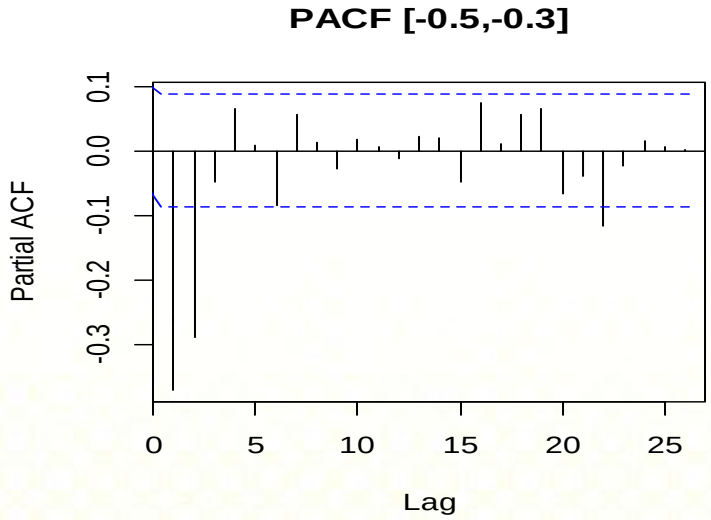


Sample ACF and PACF for ARMA(1,1)









Sample ACF and PACF for ARMA(1,1)