Statics SKMM1203

Equilibrium (2D and 3D)

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Brief concept:

A particle can be subjected only to a system of concurrent forces, and the necessary and sufficient conditions for equilibrium can be expressed mathematically as:

$$R = \sum F = 0$$

where $\sum F$ is the vector sum of all forces acting on the particle

Objectives:

- To determine required forces for equilibrium condition of concurrent force system
- To draw a free-body diagram for concurrent force system

Brief concept:

In particular, a particle is in equilibrium when the resultant of all forces acting on it equals zero. A particle is subjected to two forces as shown in Figure 3.1. It is in equilibrium condition if the two forces have same magnitude with opposite direction and act on the same line of action. If a particle is subjected to multiple loadings, equilibrium condition is achieved when the resultant of all the forces equals zero as demonstrated in Figure 3.2.
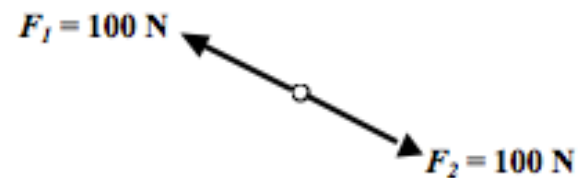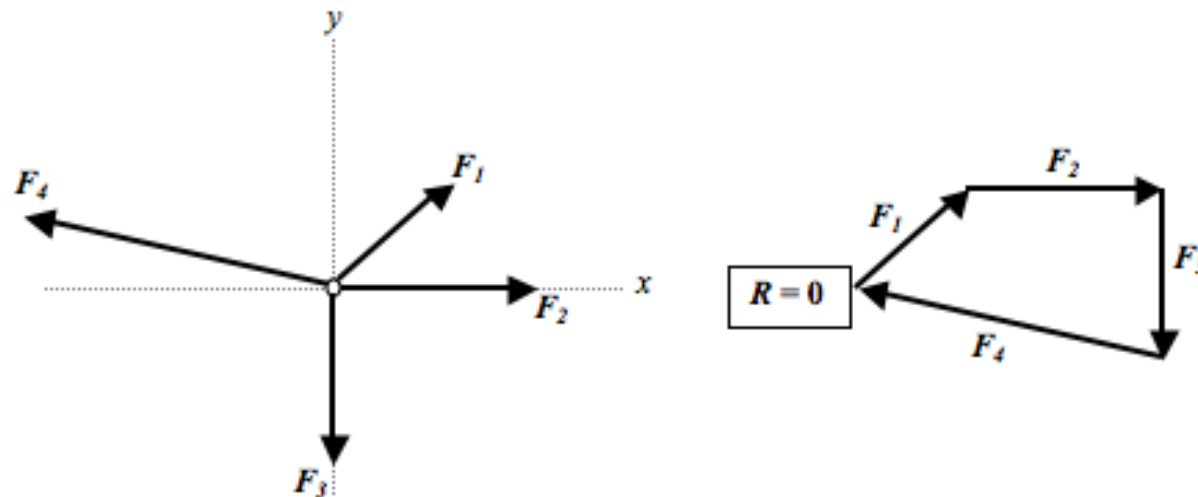
$F_1 = 100$ N

$F_2 = 100$ N

Figure 3.1

$R = 0$

Figure 3.2

Brief concept:

Two dimensional concurrent force system

For a system of coplanar (e.g x-y plane), concurrent forces can be written as

$$R = R_x i + R_y j$$
$$= \sum F_x i + \sum F_y j = 0$$

In scalar form, the equations become

$$R_x = \sum F_x = 0$$
$$R_y = \sum F_y = 0$$

That is, the sum of the rectangular components of the forces in any direction must be zero.

Brief concept:

Three dimensional concurrent force system

For a three-dimensional system of concurrent forces, the equilibrium condition can be expressed as

$$R = \sum F = R_x i + R_y j + R_z k = 0$$
$$\sum F_x i + \sum F_y j + \sum F_z k = 0$$

This equation is satisfied only if

$$R_x = \sum F_x i = 0$$
$$R_y = \sum F_y j = 0$$
$$R_z = \sum F_z k = 0$$

In scalar form, these equations become

$$R_x = \sum F_x = 0$$
$$R_y = \sum F_y = 0$$
$$R_z = \sum F_z = 0$$

Brief concept:

Solving an engineering mechanics problem usually requires identification of all external forces acting on a "body of interest". A carefully prepared drawing or sketch that shows a "body of interest" separated from all interacting bodies is known as a *free-body diagram* (FBD). It is important that all forces acting on the body of interest be shown. The actual procedure for drawing a *free-body diagram* consists of three essential steps:-

- Decide which body or combination of bodies is to be isolated and analyzed.

- Prepare a drawing or sketch of the outline of the isolated body selected.

- Represent all forces, known and unknown, that are applied by other bodies to the isolated body with vectors in their correct positions.

Examples:

Q1.

The weight of the engine is 4.4 kN and is suspended from a vertical chain at A. A second chain round the engine is attached at A, with a spreader bar between B and C. The angles, θ at B and C are 50°. Determine the tension in the chain AB and determine the compressive force acting in the spreader bar BC.
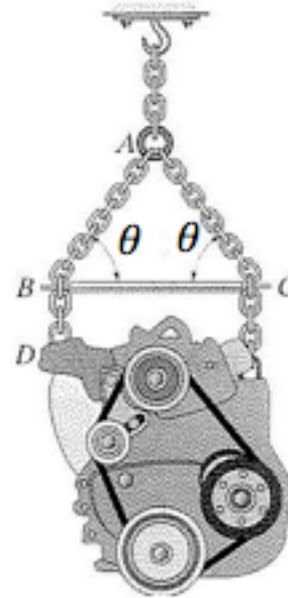
A1.
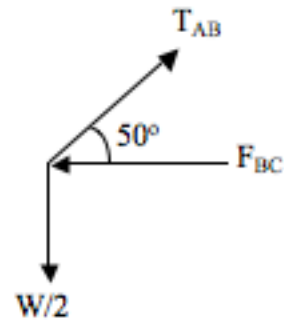
$(+\uparrow)\Sigma F_y = 0$

$T_{AB}\sin 50° - 4.4/2 = 0$
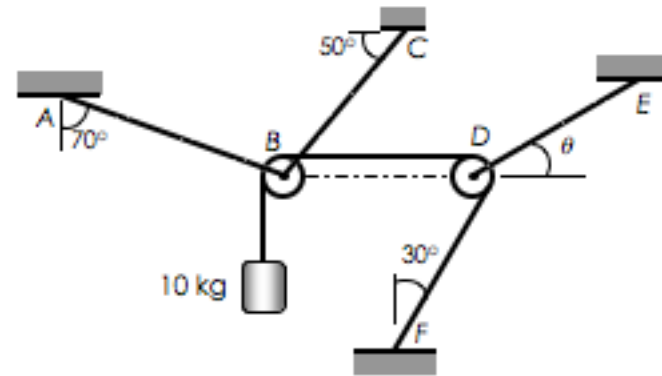
$T_{AB} = 2.87 \text{ kN}$

$(+\rightarrow)\Sigma F_x = 0$
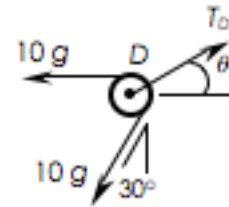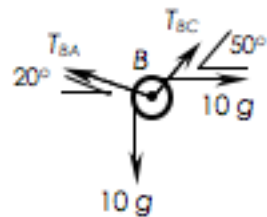
$2.87\cos 50° - F_{BC} = 0$

$F_{BC} = 1.84 \text{ kN}$

Examples:

Q2.

The 10 kg mass is supported by the cable system shown. Determine the tension in cables *AB*, *BC* and *DE*, and the angle θ.
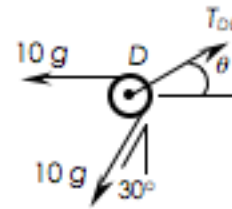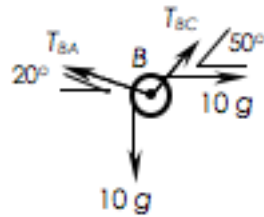

A2.

A2.

$(+\rightarrow)\Sigma F_x = 0$

$10g + T_{BC}\cos 50° - T_{AB}\cos 20° = 0$

$98.1 + 0.643 T_{BC} - 0.94 T_{AB} = 0 \qquad (1)$

$(+\uparrow)\Sigma F_y = 0$

$-10g + T_{BC}\sin 50° + T_{AB}\sin 20° = 0$
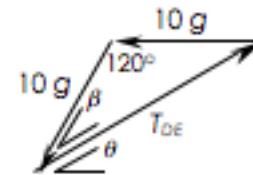
$-98.1 + 0.766 T_{BC} + 0.342 T_{AB} = 0 \qquad (2)$

cosine rule

$T_{DE}{}^2 = (10g)^2 + (10g)^2 - 2(10g)(10g)\cos 120°$

$T_{DE} = 169.9 \text{ N}$

$\beta = 30°$ since the triangle is isosceles

$(1) \div 0.643 \qquad 152.57 + T_{BC} - 1.462\ T_{AB} = 0$

$(2) \div 0.766 \qquad -128.07 + T_{BC} + 0.446 T_{AB} = 0$

$\therefore \qquad \theta = 30°$

$280.64 = 1.908 T_{AB}$

$T_{AB} = 147.1 \text{ N}$

input into (1)

$98.1 + 0.643 T_{BC} - 0.94(147.1) = 0$

Examples:

Q3.

A 100 kg traffic light is supported by a system of cables as shown in this figure. Determine the tensions in each of the three cables
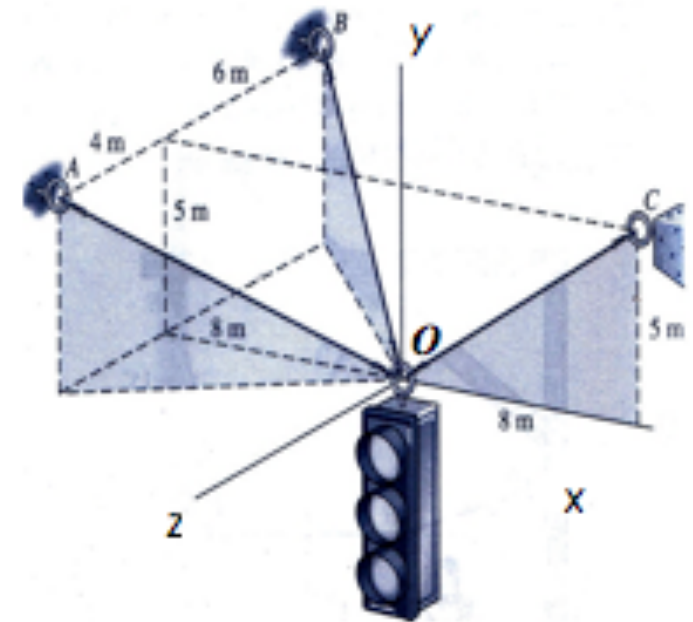
Examples:

T_{OA} $dx = -8$ m

$dy = 5$ m $\Big\}$ $d = \sqrt{(-8)^2 + 5^2 + 4^2} = 10.25$ m

$dz = 4$ m

$T_{OA} = (-8/10.25)\ T_{OA}\ \mathbf{i} + (5/10.25)\ T_{OA}\ \mathbf{j} + (4/10.25)\ T_{OA}\ \mathbf{k}$

$T_{OA} = -\ 0.78\ T_{OA}\ \mathbf{i} + 0.488\ T_{OA}\ \mathbf{j} + 0.39\ T_{OA}\ \mathbf{k}$

T_{OB} $dx = -8$ m

$dy = 5$ m $\Big\}$ $d = \sqrt{(-8)^2 + 5^2 + (-6)^2} = 11.18$ m

$dz = -6$ m

$T_{OB} = (-8/11.18)\ T_{OB}\ \mathbf{i} + (5/11.18)\ T_{OB}\ \mathbf{j} + (-6/11.18)\ T_{OB}\ \mathbf{k}$

$T_{OB} = -0.716\ T_{OB}\ \mathbf{i} + 0.447\ T_{OB}\ \mathbf{j} - 0.537\ T_{OB}\ \mathbf{k}$

T_{OC} $dx = 8$ m

$dy = 5$ m $\Big\}$ $d = \sqrt{8^2 + 5^2 + (0)^2} = 9.43$ m

$dz = 0$ m

$T_{OC} = (8/9.43)\ T_{OC}\ \mathbf{i} + (5/9.43)\ T_{OC}\ \mathbf{j}$

$T_{OC} = 0.848\ T_{OC}\ \mathbf{i} + 0.530\ T_{OC}\ \mathbf{j}$

100 kg   100 kg $= -\ 100g\ \mathbf{j}$

i component

$-0.78T_{OA} - 0.716T_{OB} + 0.848T_{OC} = 0$ $\qquad$ (1)

j component

$0.488T_{OA} + 0.447T_{OB} + 0.53T_{OC} - 100g = 0$ $\qquad$ (2)

k component

$0.39T_{OA} - 0.537T_{OB} = 0$ $\qquad$ (3)

$\qquad T_{OA} = 1.377T_{OB}$ $\qquad$ (3a)

Substitute (3a) into (1) and (2)

Then, solve simultaneous equations for (1) and (2)

$T_{OA} = 603$ N

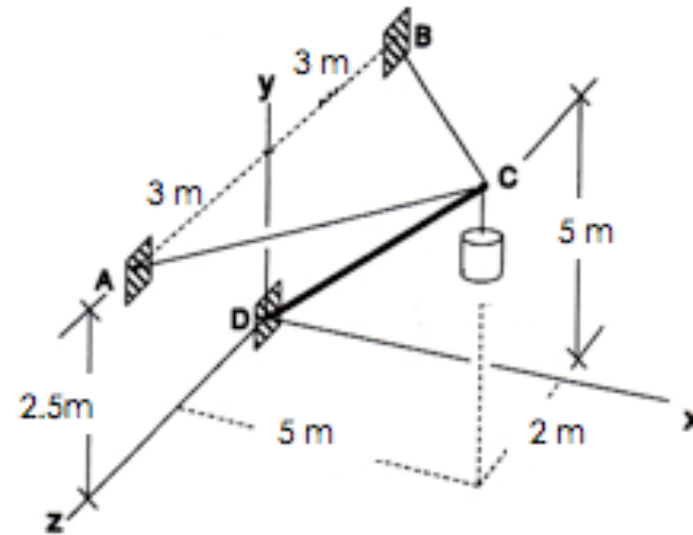$T_{OB} = 439$ N

$T_{OC} = 925$ N

Examples:

Q4.

Determine the forces in cables AC and BC, and in the strut
CD due to the weight, 10 kg hanging off point C

Examples:

A4.

$$T_{AC} \quad \left. \begin{array}{l} dx = -5 \text{ m} \\ dy = -2.5 \text{ m} \\ dz = 1 \text{ m} \end{array} \right\} \quad d = \sqrt{(-5)^2 + (-2.5)^2 + 1^2} = 5.68 \text{ m}$$

$T_{AC} = (-5/5.68)\, T_{AC}\, \mathbf{i} + (-2.5/5.68)\, T_{AC}\, \mathbf{j} + (1/5.68)\, T_{AC}\, \mathbf{k}$

$T_{AC} = -0.88\, T_{AC}\, \mathbf{i} - 0.44\, T_{AC}\, \mathbf{j} + 0.18\, T_{AC}\, \mathbf{k}$

$$T_{BC} \quad \left. \begin{array}{l} dx = -5 \text{ m} \\ dy = -2.5 \text{ m} \\ dz = -5 \text{ m} \end{array} \right\} \quad d = \sqrt{(-5)^2 + (-2.5)^2 + (-5)^2} = 7.5 \text{ m}$$

$T_{BC} = (-5/7.5)\, T_{BC}\, \mathbf{i} + (-2.5/7.5)\, T_{BC}\, \mathbf{j} + (-5/7.5)\, T_{BC}\, \mathbf{k}$

$T_{BC} = -0.67\, T_{BC}\, \mathbf{i} - 0.33\, T_{BC}\, \mathbf{j} - 0.67\, T_{BC}\, \mathbf{k}$

$$F_{CD} \quad \left. \begin{array}{l} dx = 5 \text{ m} \\ dy = 5 \text{ m} \\ dz = 2 \text{ m} \end{array} \right\} \quad d = \sqrt{5^2 + 5^2 + 2^2} = 7.35 \text{ m}$$

$F_{CD} = 5/7.35\, F_{CD}\, \mathbf{i} + 5/7.35\, F_{CD}\, \mathbf{j} + 2/7.35\, F_{CD}\, \mathbf{k}$

$F_{CD} = 0.68\, F_{CD}\, \mathbf{i} + 0.68\, F_{CD}\, \mathbf{j} + 0.27\, F_{CD}\, \mathbf{j}$

50 kg $50 \text{ kg} = -50g\, \mathbf{j}$

i component

$-0.88 T_{AC} - 0.67 T_{BC} + 0.68\, F_{CD} = 0 \qquad (1)$

j component

$-0.44 T_{AC} - 0.33 T_{BC} + 0.68 F_{CD} - 50g = 0 \quad (2)$

k component

$0.18 T_{AC} - 0.67 T_{BC} + 0.27\, F_{CD} = 0 \qquad (3)$

Thus, solve simultaneous equations for (1), (2) and (3)

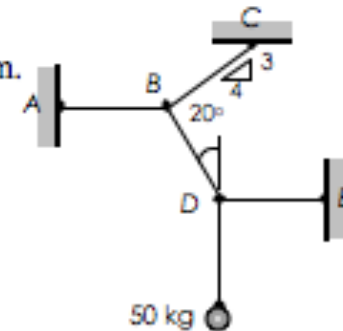$$T_{AC} = 56.49 \text{ N}$$

$$T_{BC} = 74.07 \text{ N}$$

$$T_{CD} = 145.97 \text{ N}$$

Practices:

Practice Questions:
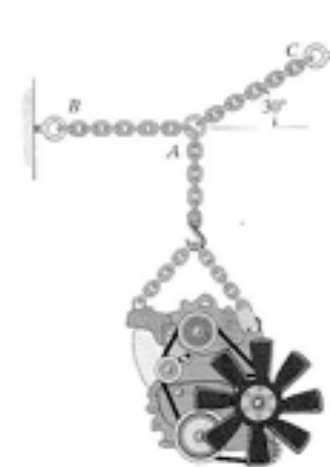
PQ 1.

Determine the tension in each cable for the system to maintain equilibrium.

50 kg

PQ 2.

Determine the maximum weight in kN of the engine that can be supported without exceeding the maximum tension given of 4.5 kN in chain AB or 6.50 kN in chain AC
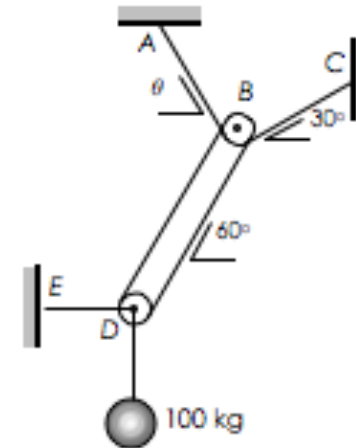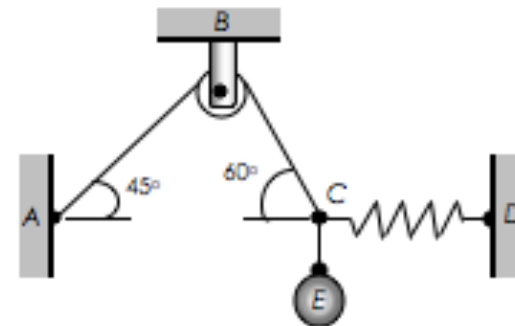
Practices:

PQ 3.

The 100 kg mass is supported by a cable and pulley system as shown. Determine the tension in each cable and the angle θ for the system to maintain equilibrium.

($T_{BD} = 566.4$ N, $T_{DE} = 566.4$ N, $T_{AB} = T_{BC} = 654$ N, $\theta = 90°$)

PQ 4.

A 6 kg mass at E is supported as shown. Determine tension in the spring and cable AB.
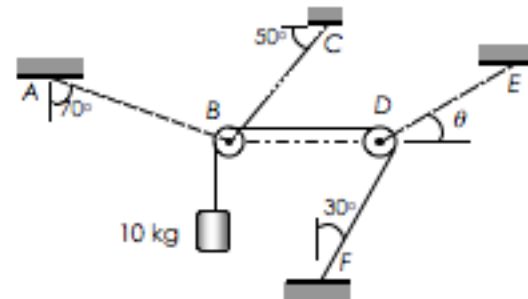
Practices:

PQ 5.
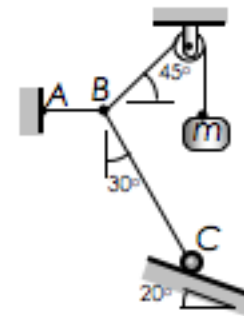
The 10 kg mass is supported by the cable system shown. Determine the tension in cables *AB*, *BC* and *DE*, and the angle θ.

$(T_{AB} = 147.1 \text{ N}, T_{BC} = 62.48 \text{ N}, T_{DE} = 169.9 \text{ N}, \theta = 30°)$

PQ 8.

A 5 kg ball is located at end *C*. Determine the suitable mass *m* required to maintain cable *AB* in the horizontal position as shown in the diagram.
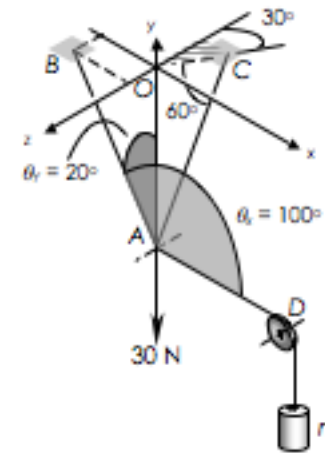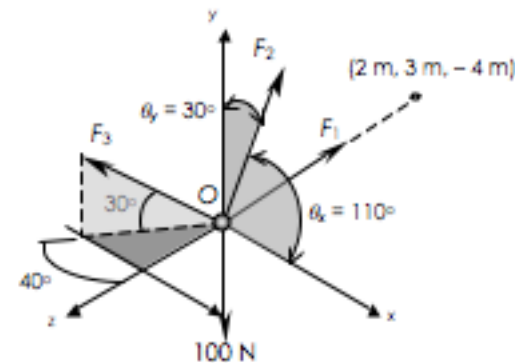
Practices:

PQ 9.

The mass m is maintained in equilibrium with the support of cables AB and AC, and a 30 N force at A. Cable AD is parallel to the x-axis. Determine the tension in cables AB and AC, and the mass m.
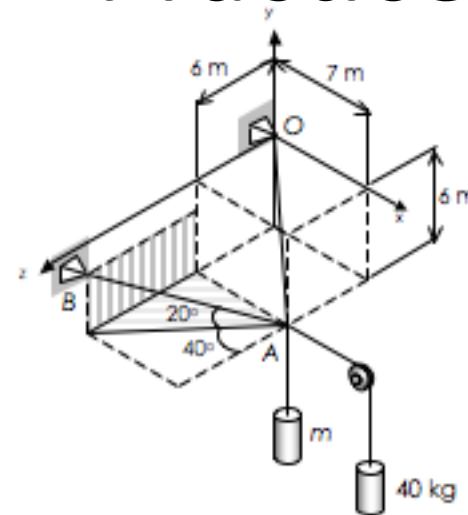

PQ 10.

Determine the forces F_1, F_2, and F_3 so that the system is in equilibrium.
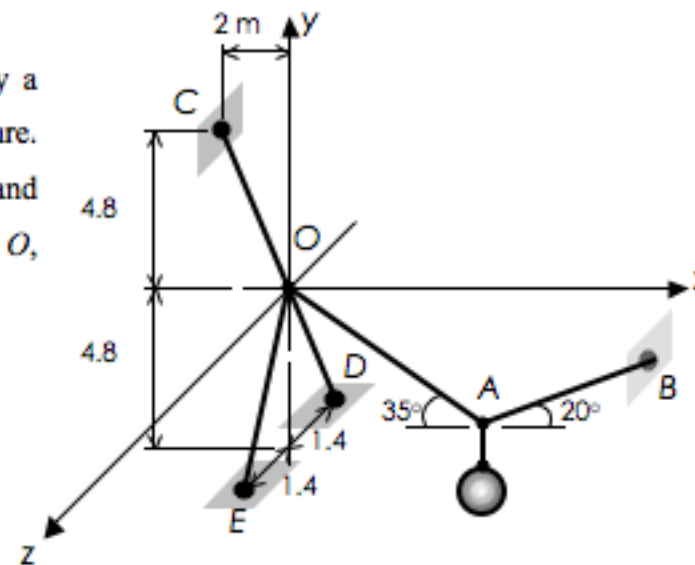
Practices:

PQ 11.

The mass *m* is supported at the position shown by cables *AO*, *AB* and the 40 kg mass. Determine the tension in cables *AO* and *AB*, and mass *m* in kg.

PQ 12.

A weight of 500 N at point *A* is supported by a system of five cables as shown in this figure. Determine tension in each cable. Points *C*, *O*, *A* and *B* are located on the same *x–z* plane while points *O*, *D* and *E* are located on the same *x–y* plane.

Practices:

PQ 16.

The mass m is supported by cables AB and BC, and a force P as shown. If the tension in both cables AB and BC equal 10 kN, determine the force P, angle θ and mass m to maintain equilibrium. The force P is situated on the x-z plane.