# Application of Statistics in Educational Research I 

## MPU1034

## One-Way ANOVA*

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## The General Idea About One-Way Analysis of Variance (ANOVA)

$\checkmark \quad z$ test is applicable for testing hypothesis of any normally distributed data generated from a large n and involved with one independent variable
$\checkmark t$ tests are applicable for testing hypothesis of data generated from:

- one independent variable involving a single group of sample
- one independent variable involving two independent groups of sample
- one two-level independent variable involving two correlated groups of sample
$\checkmark$ Strictly speaking, one-way ANOVA is an extended version of a $t$ test; it lets you perform a $t$ test for one independent variable with 3 or more levels at a time


## Strong Notes:

If you were to conduct hypothesis testing of a specific independent variable involving $\underline{3 \text { or }}$ more levels....
$\square$ You cannot draw a directly-transitive conclusion from multiple $t$ tests among the different groups (as permitted by $t$ tests)!
$\square$ Instead, you have to employ a one-way ANOVA (sometime called $F$ test) for this purpose.
$\square$ As for $t$ tests, the aim of one-way ANOVA is to determine whether the null hypothesis $\left(\mathrm{H}_{0}\right)$ can be safely rejected

## Essential Steps in Performing One-Way ANOVA

## Setting The Hypotheses

Estimating MSs*
Finding $F$ Value ${ }^{*}$
Finding Degree of Freedom*
$\square$ Finding Critical F Value*


Conducting post hoc Test*

## The Example:

You are to study the factor of unintended acceleration that cause road accident. It is hypothesised that the distance between the brake and the accelerator pedals plays a contributing factor in driver error that cause road accident. To test this hypothesis, you design an experiment in which subjects use one of three driving simulators, each with a different pedal arrangement. In the close-pedal arrangement, the distance between pedals is only 1 inch ; in the moderatepedal arrangement, it's 2 inches; and in the far-pedal arrangement, 3 inches.

You assign ten subjects to each condition and instruct them to drive their simulators for 4 hours. During these 4 hours, the

| Type 1 $X_{1}$ | Pedal arrangement | Type 3 $X_{3}$ |
| :---: | :---: | :---: |
|  | Type 2 |  |
|  | $X_{2}$ |  |
| 3 | 4 | 6 |
| 2 | 3 | 4 |
| 4 | 4 | 3 |
| 1 | 3 | 4 |
| 0 | 5 | 7 |
| 2 | 4 | 6 |
| 3 | 2 | 5 |
| 2 | 5 | 8 |
| 1 | 4 | 5 |
| 2 | 4 | 6 | number of errors made by each driver is recorded.

## The Hypotheses

The null hypothesis:
$H_{0}$ : There is no difference between the sample means of any of the level

$$
\mathrm{H}_{\mathrm{o}}: \mu_{1}=\mu_{2}=\ldots \ldots \ldots . .=\mu_{\mathrm{k}}
$$

where k is the number of levels of the independent variable

The alternative hypothesis:
$H_{1}$ : At least one of the sample means comes from a population different from that of the other sample means.
$H_{0}$ : There is no difference between the sample means of any of the three level

$$
\mathrm{H}_{0}: \mu_{1}=\mu_{2}=\mu_{3}
$$

$H_{1}$ : At least one of the three sample means comes from a population different from that of the other sample means.

## Estimating The Mean Square (MS)

$\checkmark$ The MS is the mean (the average) of the squared deviation scores used to calculate the variation.
$\checkmark$ MS is equivalent to variance estimate (est. $\sigma$ ) for the $t$ tests
$\checkmark$ The are two types of MS:
$\square$ mean square within groups $\left(\mathrm{MS}_{\mathrm{wg}}\right)$
$\boxed{\text { mean square between groups }\left(M S_{b g}\right) ~}$

## Mean Square Within Group

$$
\mathrm{MS}_{\mathrm{wg}}=\frac{\Sigma\left(X_{1}-\bar{X}_{1}\right)^{2}+\Sigma\left(X_{2}-\bar{X}_{2}\right)^{2}+\cdots+\Sigma\left(X_{k}-\bar{X}_{k}\right)^{2}}{\left(n_{1}-1\right)+\left(n_{2}-1\right)+\cdots+\left(n_{\mathrm{k}}-1\right)}
$$

where
$\bar{X}_{k} \quad$ is the mean for $\mathrm{k}^{\text {th }}$ group
$k$ is the total number of different groups

## Mean Square Between Group

$\mathrm{MS}_{\mathrm{bg}}=\frac{n_{1}\left(\bar{X}_{1}-\overline{\bar{X}}\right)^{2}+n_{2}\left(\bar{X}_{2}-\overline{\bar{X}}\right)^{2}+\cdots+n_{k}\left(\bar{X}_{k}-\overline{\bar{X}}\right)^{2}}{k-1}$
where
$\bar{X}_{k} \quad$ is the mean for $k^{\text {th }}$ group
$\overline{\bar{X}}=\frac{\sum \bar{X}}{k}=\frac{\bar{X}_{1}+\bar{X}_{2}+\cdots+\bar{X}_{k}}{k}$
$k \quad$ is the total number of different groups
note: $\mathrm{MS}_{\mathrm{bg}}$ is always bigger or equal to $\mathrm{MS}_{\mathrm{wg}}$
(Refer to Example)

|  | Pedal arrangement <br> Type 1 <br> $x_{1}$ | Type 2 <br> 3 |
| :---: | :---: | :---: |
| 3 | $x_{2}$ | $X_{3}$ |
| 2 | 4 | 6 |
| 4 | 3 | 4 |
| 1 | 4 | 3 |
| 0 | 3 | 4 |
| 2 | 5 | 7 |
| 3 | 4 | 6 |
| 2 | 2 | 5 |
| 1 | 5 | 8 |
| 2 | 4 | 6 |

$$
\begin{array}{ccc}
\sum X_{1}=20 & \Sigma X_{2}=38 & \sum X_{3}=54 \\
\bar{X}_{1}=2 & \bar{X}_{2}=3.8 & \bar{X}_{3}=5.4 \\
\overline{\bar{X}}=\frac{\bar{X}_{1}+\bar{X}_{2}+\bar{X}_{3}}{k}=\frac{2+3.8+5.4}{3}=\frac{11.2}{3}=3.733 \\
\text { or } \\
\overline{\bar{X}}=\frac{\sum X_{1}+\sum X_{2}+\Sigma X_{3}}{N_{\text {total }}}=\frac{20+38+54}{30}=\frac{112}{30}=3.733
\end{array}
$$

$$
\begin{aligned}
\mathrm{MS}_{\mathrm{bg}} & =\frac{n_{1}\left(\bar{X}_{1}-\overline{\bar{X}}\right)^{2}+n_{2}\left(\bar{X}_{2}-\overline{\bar{X}}\right)^{2}+n_{3}\left(\bar{X}_{3}-\overline{\bar{X}}\right)^{2}}{k-1} \\
& =\frac{10(2-3.733)^{2}+10(3.8-3.733)^{2}+10(5.4-3.733)^{2}}{3-1} \\
& =\frac{10(-1.733)^{2}+10(0.067)^{2}+10(1.667)^{2}}{2} \\
& =\frac{10 \cdot 3.003+10 \cdot 0.004+10 \cdot 2.779}{2} \\
& =\frac{30.03+0.04+27.79}{2}=\frac{57.86}{2}=28.93
\end{aligned}
$$

Table 13.3 Calculations of Deviations and Squared Deviations for the Number of Pedal Errors with Three Pedal Arrangements

| Type 1 |  |  | Pedal arrangement |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Type |  |  | Type |  |
| $X_{1}$ | $x_{1}-\bar{X}_{1}$ | $\left(X_{1}-\bar{X}_{1}\right)^{2}$ | $X_{2}$ | $x_{2}-\bar{X}_{2}$ | $\left(X_{2}-\bar{X}_{2}\right)^{2}$ | $X_{3}$ | $x_{3}-\bar{X}_{3}$ | $\left(X_{3}-\bar{X}_{3}\right)^{2}$ |
| 3 | 1 | 1 | 4 | 0.2 | 0.04 | 6 | 0.6 | 0.36 |
| 2 | 0 | 0 | 3 | -0.8 | 0.64 | 4 | -1.4 | 1.96 |
| 4 | 2 | 4 | 4 | 0.2 | 0.04 | 3 | -2.4 | 5.76 |
| 1 | -1 | 1 | 3 | -0.8 | 0.64 | 4 | -1.4 | 1.96 |
| 0 | -2 | 4 | 5 | 1.2 | 1.44 | 7 | 1.6 | 2.56 |
| 2 | 0 | 0 | 4 | 0.2 | 0.04 | 6 | 0.6 | 0.36 |
| 3 | 1 | 1 | 2 | -1.8 | 3.24 | 5 | -0.4 | 0.16 |
| 2 | 0 | 0 | 5 | 1.2 | 1.44 | 8 | 2.6 | 6.76 |
| 1 | -1 | 1 | 4 | 0.2 | 0.04 | 5 | -0.4 | 0.16 |
| 2 | 0 | 0 | 4 | 0.2 | 0.04 | 6 | 0.6 | 0.36 |
| $\Sigma\left(X_{1}-\bar{X}_{1}\right)^{2}=12$ |  |  | $\Sigma\left(X_{2}-\bar{X}_{2}\right)^{2}=7.60$ |  |  | $\Sigma\left(x_{3}-\bar{X}_{3}\right)^{2}=20.40$ |  |  |

(Refer to Example)

$$
\begin{aligned}
\mathrm{MS}_{\mathrm{wg}} & =\frac{\sum\left(X_{1}-\bar{X}_{1}\right)^{2}+\Sigma\left(X_{2}-\bar{X}_{2}\right)^{2}+\sum\left(X_{3}-\bar{X}_{3}\right)^{2}}{\left(n_{1}-1\right)+\left(n_{2}-1\right)+\left(n_{3}-1\right)} \\
& =\frac{12+7.60+20.40}{(10-1)+(10-1)+(10-1)}=\frac{40}{9+9+9} \\
& =\frac{40}{27}=1.481
\end{aligned}
$$

## Finding The F Value

$\checkmark \quad F$ test is used to decide whether to reject the $\mathrm{H}_{0}$ or fail to reject $\mathrm{H}_{0}$ (for one independent variable with 3 or more levels at a time).
$\checkmark$ It is done by comparing the $\mathrm{MS}_{\mathrm{bg}}$ and $\mathrm{MS}_{\mathrm{wg}}$ using the following formula:

$$
F=\frac{\mathrm{MS}_{\mathrm{bg}}}{\mathrm{MS}}
$$

Observe that $\mathrm{F} \geq 1$ as $\mathrm{MS}_{\mathrm{bg}}$ is always bigger or equal to $\mathrm{MS}_{\text {wg }}$
If $\mathrm{MS}_{\mathrm{bg}}$ and $\mathrm{MS}_{\mathrm{wg}}$ are similar, then $\mathrm{F}=1$

## $\checkmark$ In the example,

$$
F=\frac{M S_{\mathrm{bg}}}{\mathrm{MS} \mathrm{wg}_{\mathrm{wg}}}=28.930 / 1.481=19.534
$$

## Finding Degree of Freedom (df)

$\checkmark$ There are two types of degree of freedom:
a. degree of freedom for the mean square between group ( $\mathrm{dff}_{\mathrm{bg}}$ )
b. degree of freedom for the mean square within groups ( $\mathrm{df}_{\mathrm{wg}}$ )

$$
\begin{aligned}
& d f_{\mathrm{bg}}=\mathrm{k}-1 \\
& \mathrm{df} \mathrm{wg}=\left(\mathrm{n}_{1}-1\right)+\left(\mathrm{n}_{2}-1\right)+\ldots \ldots .+\left(n_{k}-1\right)
\end{aligned}
$$

where
$k \quad$ is the total number of different groups
$\mathrm{n}_{\mathrm{m}}$ is the number of observations for the $\mathrm{m}^{\text {th }}$ group
$\checkmark$ In the example,

$$
\begin{aligned}
& \mathrm{df}_{\mathrm{bg}}=3-1=2 \\
& \mathrm{df}_{\mathrm{wg}}=(10-1)+(10-1)+(10-1)=27
\end{aligned}
$$

## Drawing Up A Conclusion

$\square$ The hypothesis testing for ANOVA is concluded in a similar way as we do for $t$ tests.
$\square \quad$ In ANOVA, we refer to Table $F$ to identify the critical value of $F$ (denoted by $F_{c v}$ )
$\square$ Table $F$ is arranged so that we look for the computed degrees of freedom between groups (i.e. $\mathrm{df}_{\mathrm{bg}}$ ) in the row at the top of the table (denoted by $\mathrm{df}_{\mathrm{N}}$ ).

The corresponding degrees of freedom within groups (i.e. $\mathrm{df}_{\mathrm{wg}}$ ) is in the most left column of the table (denoted by df $\mathrm{D}_{\mathrm{D}}$ )
$\square$ In the example, with $\alpha=0.05\left(\mathrm{df}_{\mathrm{N}}=2\right.$ and $\left.\mathrm{df}_{\mathrm{D}}=27\right)$ the critical $F$ value is

$$
F_{\mathrm{cv}}=3.35
$$

$\square$ Conclusion:
Observe that, with $\alpha=0.05$, the $F=19.534$ lies in the rejection region (as $F>F_{c v}$ ).
Thus, we can reject $\mathrm{H}_{0}$ and accept $\mathrm{H}_{1}$ (i.e. there is a difference between at least two of the pedal arrangements and the difference is sigificant)
(This is vital if you use SPSS to perform ANOVA)

## Finding The p Value (optional)

$\square$ Note that $F$ is the ratio of two estimates i.e.

$$
F=\frac{\mathrm{MS}_{\mathrm{bg}}}{\mathrm{MS}}
$$

with $F \geq 1$
If the estimates are very different (i.e. the F value is large), then at least one of the samples probably comes from a population different from the other samples, so the $H_{1}$ is true (i.e. we can safely reject $\mathrm{H}_{0}$ )
$\square$ Recall that, in the example, using the Table $F$ with $\alpha=0.05, \mathrm{df}_{\mathrm{N}}=2$ and $\mathrm{df}_{\mathrm{D}}=27$, the critical value of $F$ value is $F_{c v}=3.35$

The computed $F=19.354$ is large as compared to $F_{c v}$ (i.e. much larger than 3.35 that we get from the Table $F$ at $\alpha=0.05!!$ ). Therefore, it is very probable for $\mathrm{H}_{1}$ to occur (i.e. very unlikely for $\mathrm{H}_{0}$ to occur!).

In this case, if $\boldsymbol{p}$ is the probability for $\mathrm{H}_{0}$ to occur, then $\boldsymbol{p}$ is small with $p<0.05$ (in fact, $\boldsymbol{p}$ is smaller than 0.01 as the $F_{c v}$ with $\alpha=0.01$ is 5.49)

This tells us that the probability for $\mathrm{H}_{0}$ to occur is very small (i.e. less than the prescribed $\alpha$ ). Thefore we reject $\mathrm{H}_{0}$ and accept $\mathrm{H}_{1}$

## The Source Table

$\square$ You will find a source table useful for the purpose of drawing up conclusion and reporting of ANOVA analysis.
$\square$ A source table displays the vital information of the ANOVA performed on the data; sum squares, degree of freedom, mean squares, $F$ value and $\boldsymbol{p}$.

## Source Table ( $\alpha=0.05$ )

| Source | $\boldsymbol{S S}$ | $\boldsymbol{d f}$ | $\boldsymbol{M S}$ | $\boldsymbol{F}$ | $\boldsymbol{p}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
|       <br> Between groups $\mathrm{SS}_{\mathrm{bg}}$ $d f_{\mathrm{bg}}$ $\mathrm{MS}_{\mathrm{bg}}$ $F$ $<$ or $>0.05$ <br> Within group $\mathrm{SS}_{\mathrm{wg}}$ $d f_{w g}$ $\mathrm{MS}_{\mathrm{wg}}$   <br> Total $\mathrm{SS}_{\text {total }}$ $d f_{\text {total }}$   ${ }^{2}$ |  |  |  |  |  |

note: a. Reject $\mathrm{H}_{0}$ if $p<0.05$
b. The exact $p$ value can be found if you have very detailed Table $F$ with various values of $\alpha$ (like the one adopted by SPSS)

## Source Table ( $\alpha=0.01$ )

| Source | $\boldsymbol{S S}$ | $\boldsymbol{d f}$ | $\boldsymbol{M S}$ | $\boldsymbol{F}$ | $\boldsymbol{p}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Between groups $\mathrm{SS}_{\mathrm{bg}}$ $d f_{b g}$ $\mathrm{MS}_{\mathrm{bg}}$ $F$ $<$ or $>0.01$ <br> Within group $\mathrm{SS}_{\mathrm{wg}}$ $d f_{w g}$ $\mathrm{MS}_{\mathrm{wg}}$   <br> Total $\mathrm{SS}_{\text {total }}$ $d f_{\text {total }}$    |  |  |  |  |  |

note: a. Reject $H_{o}$ if $p<0.01$
b. The exact $p$ value can be found if you have very detailed Table $F$ with various values of $\alpha$ (like the one adopted by SPSS)

## Source Table ( $\alpha=0.05$ )

Table 13.5 Source Table for Analysis of Variance of Pedal Error Data

| Source | Sum of squares | $d f$ | Mean square | $F$ | $p$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Between | 57.867 | 2 | 28.934 | 19.537 | $<.05$ |
| Within | 40.000 | 27 | 1.481 |  |  |
| Total | 97.867 | 29 |  |  |  |

## Post Hoc Test

$\square \quad$ The $F$ test tells us whether we can safely reject the $H_{0}$. In the case where we reject $\mathrm{H}_{0}$, it indicates that there is some difference between at least two and possibly more of the groups, but it does not reveal where that difference lies.

However, there are several tests that can do so. These are called post hoc tests. Post hoc is Latin for after the fact.
$\square$ These tests are only conducted after you have determined that you have an $F$ ratio that is significant.
$\square$ The one we discuss next is called the Tukey's HSD, which stands for Tukey's honestly significant difference.

## Performing One-Way ANOVA Tests Using SPSS

| Group | Type | Group | Type |
| :---: | :---: | :---: | :---: |
| 1 | 3 | 2 | 4 |
| 1 | 2 | 2 | 2 |
| 1 | 4 | 2 | 5 |
| 1 | 1 | 2 | 4 |
| 1 | 0 | 2 | 4 |
| 1 | 2 | 3 | 6 |
| 1 | 3 | 3 | 4 |
| 1 | 2 | 3 | 3 |
| 1 | 1 | 3 | 4 |
| 1 | 2 | 3 | 7 |
| 2 | 4 | 3 | 6 |
| 2 | 3 | 3 | 5 |
| 2 | 4 | 3 | 8 |
| 2 | 3 | 3 | 5 |
| 2 | 4 | 3 | 6 |

## ONEWAY

Type BY Group
/MISSING ANALYSIS /POSTHOC = TUKEY ALPHA(.05).

## SPSS Generated One-Way ANOVA

| Type | ANOVA |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | Sum of Squares | df | Mean Square | F | Sig. |
| Between <br> Groups | 57.8667 | 2 | 28.9333 | 19.530 | 0.000 |
| Within Groups | 40.000 | 27 | 1.4815 |  |  |
| Total | 97.8667 | 29 |  |  |  |

Sig. $=p$

## How to Draw A Conclusion About The Test?

| Method 1 | Check the value of significant <br> $p ;$ Reject $H_{0}$ if $p<\alpha$ |
| :--- | :--- |
| Method 2 | Check the value of $F$; Reject <br> $H_{0}$ if $F$ falls in the rejection <br> region (refer to $F_{c .,}$ identified <br> from the Table $F$ ) |

## Conclusion: Reject $\mathrm{H}_{0}$ (i.e. there is significant difference between the means)

## Post Hoc : Tukey's HSD

## Multiple Comparisons

Dependent Variable: Ty pe
Tukey HSD

| (I) Group | (J) Group | Mean Difference (I-J) | Std. Error | Sig. | 95\% Conf idence Interval |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | Lower Bound | Upper Bound |
| Group 1 | Group 2 | -1.800* | . 544 | . 007 | -3.15 | -. 45 |
|  | Group 3 | -3.400* | . 544 | . 000 | -4.75 | -2.05 |
| Group 2 | Group 1 | 1.800* | . 544 | . 007 | . 45 | 3.15 |
|  | Group 3 | -1.600* | . 544 | . 018 | -2.95 | -. 25 |
| Group 3 | Group 1 | $3.400^{*}$ | . 544 | . 000 | 2.05 | 4.75 |
|  | Group 2 | $1.600^{*}$ | . 544 | . 018 | . 25 | 2.95 |

*. The mean difference is significant at the .05 lev el.
Conclusion: There exists significant difference for each pair!
Observe that $p<\alpha$ for:
Group 1 vs Group 2 (with $p=0.007$ )
Group 1 vs Group 3 (with $p=0.000$ )
Group 2 vs Group 3 (with $p=0.018$ )

The F Distribution


- There are different set of formulas to solve ANOVA questions. In the next slides, you can seed the 4 steps solving ANOVA questions based on other formulas.
- source: Gravetter (2012)



## STAGE 1

- State the hypothesis

$$
H_{0}: \mu_{1}=\mu_{2}=\mu_{3}
$$

## STEP 2



## STEP 3



## STEP 4



PERFORMING ANOVA (USING SPSS)

## CALCULATING ANOVA (HYPOTHESIS TESTING) USING SPSS SOFTWARE

1. State hypothesis
$H_{o}: \mu 1=\mu 2=\mu 3$
2. Open the SPSS software
3. Insert variable
a. >variable view
b. >create data
c. >Group ....-values $=1>$ walk in shopping
$=2>$ online shopping
= $3>$ sales day shopping
d. >Sales
4. Insert the number
a. >data view $>$ Group ...- just put 1,2 and 3
>Sales ... - put in sales number
5. To find the answer
a. >analyze
b. >compare means
c. >one way ANOVA
i. >dependent list >sales
ii. >factor >group
d. $>\mathrm{OK}$
e. ...wait for the processing time....
f. >result appeared
6. Interpreting the result
a. Look at result's table
b. >sig
c. If "sig" is bigger than $0.05=$ accept the $\mathrm{H}_{0}$
d. If "sig" is smaller than $0.05=$ reject the $\mathrm{H}_{0}$
