Application of Statistics in Educational Research I

MPU1034

CORRELATED-SAMPLES *t* TEST* (paired-samples)

Prof. Dr. Mohd Salleh Abu
Dr. Hamidreza Kashefi

main source: Vernoy & Vernoy (1997)

Example:

Suppose that a group of 10 students are undergoing Stroop Effect Test. In this test, each of them was asked to time how long it takes you to name the shapes from the *shapes-alone* list. After completing this task, he/she was asked to time how long it takes you to name the shapes from the combined *words-and-shapes* list (refer to data).

Test the existence of significant difference of time taken in completing the two modes of tasks using one-tailed *t* test at $\alpha = 0.05$

Subject	List 1 (shapes only)	List 2 (words and shapes)
H. J.	22	27
T. B.	27	25
J. J.	23	32
M. V.	29	32
D. J.	32	51
P. T.	24	33
N. Z.	33	30
A. H.	23	29
K. L.	15	24
S. E.	12	20

The General Idea

- ✓ Not all *t* tests are conducted between independent sample means. Frequently, the two samples are positively correlated with each other, as in a within-subjects design where each subject participates in each of the experimental conditions.

  For example, where we asked you to time how long it takes you to name the shapes from the shapes-alone list, and then to time how long it takes you to name the shapes from the combined words-and-shapes list. If this were an experiment, you as a subject would have participated in each condition, and you would have been a member of two correlated samples.

- ✓ In this particular case, we still can use a *t* test to analyze the results to be sure that the observed difference is not due to mere chance.

  This *t* test, however, must be different from those previously described; these two samples are not independent because the same subjects participated in both conditions.

  We call this test as correlated-samples or paired-sample *t* test.

- ✓ The major difference between a *t* test for independent-samples and a *t* test for correlated-samples *is* that in the latter, the correlation between the samples can be used to reduce the size of the standard error of the difference between the sample means.

  Reducing the standard error of the difference can be a real advantage because the smaller the standard error of the difference, the larger the *t.*

- ✓ The estimated standard error of the difference between correlated sample means can be computed using:

$$\text{est. } \sigma_{\text{diff}} = \sqrt{\frac{\dfrac{\Sigma D^2}{n} - \overline{D}^2}{n-1}} \qquad (12.14)$$

where

D^2 is the squared difference of scores (for each pair)

$\overline{D}$ is the mean of the difference scores

n is the number of pair

- ✓ The estimated standard error of the difference between correlated sample means can also be computed using (in the case when the correlation coefficient r is known; refer to Vernoy & Vernoy p 271):

$$\text{est. } \sigma_{\text{diff}} = \sqrt{(\text{est. } \sigma_{\bar{X}_1})^2 + (\text{est. } \sigma_{\bar{X}_2})^2 - (2 \cdot r \cdot \text{est. } \sigma_{\bar{X}_1} \cdot \text{est. } \sigma_{\bar{X}_2})} \qquad (12.12)$$

where

$$\text{est. } \sigma_{\bar{X}} = \frac{S}{\sqrt{n-1}}$$

- ✓ The corresponding *t* score (with respect to the sample mean difference) is computed by:

$$t = \frac{\bar{D} - (\mu_1 - \mu_2)}{\text{est. } \sigma_{diff}}$$

- ✓ The degree of freedom is given by:

$$df_{\text{correlated samples}} = \text{number of pairs} - 1$$

PERFORMING CORRELATED-SAMPLES *t* TEST (MANUAL)

H_o : There is no significant difference in terms of time taken to name shapes from the *shapes-alone* list as compared with those from the combined *words-and-shapes* list

or in short

$$H_o : \mu_1 = \mu_2$$

H_1 : There is significant difference in terms of time taken to name shapes from the *shapes-alone* list as compared with those from from the combined *words-and-shapes* list

or in short

$$H_1 : \mu_1 \neq \mu_2$$

Table 12.4 The Data Listed in Table 12.3, Plus Differences (D) and Differences Squared (D^2) Between Time (in seconds) Needed to Name Shapes from Two Different Lists

Subject	List 1 (shapes only)	List 2 (words and shapes)	D	D^2
H. J.	22	27	−5	25
T. B.	27	25	2	4
J. J.	23	32	−9	81
M. V.	29	32	−3	9
D. J.	32	51	−19	361
P. T.	24	33	−9	81
N. Z.	33	30	3	9
A. H.	23	29	−6	36
K. L.	15	24	−9	81
S. E.	12	20	−8	64
			$\Sigma D = -63$	$\Sigma D^2 = 751$
			$\overline{D} = -6.3$	

Computing est. σ_{diff}:

$$\text{est. } \sigma_{diff} = \sqrt{\frac{\frac{\sum D^2}{n} - \bar{D}^2}{n-1}}$$

$$= \sqrt{\frac{\frac{751}{10} - (-6.3)^2}{10-1}}$$

$$= 1.983$$

Computing t score:

$$t = \frac{\bar{D} - (\mu_1 - \mu_2)}{\text{est. } \sigma_{diff}}$$

If H_0 is true (i.e. if $\mu_1 = \mu_2$)

$$t = \frac{\bar{D}}{\text{est. } \sigma_{diff}} = \frac{-6.3}{1.983}$$

$$= -3.177$$

Computing Degree of Freedom *df*

df = number of pairs – 1 = 10 – 1 = 9

Identifying critical value of *t* for the rejection of H_o:

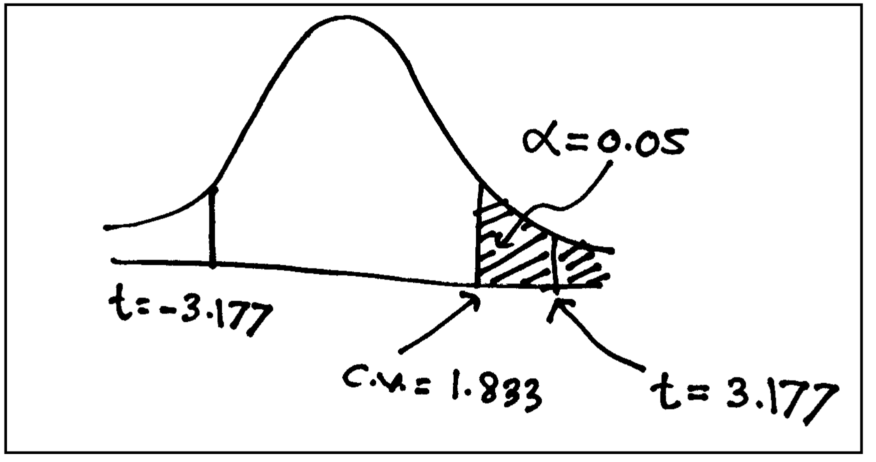
- ✓ Because our research hypothesis is one-tailed (we predicted that it would take longer to name the shapes when the words are present), we can look up the critical value of *t* in Table T for a one-tailed test with 9 degrees of freedom. That critical *t*-value is 1.833.

- ✓ Before you even look up the *t* value in the table, you need to analyze what your hypothesis predicts. If it predicts that the second mean will be greater than the first mean, you should expect a negative t-value because the formula requires that you subtract the second mean from the first.

  If, on the other hand, your hypothesis predicts that the first mean will be greater than the second, you should expect a positive t-value.

  Remember, when the research hypothesis predicts a negative *t* value and your computed *t*-value is negative, or when the research hypothesis predicts a positive *t*-value and your computed *t*-value is positive, you can ignore the sign when looking up the value in Table T and compare the absolute value of *t* to the critical value listed in the table.

5	0.267	0.727	1.476	2.015	2.571
6	0.265	0.718	1.440	1.943	2.447
7	0.263	0.711	1.415	1.895	2.365
8	0.262	0.706	1.397	1.860	2.306
9	0.261	0.703	1.383	1.833	2.262



Conclusion:

We reject H_o and accept H_1

(i.e. there is significant difference in terms of time taken to name shapes from the *shapes-alone* list as compared with those from the combined *words-and-shapes* list)

PERFORMING CORRELATED-SAMPLES *t* TEST (USING SPSS)

Variables Tested : Shapes versus Words+Shapes

t Test : Correlated-Samples

Groups Tested : Group 1 (Shapes only) Group 2 (Words+Shapes)

SPSS Output : (α = 0.05 ; 2-tailed)

Correlated-Samples Statistics (Shapes vs Words+Shapes)

	Samples	Mean	N	Std. Dev.	SE Mean
Pair 1	Shapes Only	24.00	10.00	6.75	2.13
	Words+Shapes	30.30	10.00	8.35	2.64

Correlation (Shapes vs Words+Shapes)

N	Correlation	Sig.
10	0.6739	0.0326

Correlated-Samples Test ($\alpha = 0.05$; 2-tailed)

Mean	Std. Dev.	SE Mean	95% Confidence Interval of Confidence		t	df	Sig. (2-tailed)
			Lower	Upper			
-6.300	6.273	1.984	-10.787	-1.813	-3.176	9	0.011

How to Draw A Conclusion About The Test?

Method 1	Check the significant value of *p* ; Reject H_0 if $p < \alpha$
Method 2	Check the value of *t* ; Reject H_o if *t* falls in the rejection region (refer to $t_{c.v}$ identified from the Table *t*)