Application of Statistics in Educational Research I

MPU1034
INDEPENDENT-SAMPLES t TEST*

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main source: Vernoy & Vernoy (1997)

<u>Example</u>: (Refer to Dr. Tee's language-training experiment discussed earlier)

Suppose that instead of having us compare the sample mean to the population mean, Dr. Tee asks us to compare her experimental group to a control group of 12 similar children who did not receive language training (refer to data).

Test the existence of significant increase of Dr. Tee's language-training between the experimental and control groups using one-tailed t test at $\alpha = 0.05$

Research group Number of words in vocabulary	
Child	X_1
C. W.	197
D. J.	223
P. V.	241
J. I.	183
T. B.	222
B. C.	231
R. A.	297
B. B.	220
D. T.	188
P. P.	231
C. D.	210
M. L.	234

Control group Number of words in vocabulary	
Child	X_2
P. J.	206
D. M.	199
Q. C.	205
M. V.	203
B. T.	223
Z. S.	189
K. H.	221
B. S.	195
G. S.	218
A. G.	177
H. D.	203
K. T.	174

The General Idea

✓ Note that most psychological research is not designed in such a way that the mean of one sample is compared to the population mean. In practice, most experimenters like to use control groups in their research. Often control groups are used as substitutes for population values.

For example, in Dr. Tee's vocabulary-learning experiment, we could have compared the 12 infants in the experimental group to a similar control group that did not go through the language-training program. Then, instead of comparing the mean of the research sample to the mean of the population, we could have compared the mean of the research sample to the mean of the control sample. This is normally done by computing the difference between the two means and then comparing this difference to the mean of the sampling distribution of differences between means.

✓ If there are two independent samples from populations that have different means, with population 1 being the control population and population 2 being the research population, then the distribution of differences between the sample means should have a mean equal to the difference between the two population means:

$$\mu_1 - \mu_2$$

Thus, when we are comparing two independent samples,

H_o = The two samples derive from populations with equal means (i.e. $\mu_1 - \mu_2 = 0$)

H_1 = The two samples derive from populations with equal means (i.e. $\mu_1 - \mu_2 \neq 0$)

✓ We can use a independent-samples t test to test these hypotheses. Note that in this case, the score we are testing is the *difference* between the two sample means

✓ The standard error of the difference between two independent samples (σ_{diff}) is computed using:

$$\sigma_{diff} = \sqrt{\sigma^2_{\bar{x}_1} + \sigma^2_{\bar{x}_2}} \qquad (12.7)$$

✓ The estimated standard error of mean (est. σ_{diff}) is estimated using:

$$\text{est. } \sigma_{diff} = \sqrt{(\text{est. } \sigma_{\bar{x}_1})^2 + (\text{est. } \sigma_{\bar{x}_2})^2} \qquad (12.8)$$

$$\text{est. } \sigma_{diff} = \sqrt{\frac{S_1^2}{(n_1 - 1)} + \frac{S_2^2}{(n_2 - 1)}} \qquad (12.9)$$

✓ The corresponding *t* score (with respect to the sample mean difference) is computed by:

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\text{est. } \sigma_{diff}}$$

PERFORMING INDEPENDENT-SAMPLES t TEST (MANUAL)

Table 12.2 Number of Words in Vocabulary for Research Group of 12 Toddlers Using the Language-Training System and Control Group of 12 Toddlers Without the Language-Training System

	Research group Number of words in vocabulary			Control group Number of words in vocabulary	
Child	X_1	X_1^2	Child	X_2	X_2^2
C. W.	197	38,809	P. J.	206	42,436
D. J.	223	49,729	D. M.	199	39,601
P. V.	241	58,081	Q. C.	205	42,025
J. I.	183	33,489	M. V.	203	41,209
T. B.	222	49,284	B. T.	223	49,729
B. C.	231	53,361	Z. S.	189	35,721
R. A.	297	88,209	K. H.	221	48,841
B. B.	220	48,400	B. S.	195	38,025
D. T.	188	35,344	G. S.	218	47,524
P. P.	231	53,361	A. G.	177	31,329
C. D.	210	44,100	H. D.	203	41,209
M. L.	234	54,756	K. T.	174	30,276

$$\Sigma X_1 = 2677 \qquad \Sigma X_1^2 = 606,923 \qquad\qquad \Sigma X_2 = 2413 \qquad \Sigma X_2^2 = 487,925$$

$$\overline{X}_1 = 223.083 \qquad\qquad\qquad \overline{X}_2 = 201.083$$

$$S_1 = 28.474 \qquad\qquad\qquad S_2 = 15.030$$

H_o : $\mu_1 - \mu_2 = 0$

H_1 : $\mu_1 - \mu_2 \neq 0$

Directly Computable Indices

$$\bar{X}_1 = 223.083$$

$$S_1 = 28.474$$

$$\bar{X}_2 = 201.083$$

$$S_2 = 15.030$$

Use

$$S = \sqrt{\frac{\sum X^2 - \bar{X}^2}{n}}$$

Computing est. σ_{diff}

$$\text{est. } \sigma_{diff} = \sqrt{\frac{s_1^2}{(n_1-1)} + \frac{s_2^2}{(n_2-1)}}$$

$$= \sqrt{\frac{28.474^2}{(12-1)} + \frac{15.030^2}{(12-1)}}$$

$$= 9.708$$

Computing t score:

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\text{est. } \sigma_{diff}}$$

If H_o is true (i.e. $\mu_1 - \mu_2 = 0$),

$$t = \frac{(\bar{X}_1 - \bar{X}_2)}{\text{est. } \sigma_{diff}} = \frac{223.083 - 201.083}{9.708}$$

$$= 2.266$$

Computing df:

For independent-samples,

$$df = (n_1 - 1) + (n_2 - 1)$$

$$= (12 - 1) + (12 - 1)$$

$$= 22$$

Identifying critical value of t for the rejection of H_o:

By looking in Table T, we find that for a one-tailed research hypothesis with 22 degrees of freedom, the critical value of t equals **1.717.**

20	0.257	0.687	1.325	1.725	2.086
21	0.257	0.686	1.323	1.721	2.080
22	0.256	0.686	1.321	1.717	2.074
23	0.256	0.685	1.319	1.714	2.069
24	0.256	0.685	1.318	1.711	2.064

Conclusion:

Our computed t score of **2.266** is greater than the table value, so we can reject the null hypothesis and accept the research hypothesis. This means that the language-training method does indeed lead to a more expanded vocabulary for 2-year-olds who participated in the program than for those in the control group who did not participate.
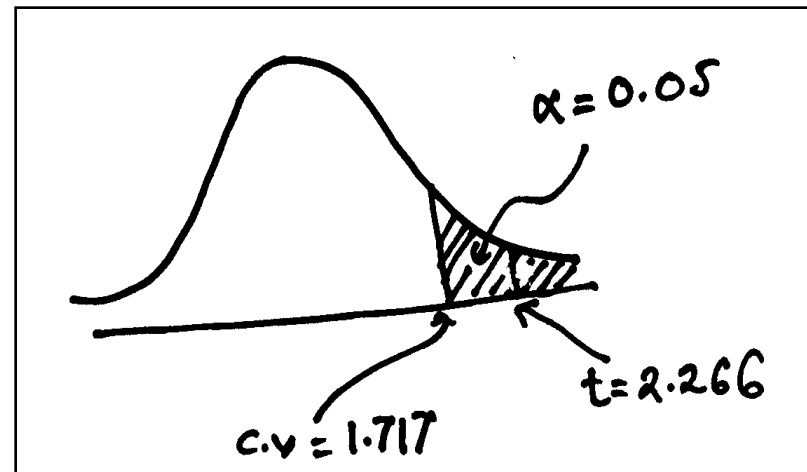
PERFORMING INDEPENDENT-SAMPLES t TEST (USING SPSS)

Variable Tested: No_Words

t Test   :   Independent-Samples

Groups Tested   :   Research Group vs Control Group

SPSS Output   :   (α = 0.05 ; 2-tailed)

Assumption	Levene's Test for Equality of Variances	
	F	Sig.
Equal variances assumed	1.35	0.26
Equal variances not assumed		

T Test for Equality of Means

t	df	Sig. (2-tailed)	Mean_Diff	SE_Diff	95% Confidence Interval of the Difference	
					Lower	Upper
2.266	22	0.034	22	9.708	1.867	42.133
2.266	16.689	0.037	22	9.708	1.489	42.511

How to Draw A Conclusion About The Test?

Method 1	Check the significant value of p ; Reject H_0 if $p < \alpha$
Method 2	Check the value of t ; Reject H_o if t falls in the rejection region (refer to $t_{c.v}$ identified from the Table t)