Application of Statistics in Educational Research I

MPU1034

SINGLE-SAMPLE *t* TEST*

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main source: Vernoy & Vernoy (1997)

The General Idea

- ✓ Performing hypothesis testing on a single group of sample
- ✓ Similar test as z test but applied on a small size sample
- ✓ Can be approximated by z test if n is large enough
- ✓ Hypotheses to be tested:

$$H_o: \mu_{\bar{X}} = \mu$$

$$H_1: \mu_{\bar{X}} \neq \mu$$

Note: We MUST know μ to perform a single-sample *t* test!

Computing *t* Score

✓ The formula for computing the *t* score is essentially the same as that for computing the z score:

$$t = \frac{\overline{X} - \mu_{\overline{X}}}{\sigma_{\overline{X}}}$$

(if the standard deviation of population is known)

or

$$t = \frac{\overline{X} - \mu_{\overline{X}}}{\text{est. } \sigma_{\overline{X}}}$$

(if the standard deviation of population is unknown)

where $\text{est. } \sigma_{\overline{X}}$ is the estimated std error of mean

Estimating The Std. Deviation & Std. Error

Question: How to compute the standard error of the mean est. $\sigma_{\bar{X}}$?

✓ If we know the standard deviation of the population, we should use it; in this case,

$$t = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}}$$

where

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

, σ being the std dev. of population

✓ If we do not, we must estimate it. We can do this in two ways:

- Use Formula 10.6 to obtain the estimated standard deviation of the population and then use Formula 12.1, which follows, to compute the standard error of the mean:

$$\text{est. } \sigma = \sqrt{\frac{\Sigma(X - \overline{X})^2}{n - 1}} = \sqrt{\frac{\Sigma X^2 - (n \cdot \overline{X}^2)}{n - 1}} \quad (10.6)$$

$$\text{est. } \sigma_{\overline{X}} = \frac{\text{est. } \sigma}{\sqrt{n}} \quad (12.1)$$

- If we don't have access to raw data, we must use another method, which involves the formula 12.2:

$$\text{est. } \sigma_{\overline{X}} = \frac{s}{\sqrt{n - 1}} \quad (12.2)$$

where $\text{est. } \sigma$ is the estimated std dev. of population

$\text{est. } \sigma_{\overline{X}}$ is the estimated std error of mean

s is the sample standard deviation

Example:

Dr. Tee has developed a language-training system that, she claims, significantly increases the number of new words acquired by infants. Average children in this part of the world begin to speak a few basic words by the time they are 1 year old; by the time they are 2, average toddlers have a vocabulary of 210 words. To test her system, Dr. Tee randomly selects 12 sets of parents who are willing to use her language-training system with their newborn infants for 2 years. At the end of the 2-year test period, she tabulates the number of words in each toddler's vocabulary. The results are as displayed.

Test the existence of significant increase of Dr. Tee's language-training using one-tailed *t* test at $\alpha = 0.05$ (as compared with the population!)

Child	Number of words in vocabulary X
C. W.	197
D. J.	223
P. V.	241
J. I.	183
T. B.	222
B. C.	231
R. A.	297
B. B.	220
D. T.	188
P. P.	231
C. D.	210
M. L.	234

H_o : The language-training system will make no difference in the size of a child's vocabulary as measured at age 2 years.

H_1 : The language-training system will increase the size of a child's vocabulary as measured at age 2 years.

$H_o: \mu_{\bar{X}} = \mu$

$H_1: \mu_{\bar{X}} \neq \mu$

Available Indices:

$\bar{X}$ = mean of sample = 223.083

s = std. dev. of sample = 28.474

n = sample size = 12

df = degree of freedom = 12−1
(for single-sample)

μ = mean of population = 210

Non-Available Index:

σ = std. dev. of population

Indices Required (via estimation):

est. $\sigma_{\bar{X}}$ = estimated std error of mean

(est. σ = estimated std dev. of population)

Table 12.1 Number of Words in Vocabulary for 12 Toddlers Using the Language-Training System

Child	Number of words in vocabulary X	X^2
C. W.	197	38,809
D. J.	223	49,729
P. V.	241	58,081
J. I.	183	33,489
T. B.	222	49,284
B. C.	231	53,361
R. A.	297	88,209
B. B.	220	48,400
D. T.	188	35,344
P. P.	231	53,361
C. D.	210	44,100
M. L.	234	54,756
	$\Sigma X = 2677$	$\Sigma X^2 = 606{,}923$
	$\bar{X} = 223.083$	
	$S = 28.474$	

Computing est. $\sigma_{\bar{X}}$

Method 1:

$$\text{est. } \sigma_{\bar{X}} = \frac{s}{\sqrt{n-1}} = \frac{28.474}{\sqrt{11}}$$

$$= 8.584$$

Method 2:

$$\text{est. } \sigma = \sqrt{\frac{\sum X^2 - (n\bar{X}^2)}{n-1}}$$

$$= \sqrt{\frac{606,923 - 12(223.083)^2}{12-1}}$$

$$= 29.742$$

and therefore

$$\text{est. } \sigma_{\bar{X}} = \frac{\text{est. } \sigma}{\sqrt{n}} = \frac{29.742}{\sqrt{12}}$$

$$= 8.586$$

$$\cong 8.584$$

Computing t score:

$$t = \frac{\bar{X} - \mu_{\bar{X}}}{\text{est. } \sigma_{\bar{X}}}$$ with $H_0 : \mu_{\bar{X}} = \mu$

$H_1 : \mu_{\bar{X}} \neq \mu$

where

$\bar{X} = 223.083$

$\mu = 210$

If H_0 is true (i.e. $\mu_{\bar{X}} = \mu$)

$$t = \frac{\bar{X} - \mu}{\text{est. } \sigma_{\bar{X}}}$$

$$= \frac{223.083 - 210}{8.584} \approx 1.524$$

Computing df:

For a single-sample t test,

$$df = n - 1 = 12 - 1 = 11$$

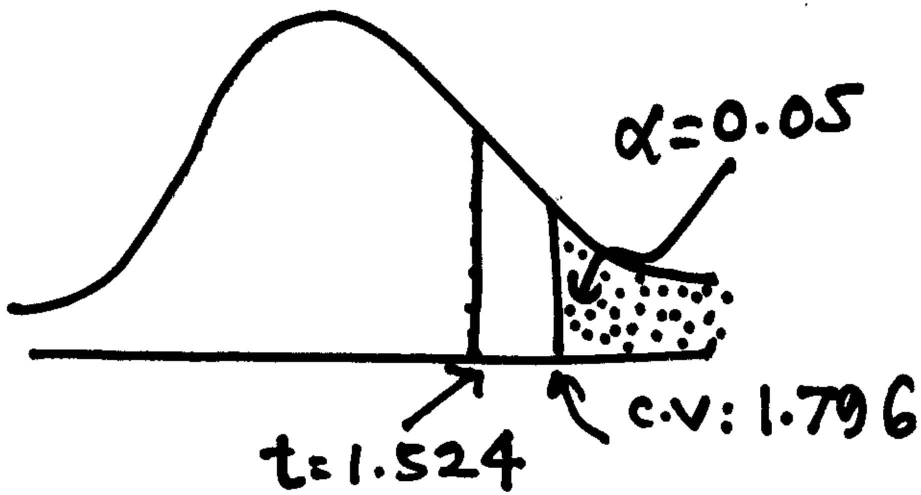
10	0.260	0.700	1.372	1.812
11	0.260	0.697	1.363	1.796
12	0.259	0.695	1.356	1.782
13	0.259	0.694	1.350	1.771
14	0.258	0.692	1.345	1.761

Finding the critical value of t for rejection of H_0 (one-tailed, $\alpha = 0.05$)

From the t table (with df = 12), the critical value of t at 0.05 level is

$$t = 1.796$$

Conclusion:



Fail to reject H_0 i.e. Dr. Tee's language training system will make no significant difference in the size of a child's vocabulary.

Performing Single-Sample *t* Test Using SPSS*

Strong Note:

You must put $\mu = 210$ as the test value for this particular case!

The Syntax:

```
T-TEST
  /TESTVAL = 210
  /MISSING = ANALYSIS
  /VARIABLES = No_Words
  /CRITERIA = CI(.95) .
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One-Sample Statistics:

Variable	N	Mean	Std. Dev.	SE Mean
No_Words	12	223.083	29.740	8.585

One-Sample Test (Test Value μ = 210 , α = 0.05 2-tailed)

Variable	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
					Lower	Upper
No_Words	1.524	11	0.156	13.083	-5.812	31.979

How to Draw A Conclusion About The Test?

Method 1	Check the significant value of p ; Reject H_0 if $p < \alpha$
Method 2	Check the value of t ; Reject H_o if t falls in the rejection region (refer to $t_{c.v}$ identified from the Table t)