SSCE1993 ENGINEERING MATHEMATICS

MULTIVARIABLE FUNCTIONS

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What is $y = f(x)$?

y *is a single variable function of* x

y is the dependent variable

x is the independent variable

f is a rule or formula that x must follow that will give a **unique** value of y

What is the domain and range of the function $y = f(x)$?

The domain of f is the set of all values x that are real and satisfying $y = f(x)$ all non-real values of y are avoided.

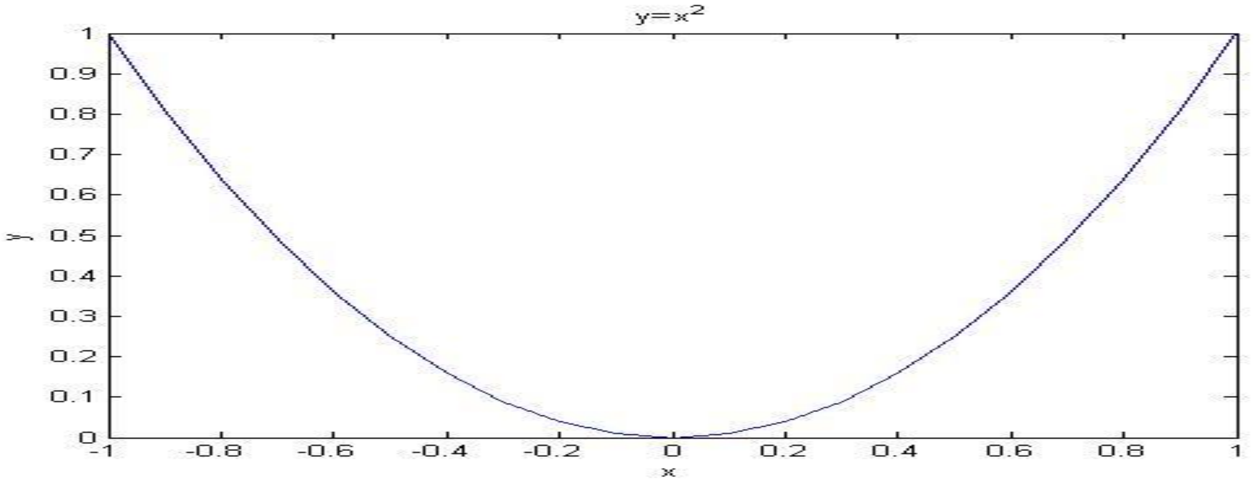
The range of f is the set of all real values of y satisfying $y = f(x)$ for all x in its domain.

The domain and the range of f are denoted by

$$D_f = \{x / x \in R\} \quad R_f = \{y / y \in R\}$$

We can represent $y = f(x)$ in a two dimensional coordinate systems (2D).

Example: The graph of $y = x^2$



The graph of $y = f(x)$ is a curve in 2D.

Classroom activity: Sketch the following curves in 2D.

1) $y = x$, 2) $y = 2x + 1$, 3) $y = x^3$, 4) $y = \dfrac{1}{x}$,

5) $y = 4 - x^2$, 6) $x = y^2$, 7) $x^2 + y^2 = 4$,

8) $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$, 9) $\dfrac{x^2}{4} - \dfrac{y^2}{9} = 1$.

What is $z = f(x, y)$?

z *is a two variable function of* (x, y)

z is the dependent variable

(x, y) is the independent variable

f is a rule or formula that (x, y) must follow that will give a **unique** value of z

What is the domain and range of the function $z = f(x,y)$?

The domain of f is the set of all values (x,y) that are real and satisfying $z = f(x,y)$ such that all the non-real values of z are avoided.

The range of f is the set of all real values of z satisfying $z = f(x,y)$ for all (x,y) in its domain, and they can be denoted by

$$D_f = \{(x,y) / x \in R, y \in R\} \quad R_f = \{z / z \in R\}$$

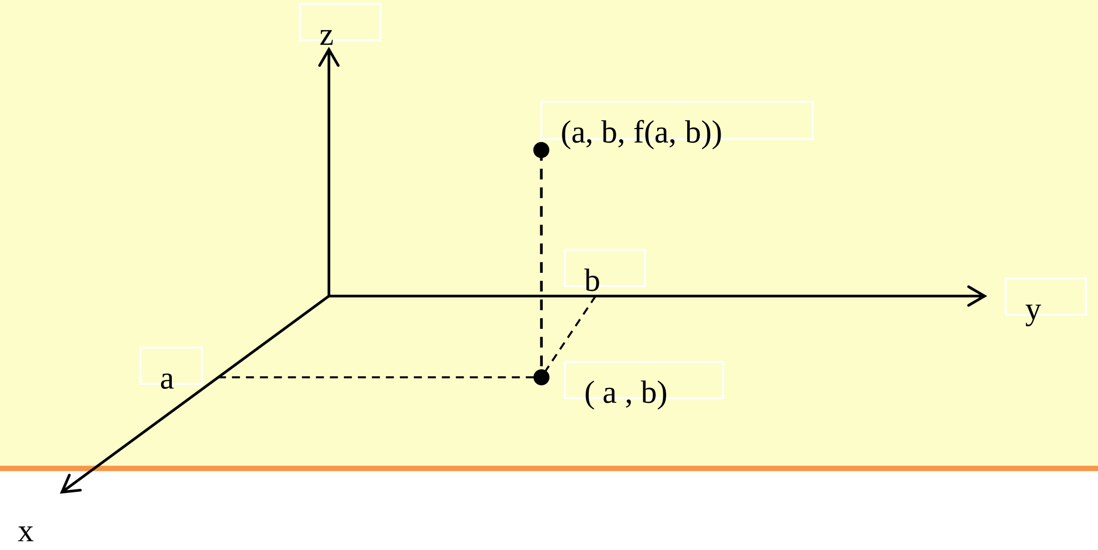
Some Examples of $z = f(x, y)$.

$$z = f(x, y) = \sqrt{x^2 + y^2},$$

$$z = f(x, y) = x^2 + y^2,$$

$$z = f(x, y) = \sqrt{4 - x^2 - y^2}.$$

The Three Dimensional Coordinate System (3D)



The graph of $z = f(x, y)$ is a **surface** in 3D.

Class Activity :Sketch the following surfaces in 3D.

$$y = x,\ y = 2x + 1,\ y = x^3,\ y = \frac{1}{x},\ y = 4 - x^2,\ x = y^2,\ x^2 + y^2 = 4,$$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1,\ \frac{x^2}{4} - \frac{y^2}{9} = 1$$

$$z = \sqrt{x^2 + y^2},\ z = x^2 + y^2,\ z = \sqrt{4 - x^2 - y^2}.$$

$$x^2 + y^2 + z^2 = 4,\ \frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1,\ \frac{x^2}{4} - \frac{y^2}{9} + z^2 = 1$$

$z = f(x, y)$ can be represented as a set of level curves in 2D

When we substitute w with several constant c where $c \in R_f$ we obtained several curves $c = f(x, y)$ that can be sketch in one 2D graph. These curves in 3D are known as a set of level curves for $z = f(x, y)$

and each curve is labeled $z = c$

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Class Activity :Sketch the level curves in 2D for the following $z = f(x, y)$ for $z = c$.

$$z = x^2 + y^2;\ z = 0, 1, 2, 3, 4$$

$$z = y / x;\ z = -2, -1, 0, 1, 2$$

$$z = x^2 + y;\ z = -2, -1, 0, 1, 2$$

$$z = x^2 + 9y^2;\ z = 0, 1, 2, 3, 4$$

What is $w = f(x, y, z)$?

w *is a three variable function of* (x, y, z)

w is the dependent variable

(x, y, z) is the independent variable

f is a rule or formula that (x, y, z) must follow that will give a **unique** value of w

What is the domain and range of the function $w = f(x, y, z)$

The domain of f is the set of all values (x, y, z) that are real and satisfying $w = f(x, y, z)$ such that all the non-real values of w are avoided.

The range of f is the set of all real values of w satisfying $w = f(x, y, z)$ for all (x, y, z) in its domain, and they can be denoted by

$$D_f = \{(x, y, z) / x \in R, y \in R, z \in R\} \quad R_f = \{z / z \in R\}$$

$w = f(x, y, z)$ can be represented as a set of level surfaces in 3D

When we substitute w with several constant c where $c \in R_f$ we obtained several surfaces $c = f(x, y, z)$ that can be sketch in one 3D graph. These surfaces in 3D are known as a set of level surfaces for $w = f(x, y, z)$ and each surface is labeled $w = c$.

Class Activity :Sketch the level surfaces in 3D for the following $w = f(x, y, z)$ for $w = c$.

$$w = f(x, y, z) = x^2 + y^2 + z^2;\ w = 0, 1, 4, 9$$

$$w = f(x, y, z) = 4x^2 + y^2 + 4z^2;\ w = 16$$

$$w = f(x, y, z) = x^2 - y^2 + z^2;\ w = 0$$

$$w = f(x, y, z) = 2x - 4y + z;\ w = 1$$

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