Theory of Computer Science – SCJ 3203

Turing Machine

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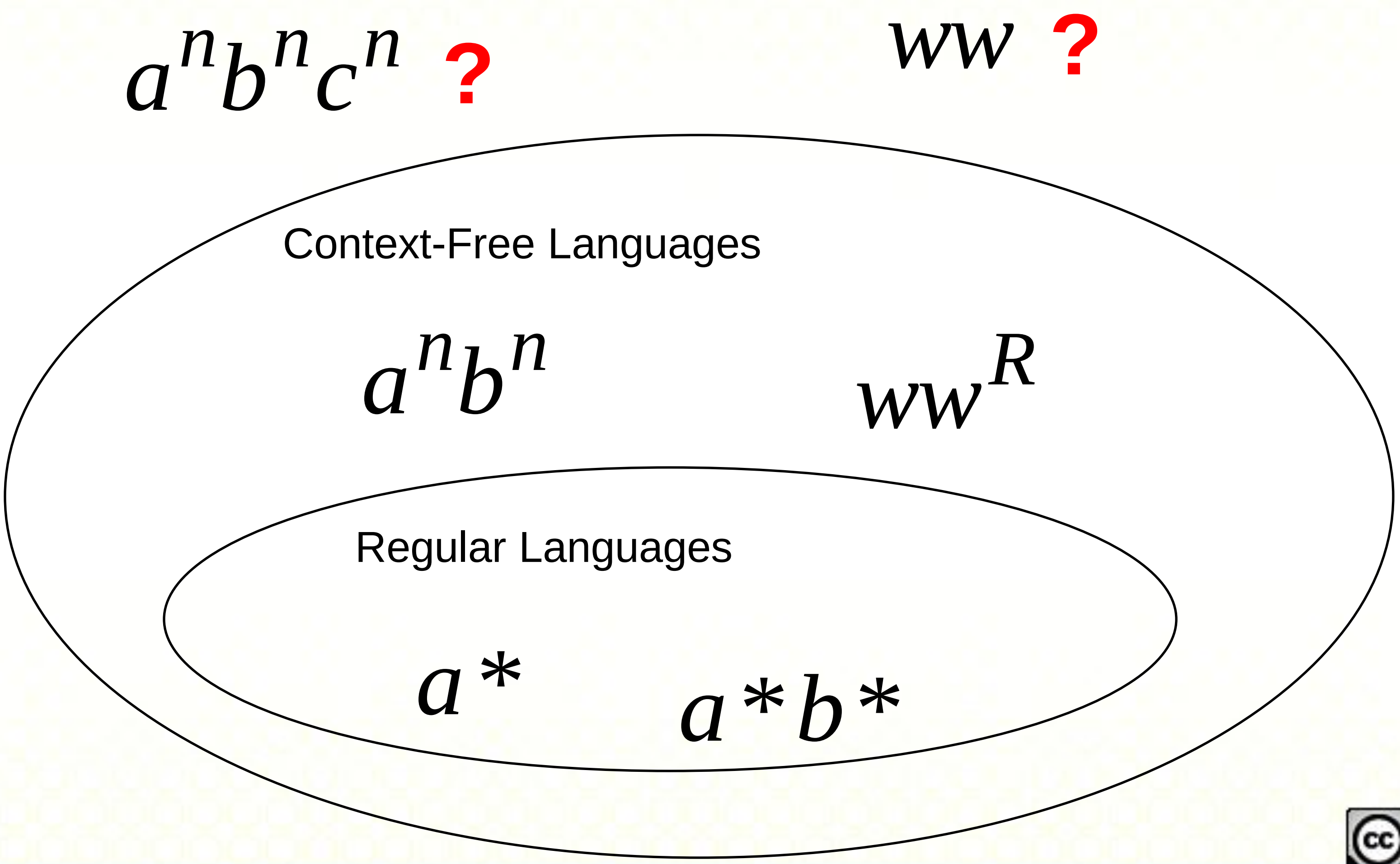
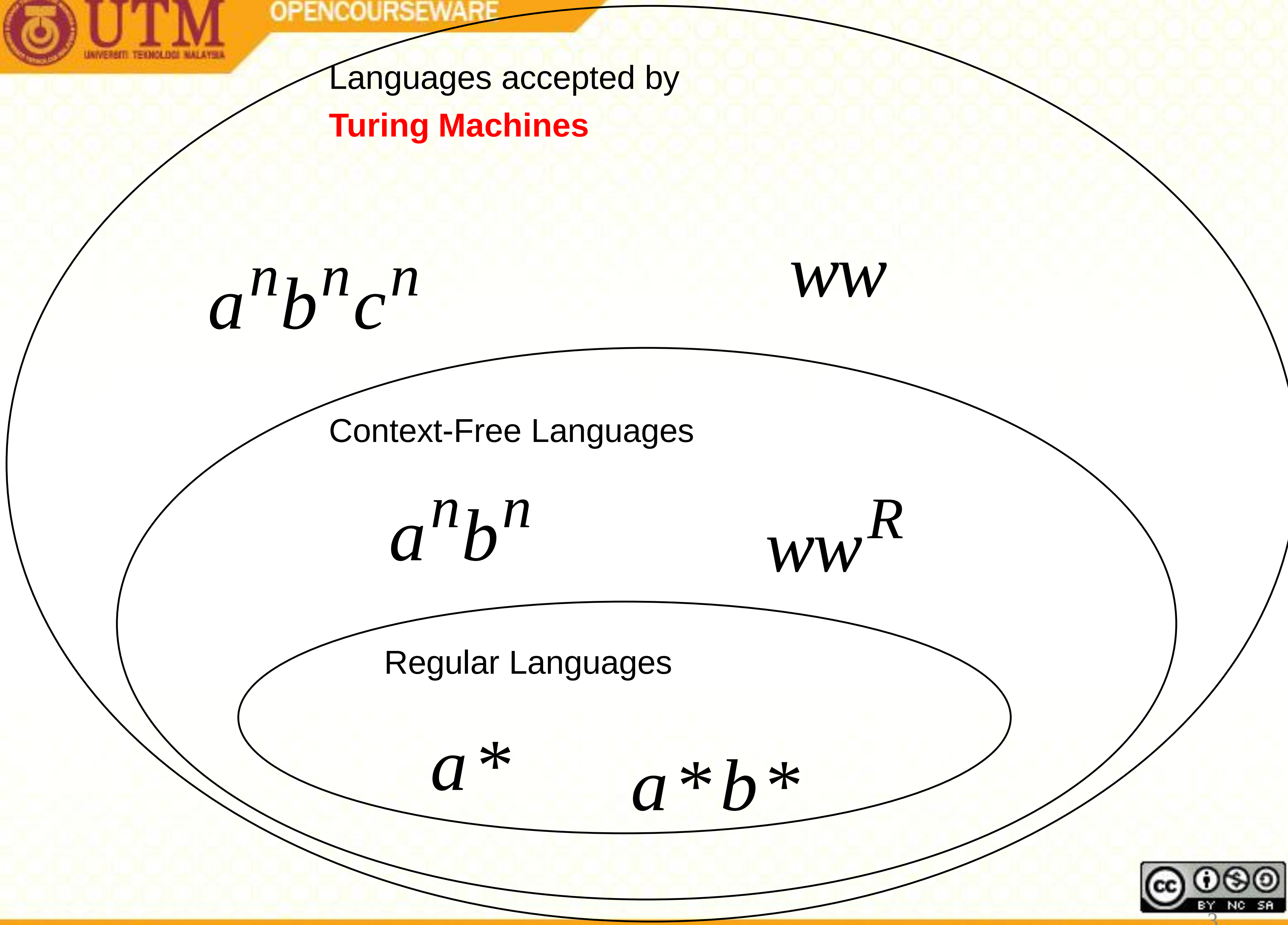
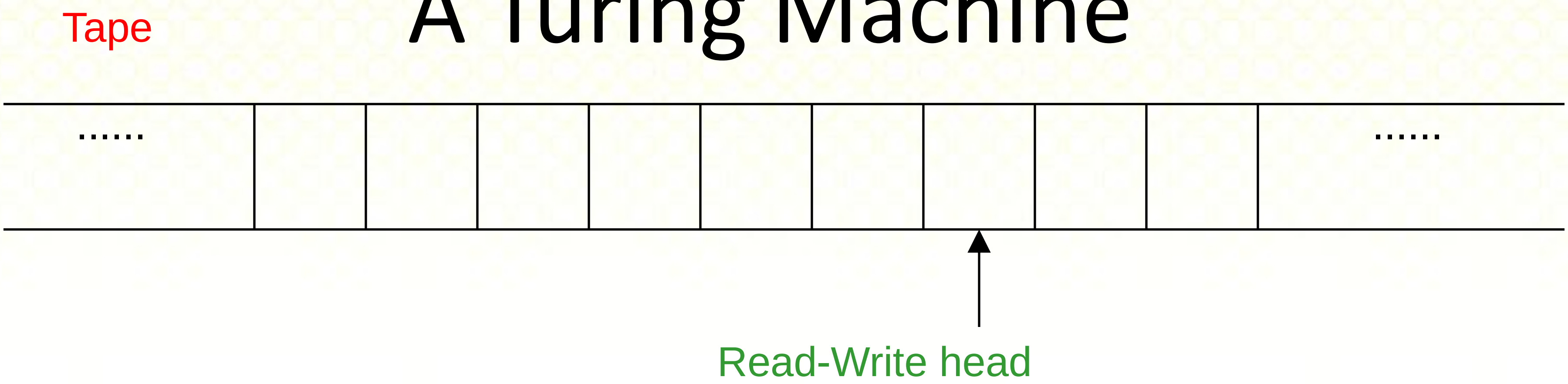
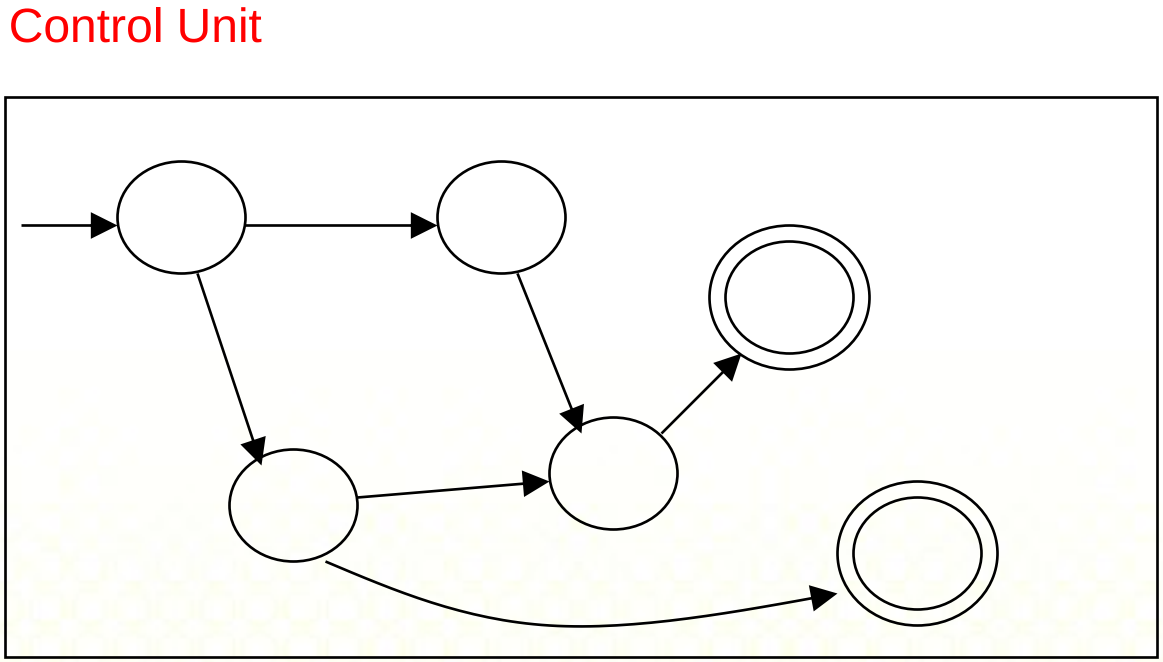
The Language Hierarchy




Languages accepted by
Turing Machines
$a^n b^n c^n$
ww
Context-Free Languages
$a^n b^n$
ww^R
Regular Languages
$a*$
$a*b*$

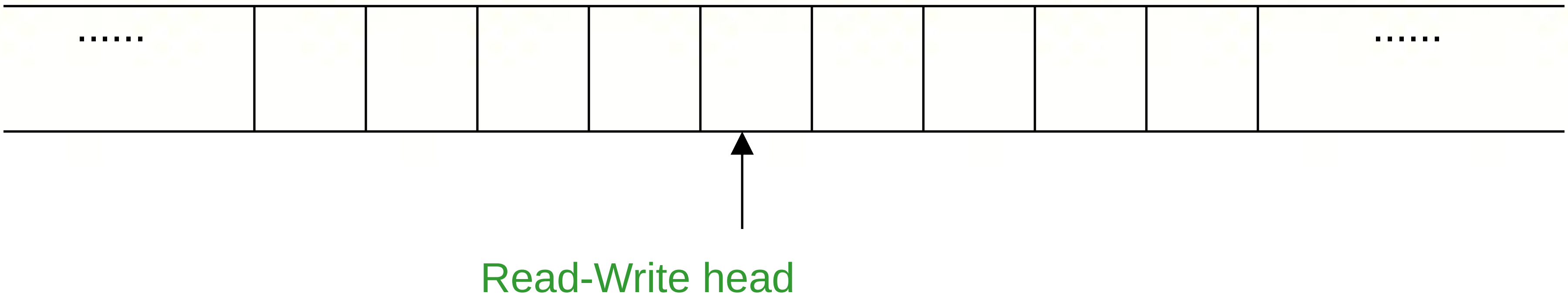
A Turing Machine



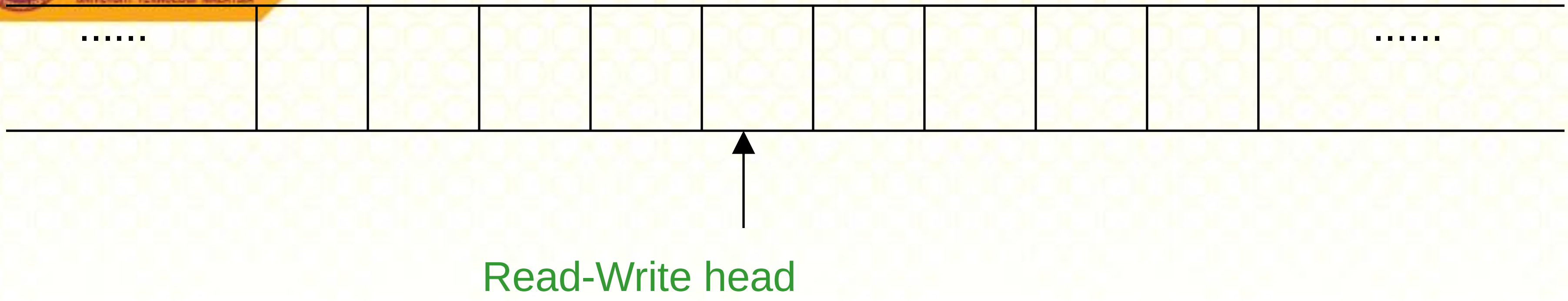


The Tape

No boundaries -- infinite length



The head moves Left or Right

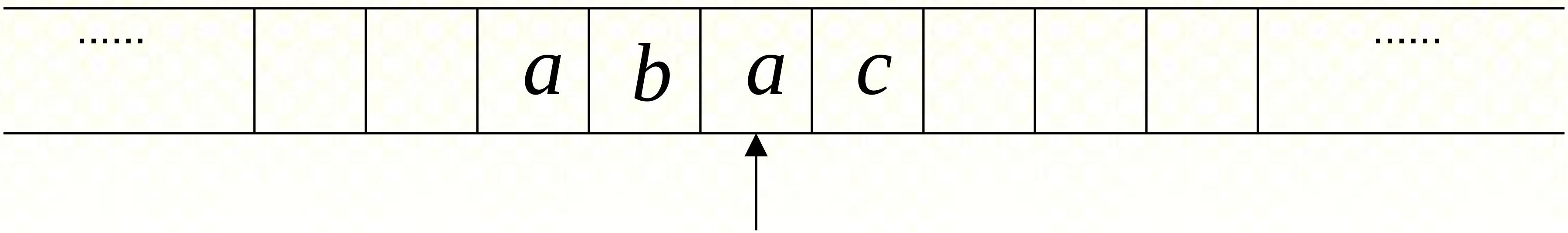


The head at each time step:

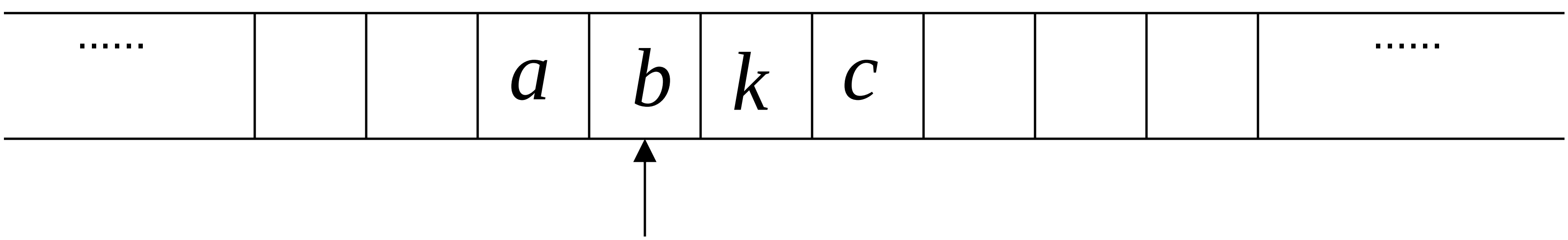
1. Reads a symbol
2. Writes a symbol
3. Moves Left or Right

Example:

Time 0



Time 1

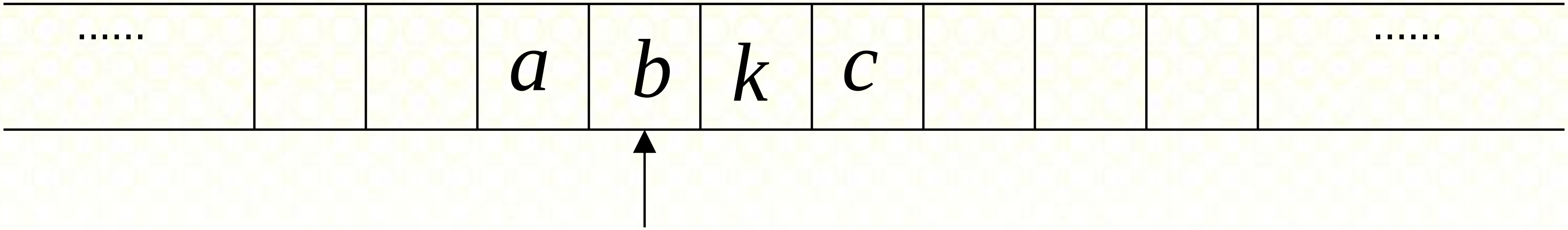


1. Reads a

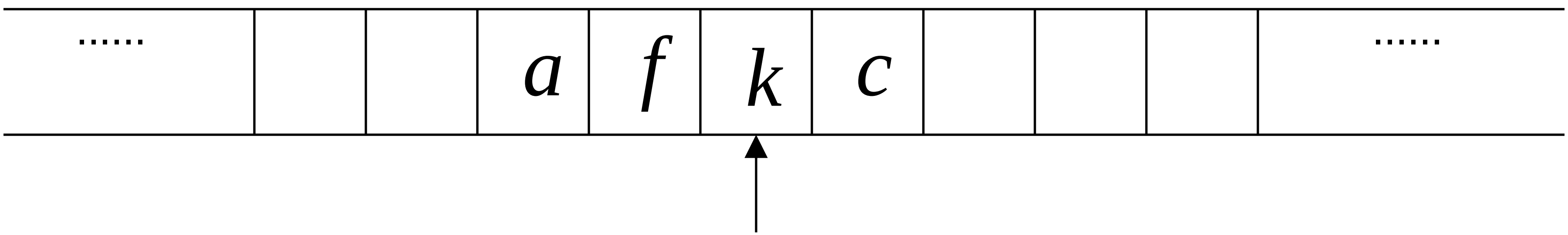
2. Writes k

3. Moves Left

Time 1



Time 2

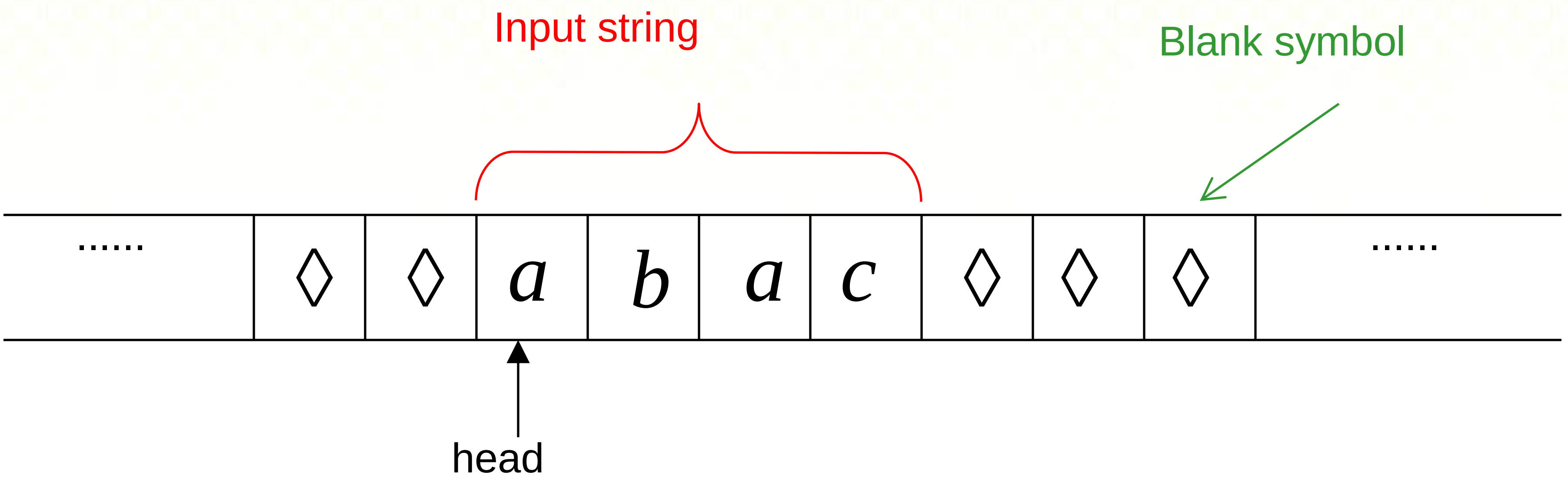


1. Reads b

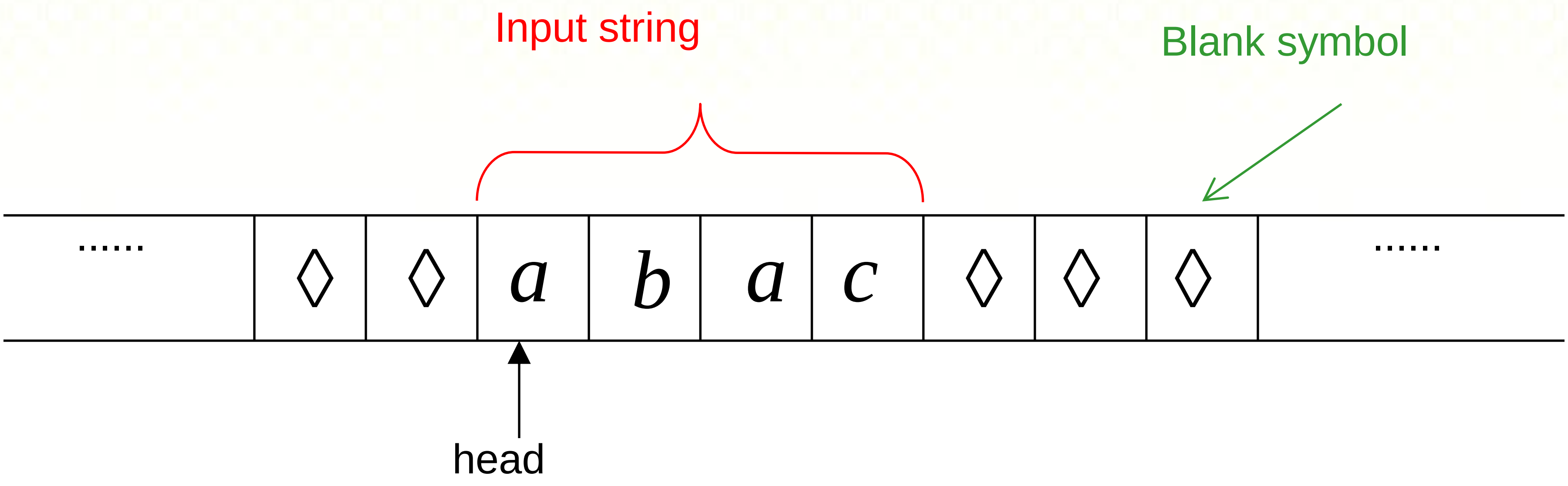
2. Writes f

3. Moves Right

The Input String

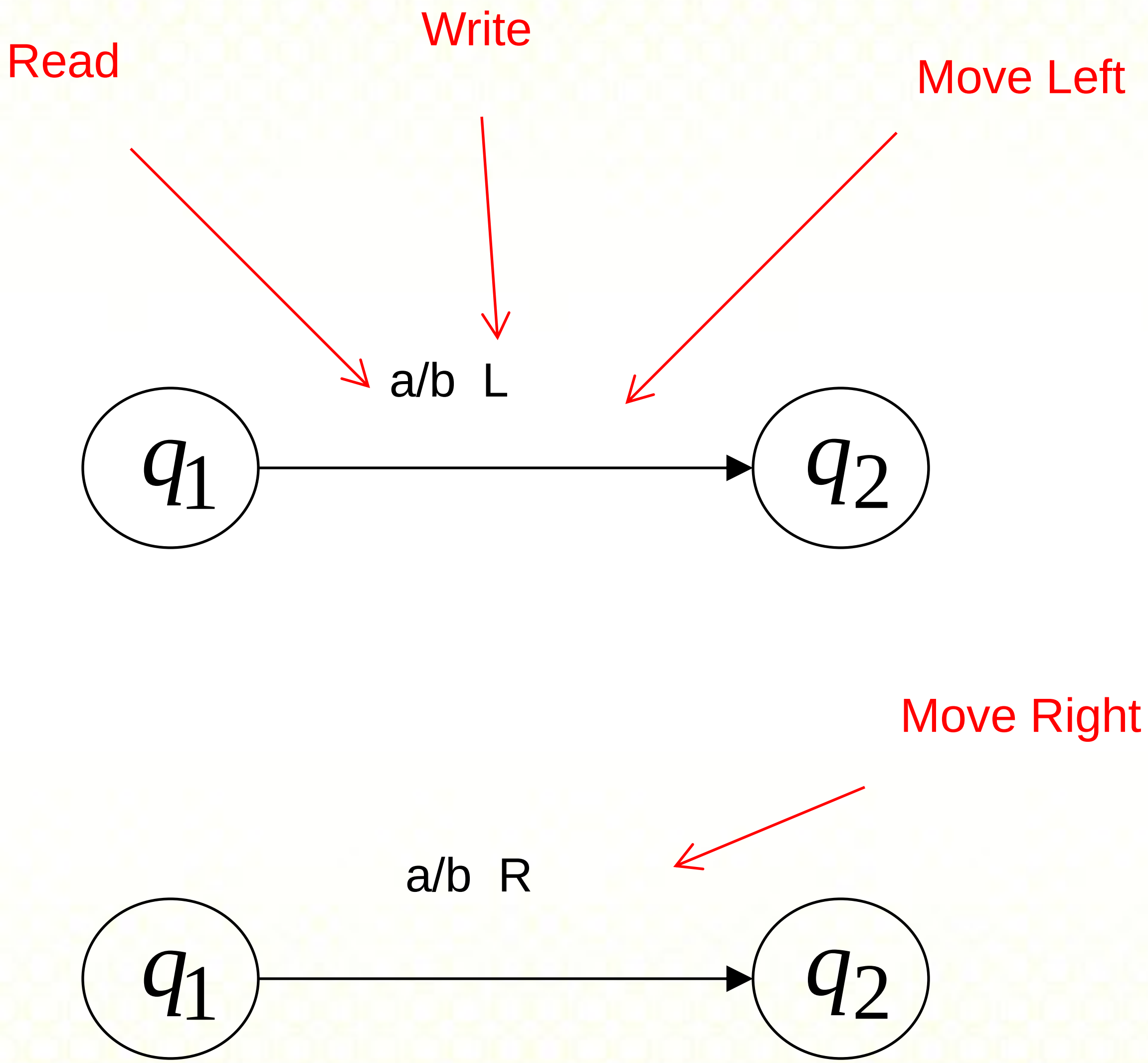


Head starts at the leftmost position of the input string



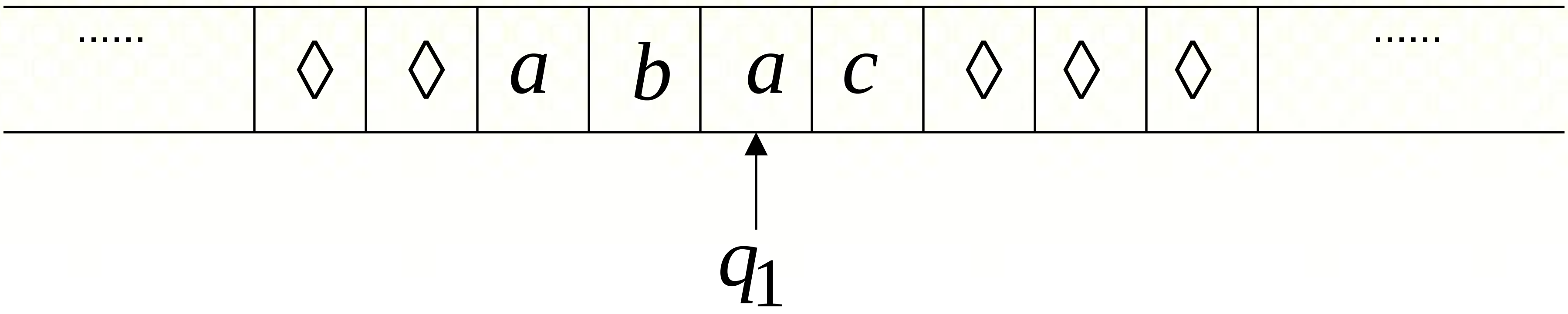
Remark: the input string is never empty

States & Transitions

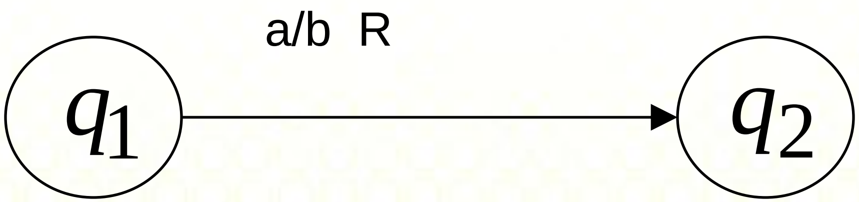


Example:

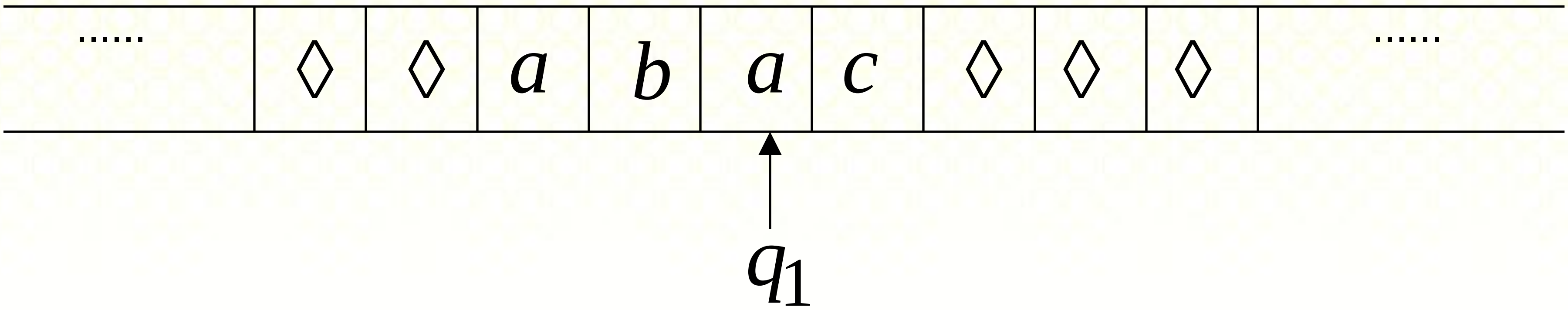
Time 1



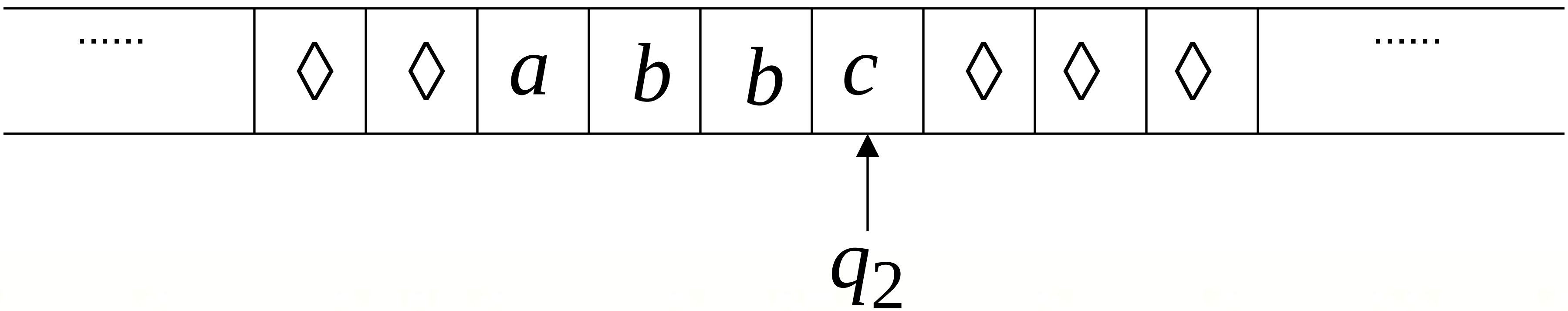
current state

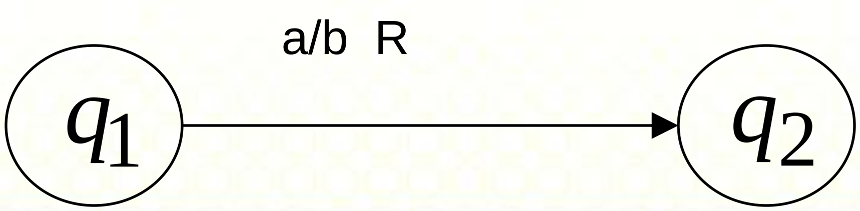


Time 1



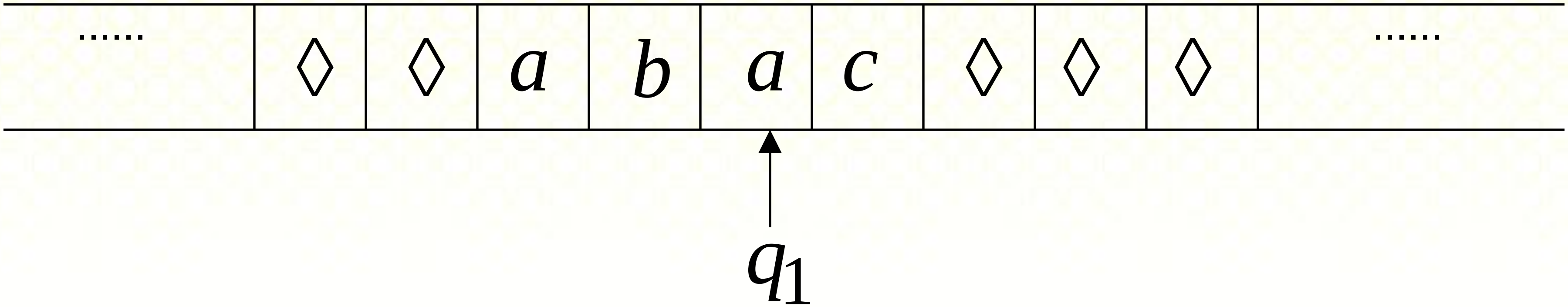
Time 2



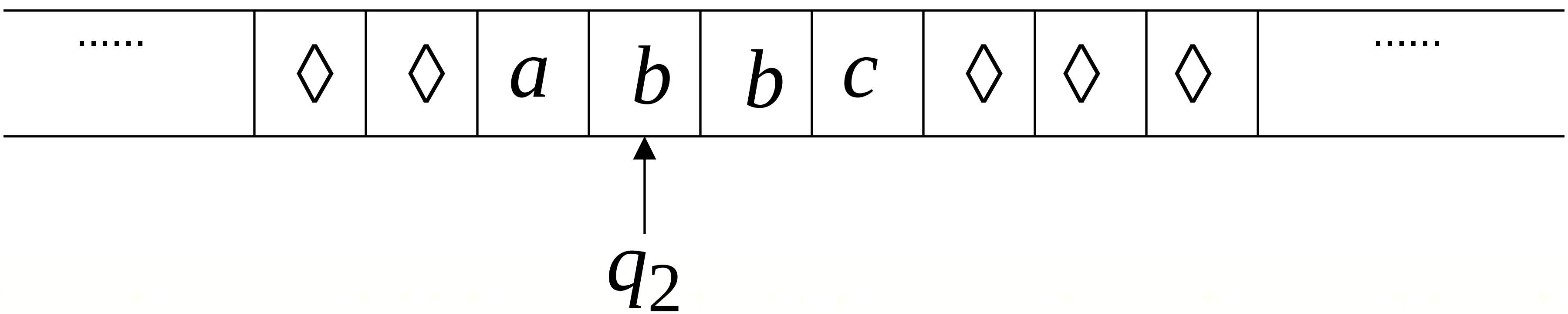


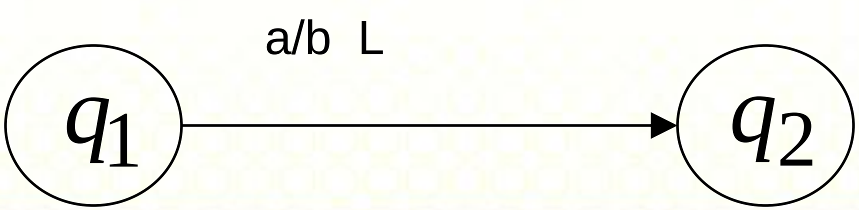
Example:

Time 1



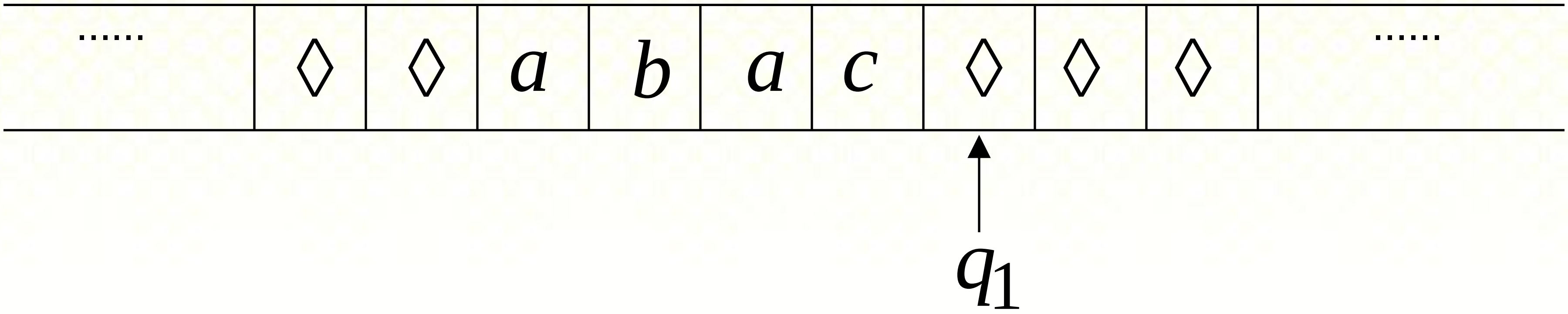
Time 2



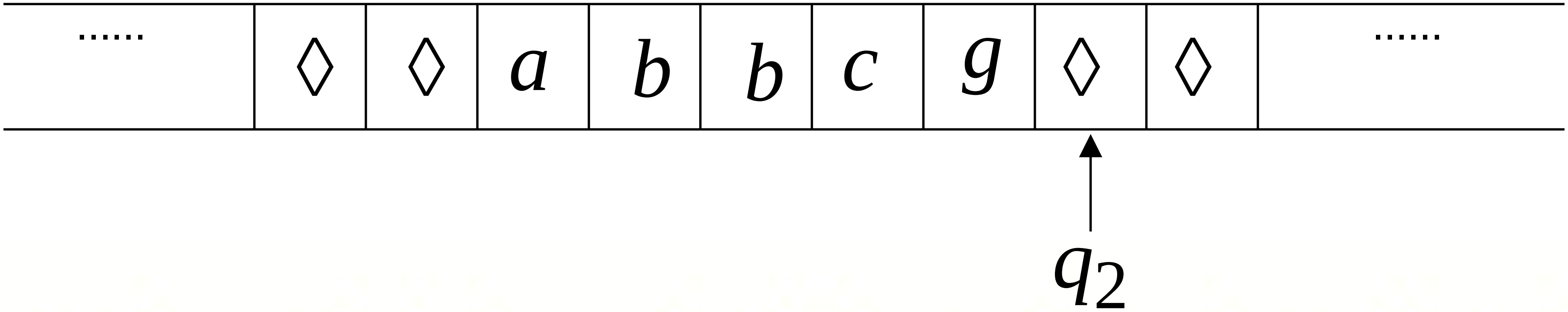


Example:

Time 1



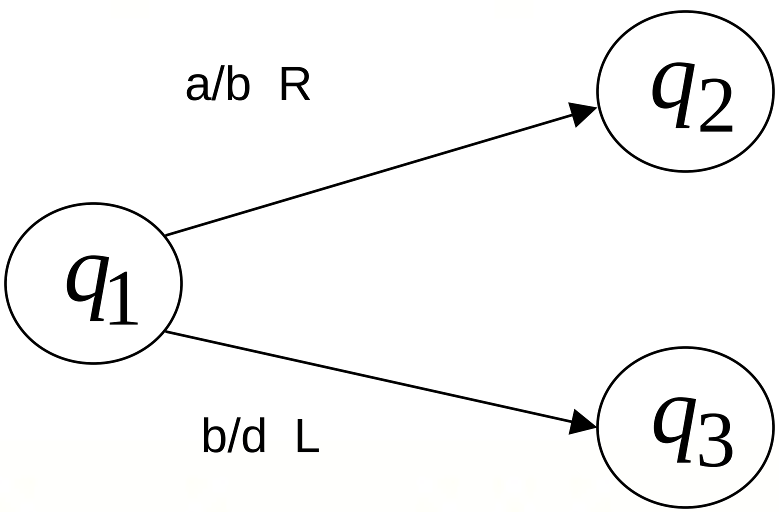
Time 2



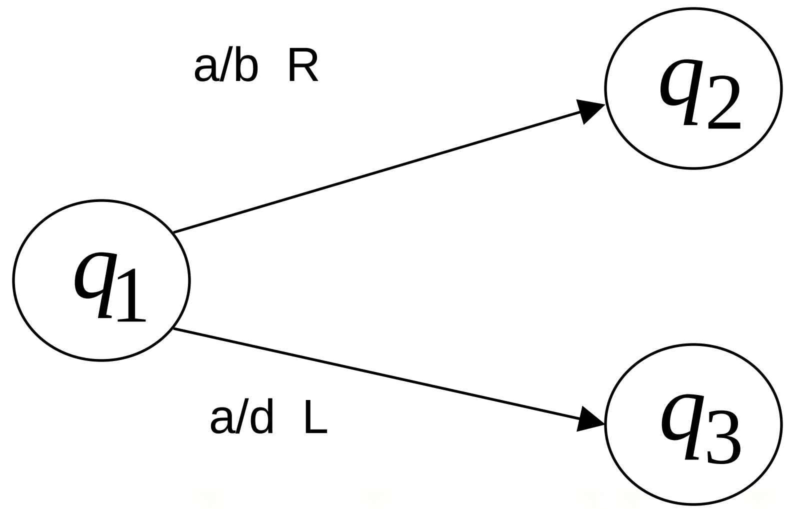
Determinism

Turing Machines are deterministic

Allowed



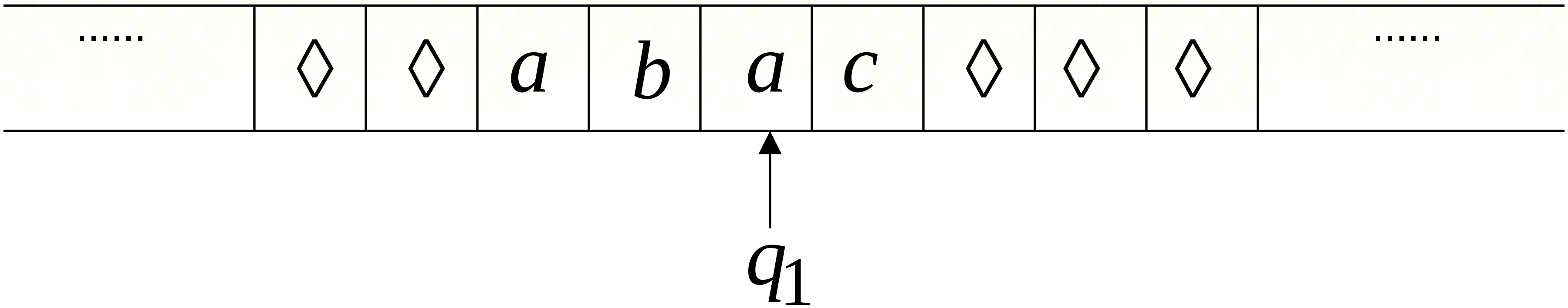
Not Allowed

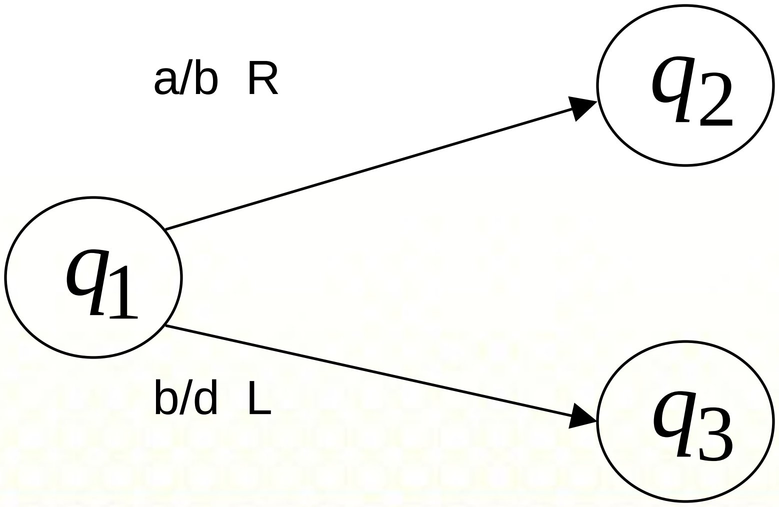


No lambda transitions allowed

Partial Transition Function

Example:





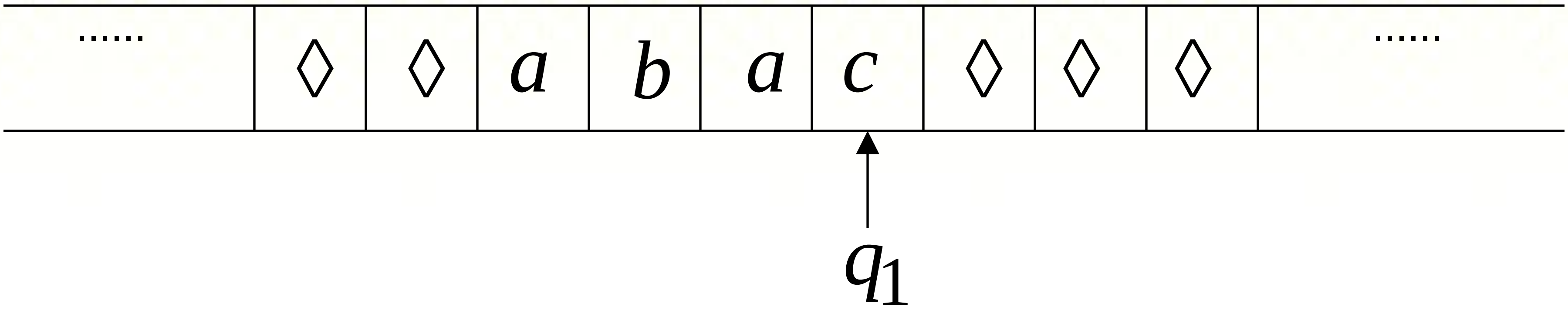
Allowed:

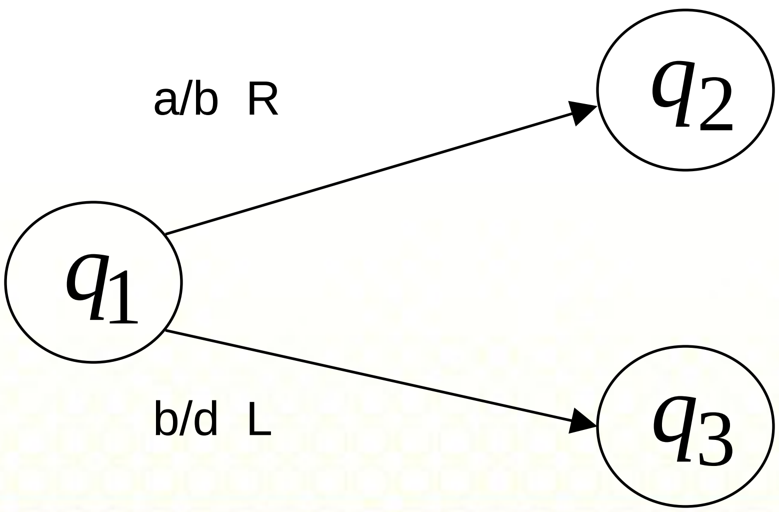
No transition
for input symbol c

Halting

The machine ***halts*** if there are
no possible transitions to follow

Example:

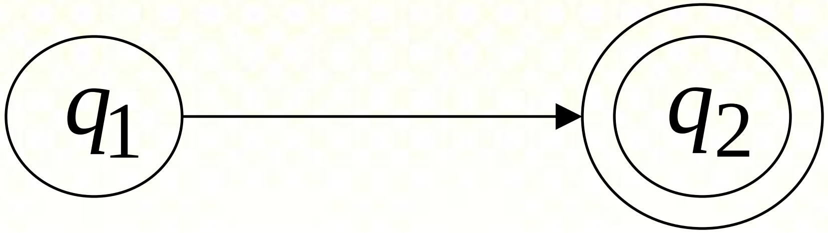




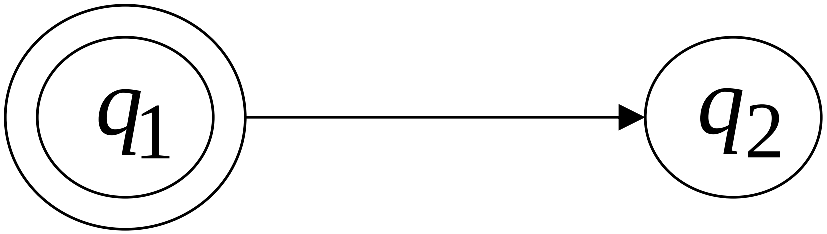
No possible transition

HALT!!!

Final States



Allowed



Not Allowed

- Final states have no outgoing transitions
- In a final state the machine halts

Acceptance

Accept Input → If machine halts in a final state

Reject Input → If machine halts in a non-final state

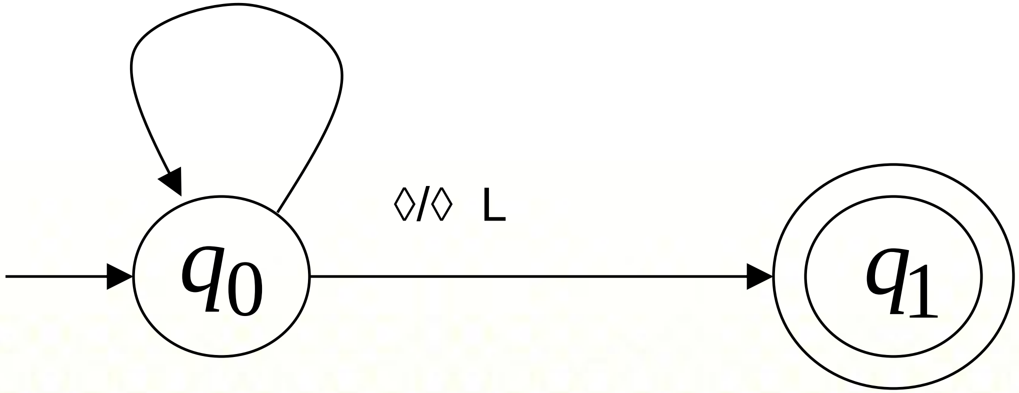
or

If machine enters an *infinite loop*

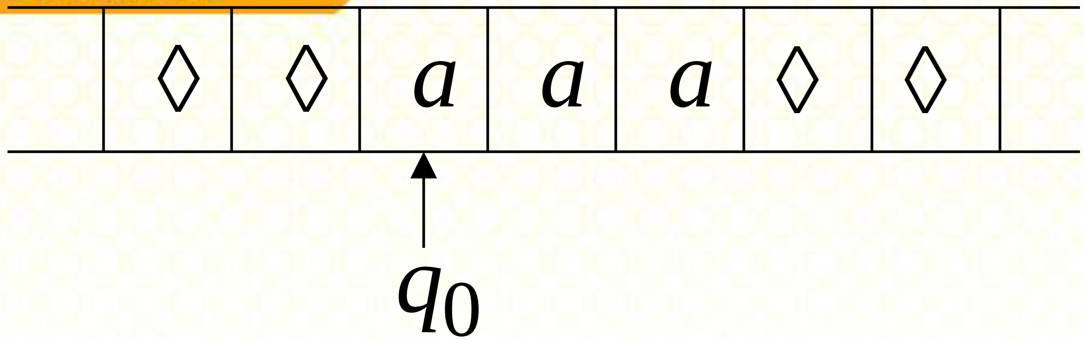
Turing Machine Example

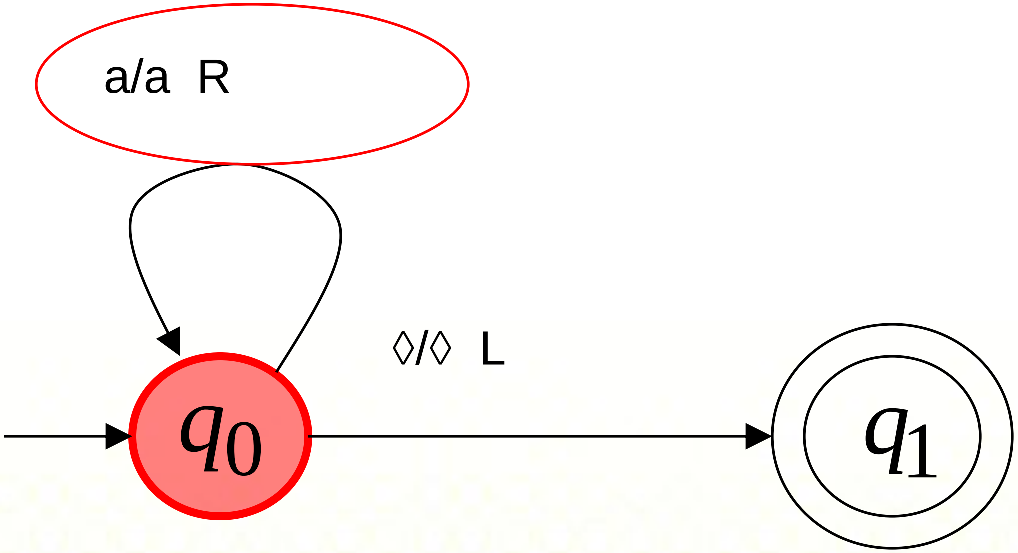
A Turing machine that accepts the language:

$$aa^*$$

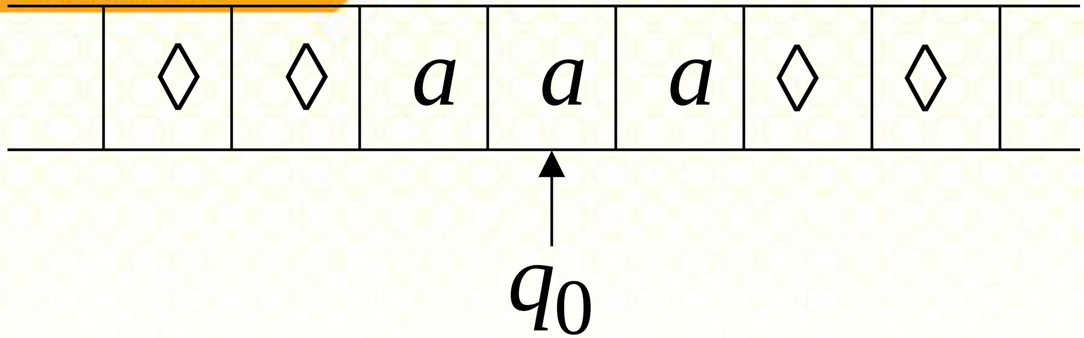


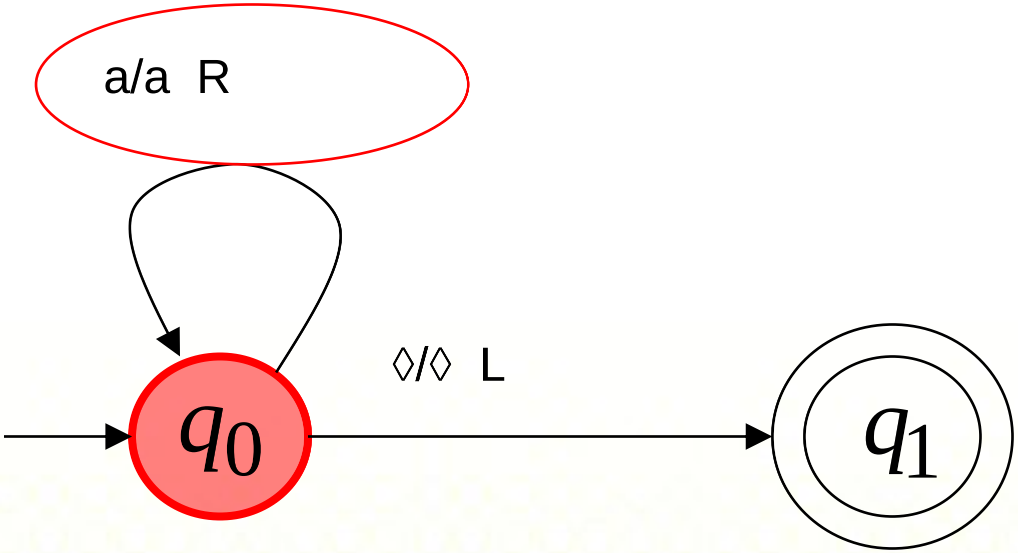
Time 0



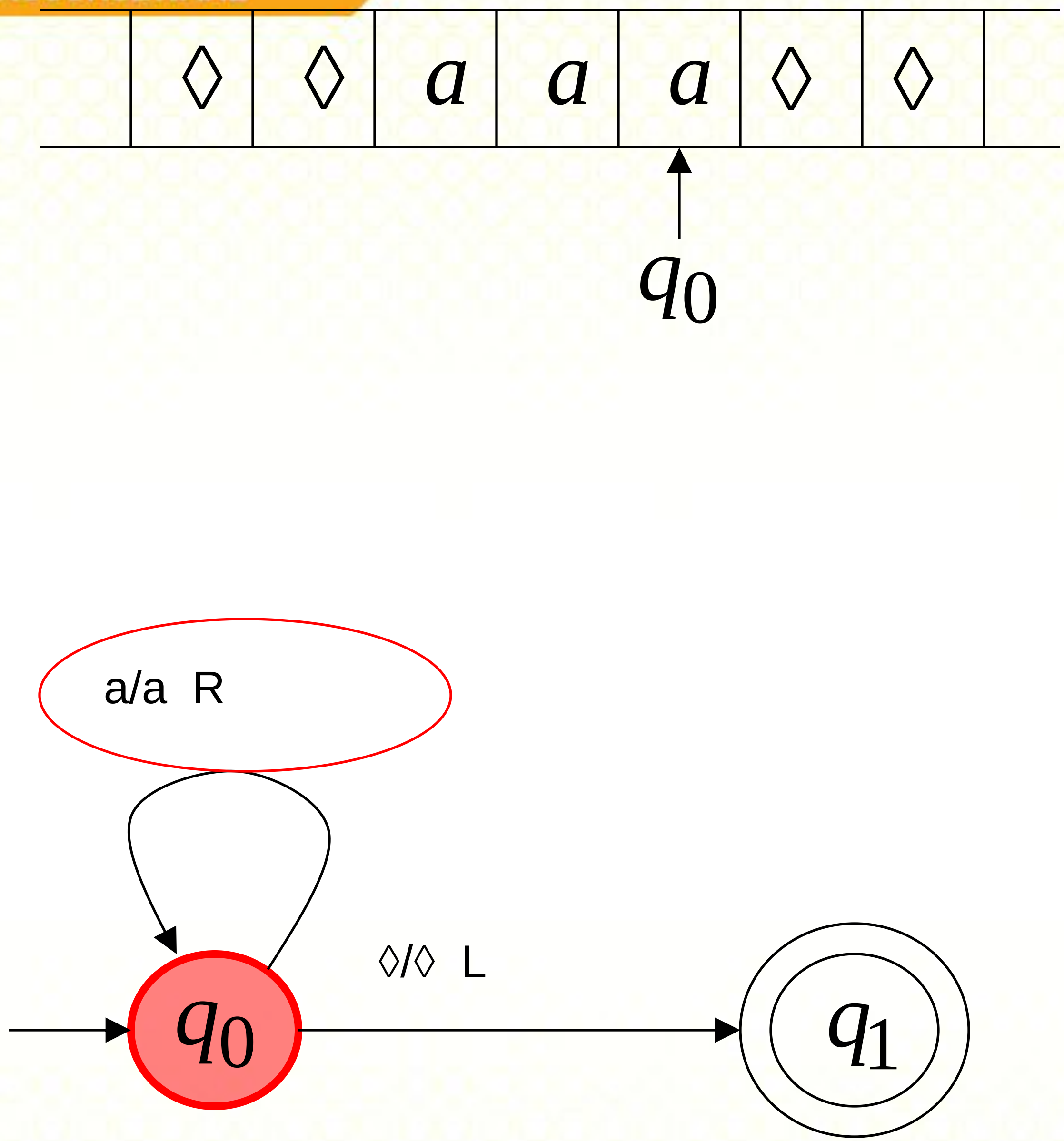


Time 1

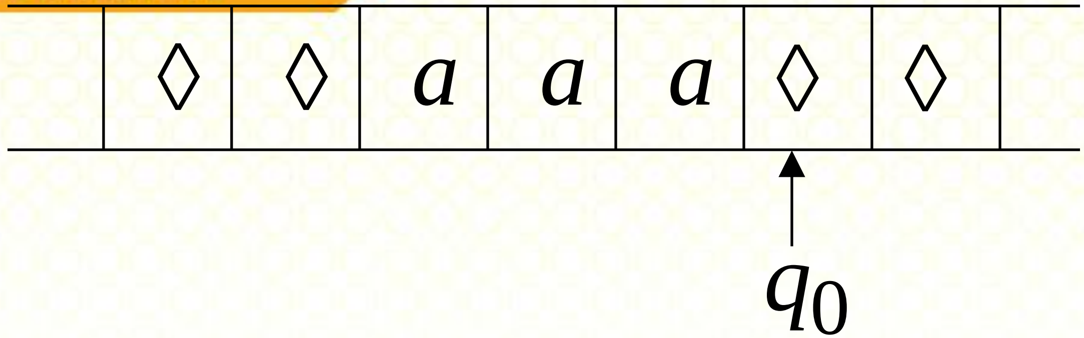


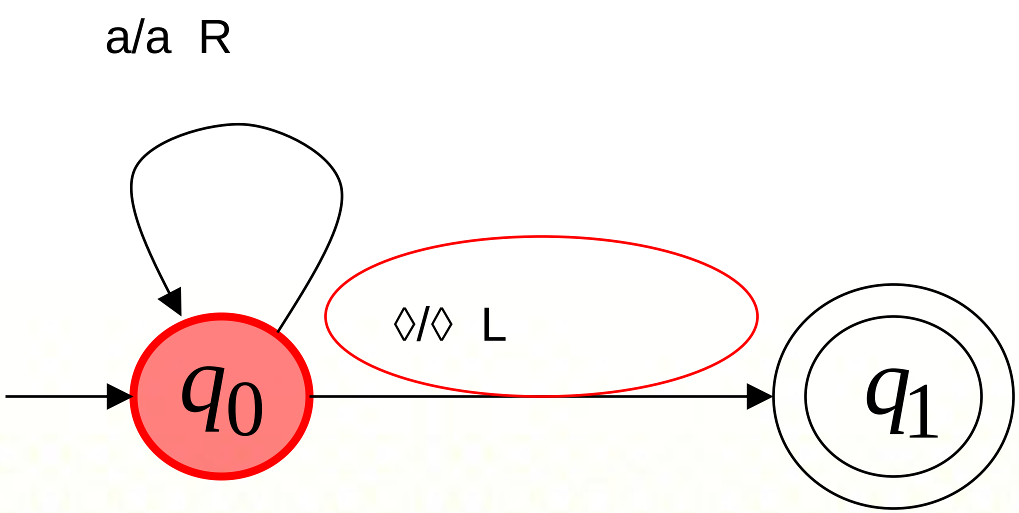


Time 2

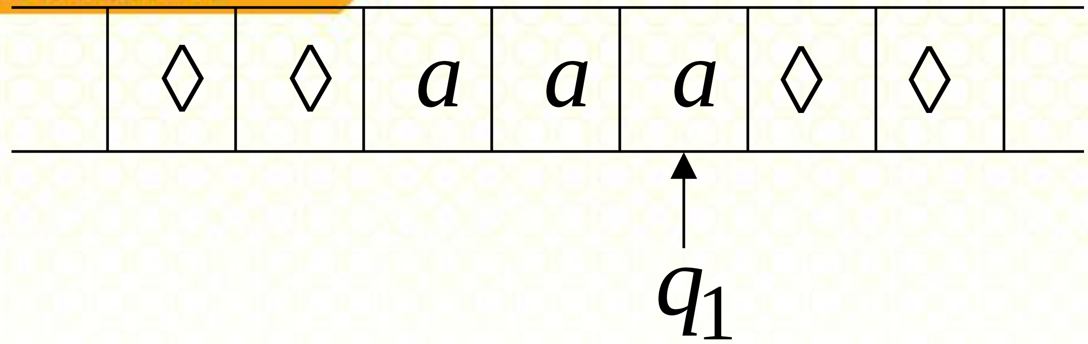


Time 3

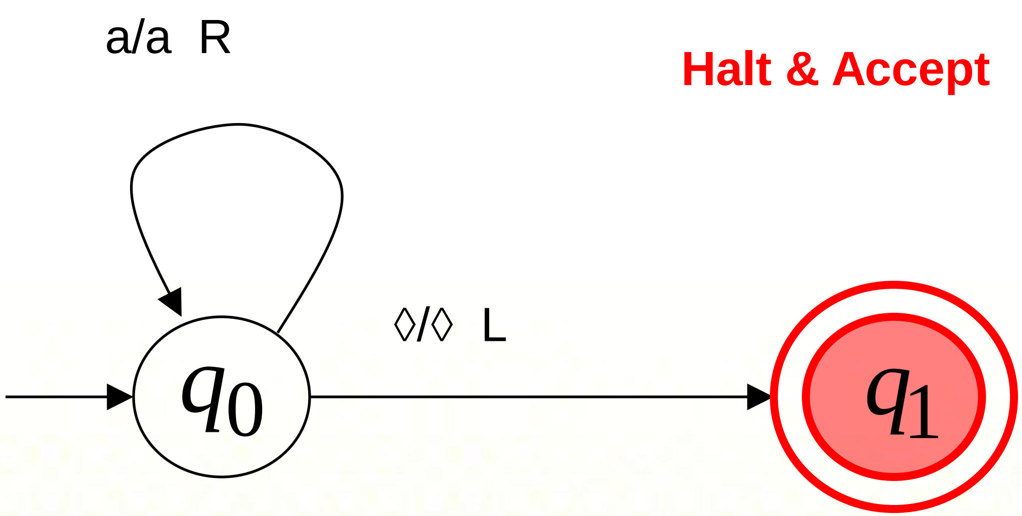




Time 4

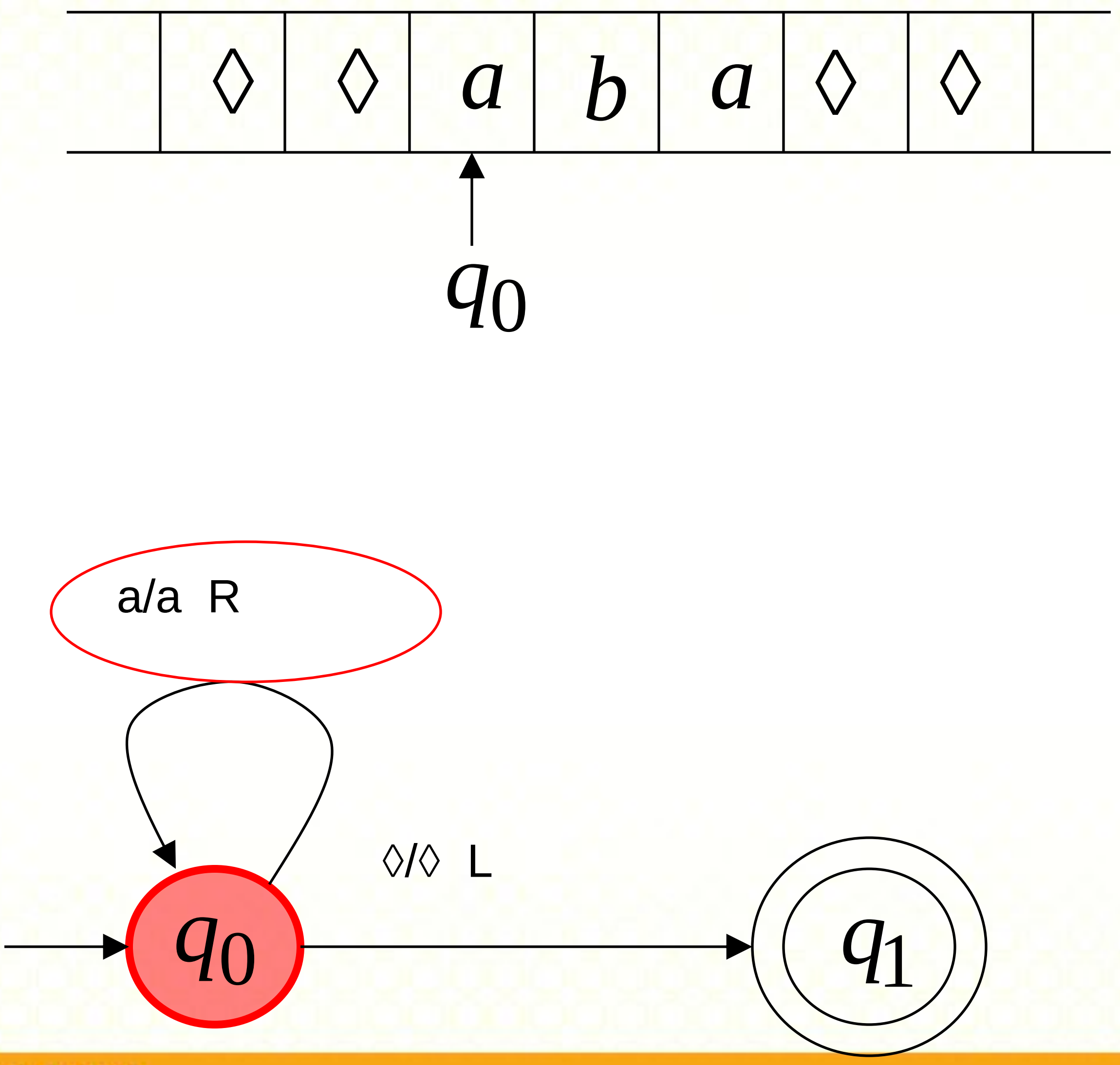


Halt & Accept

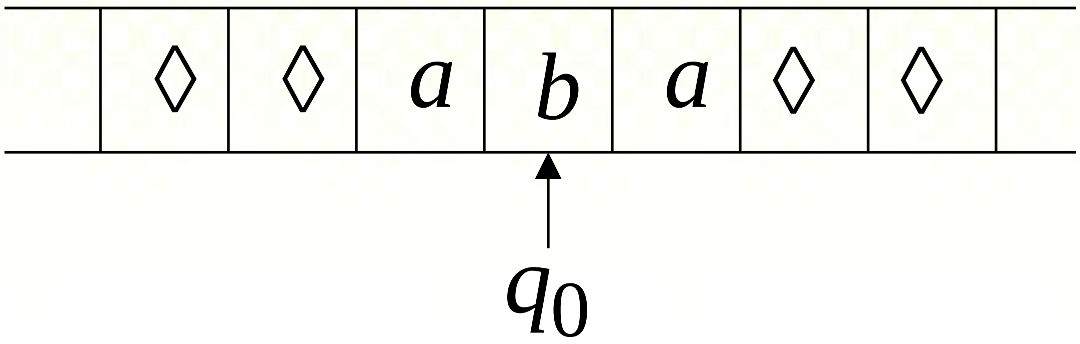


Rejection Example

Time 0

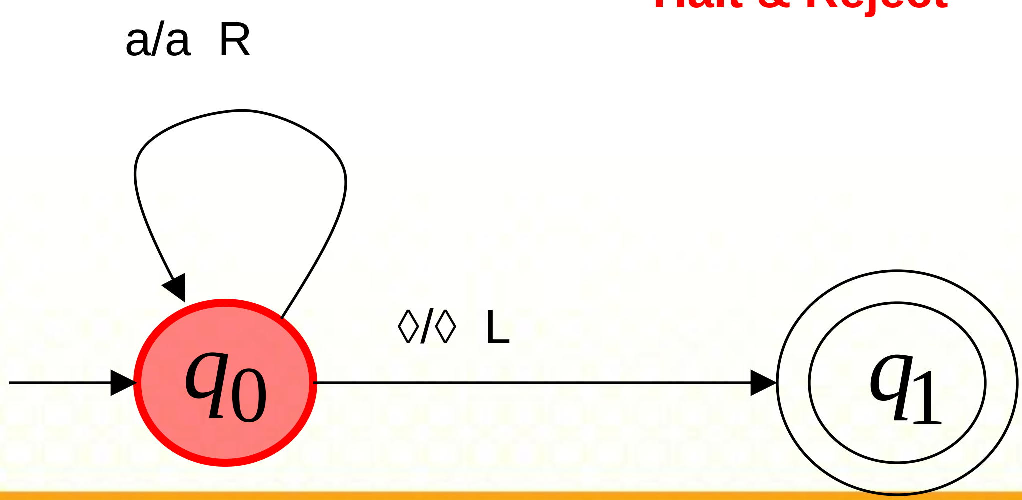


Time 1



No possible Transition

Halt & Reject

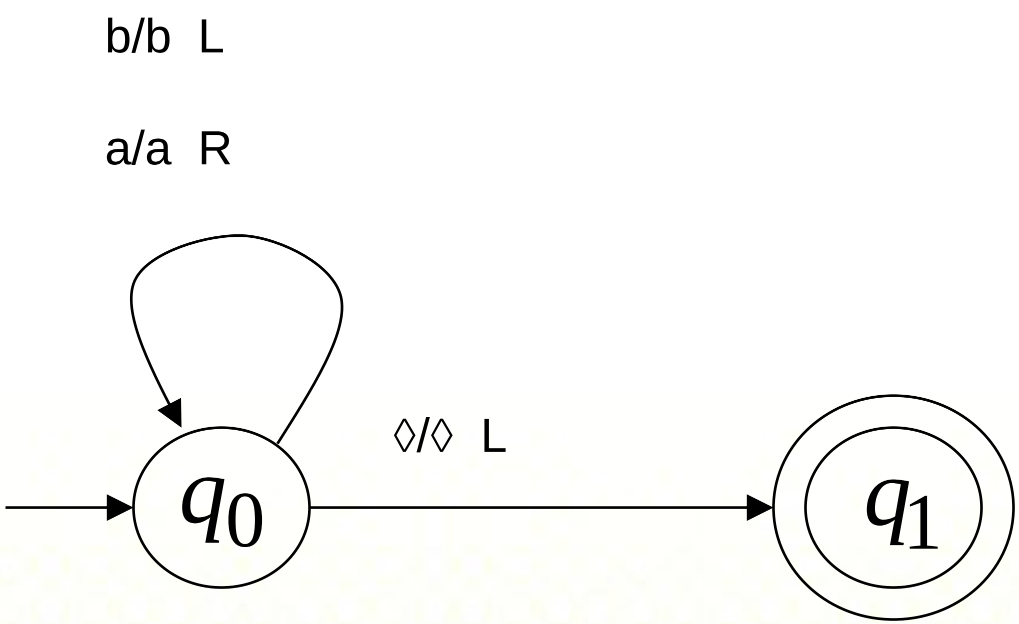


Infinite Loop Example

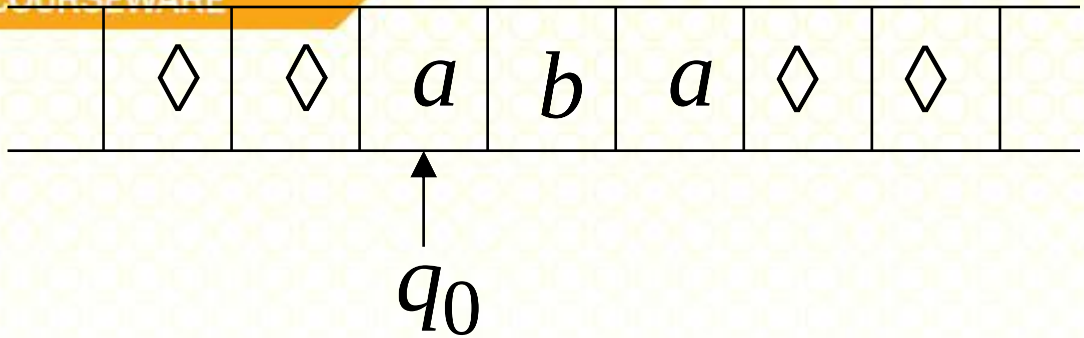
A Turing machine

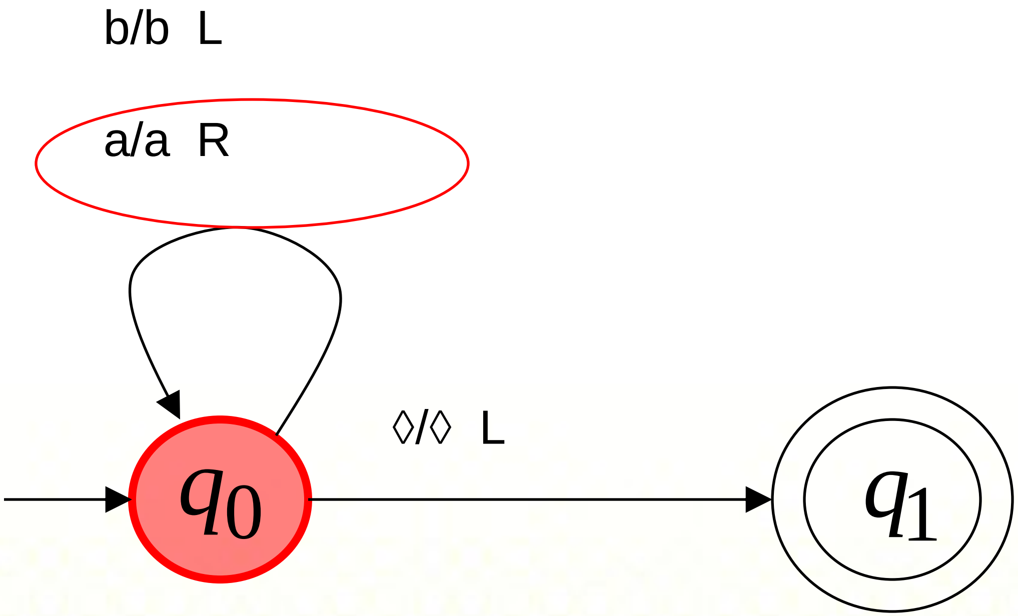
for language

$$aa^* + b(a+b)^*$$

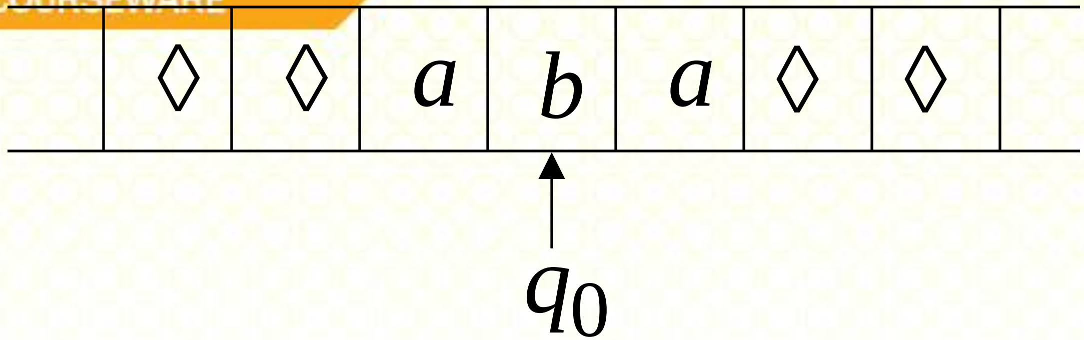


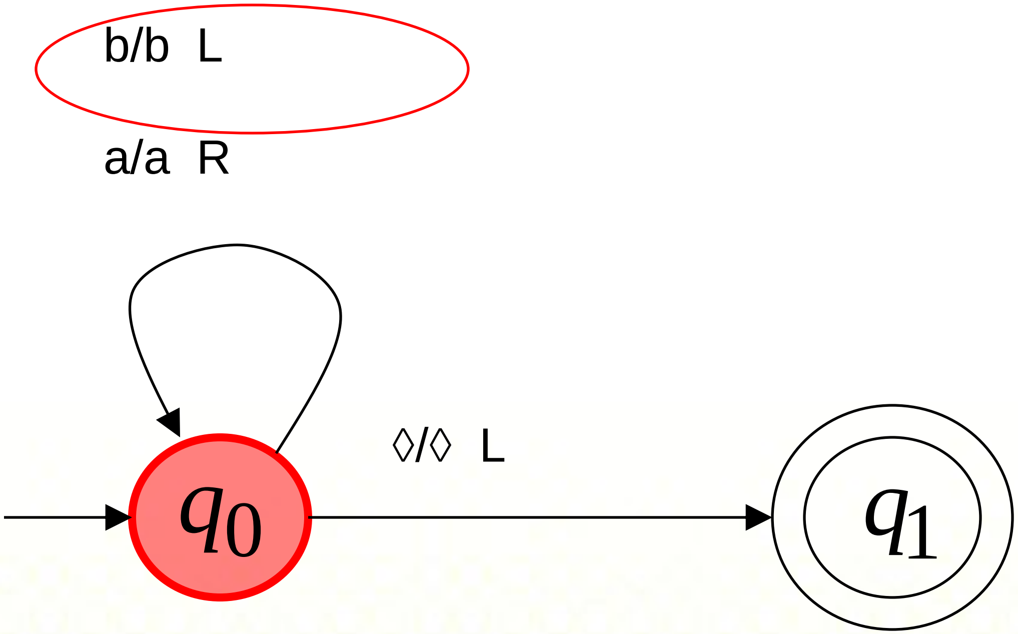
Time 0



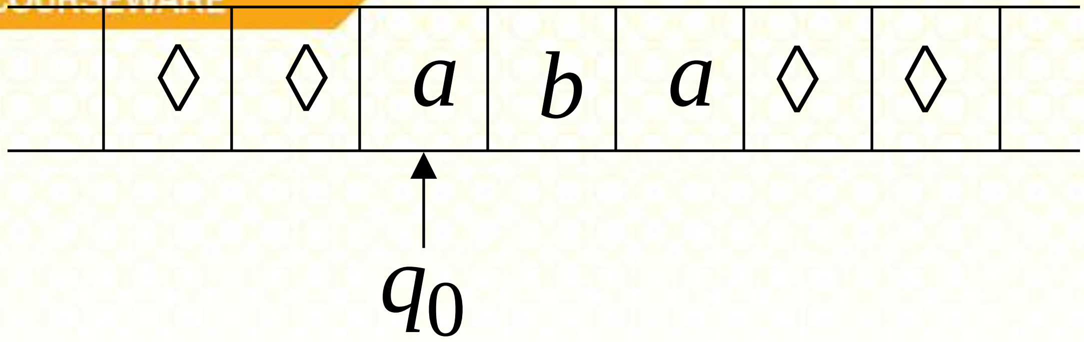


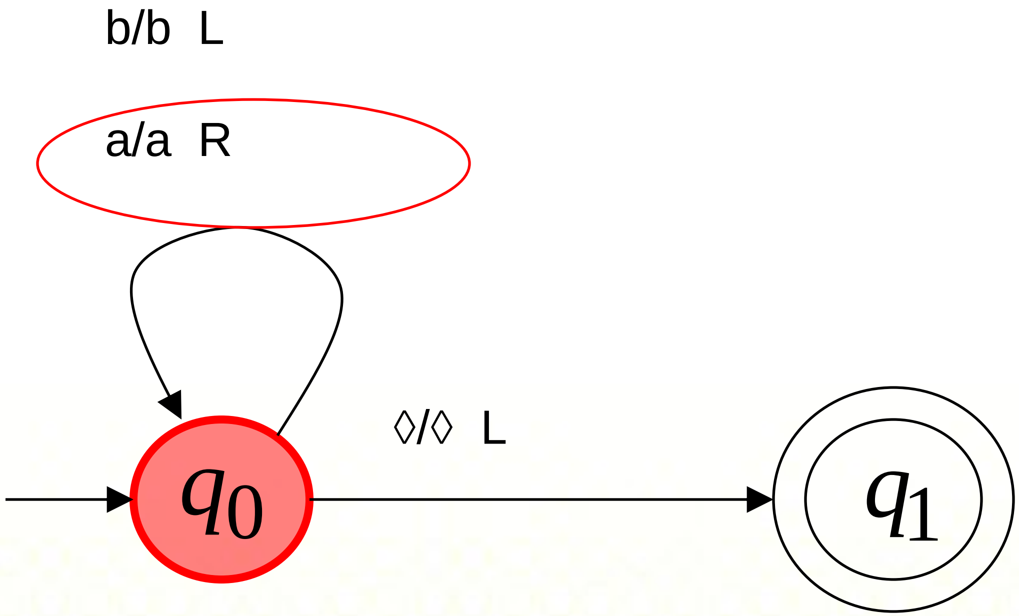
Time 1





Time 2



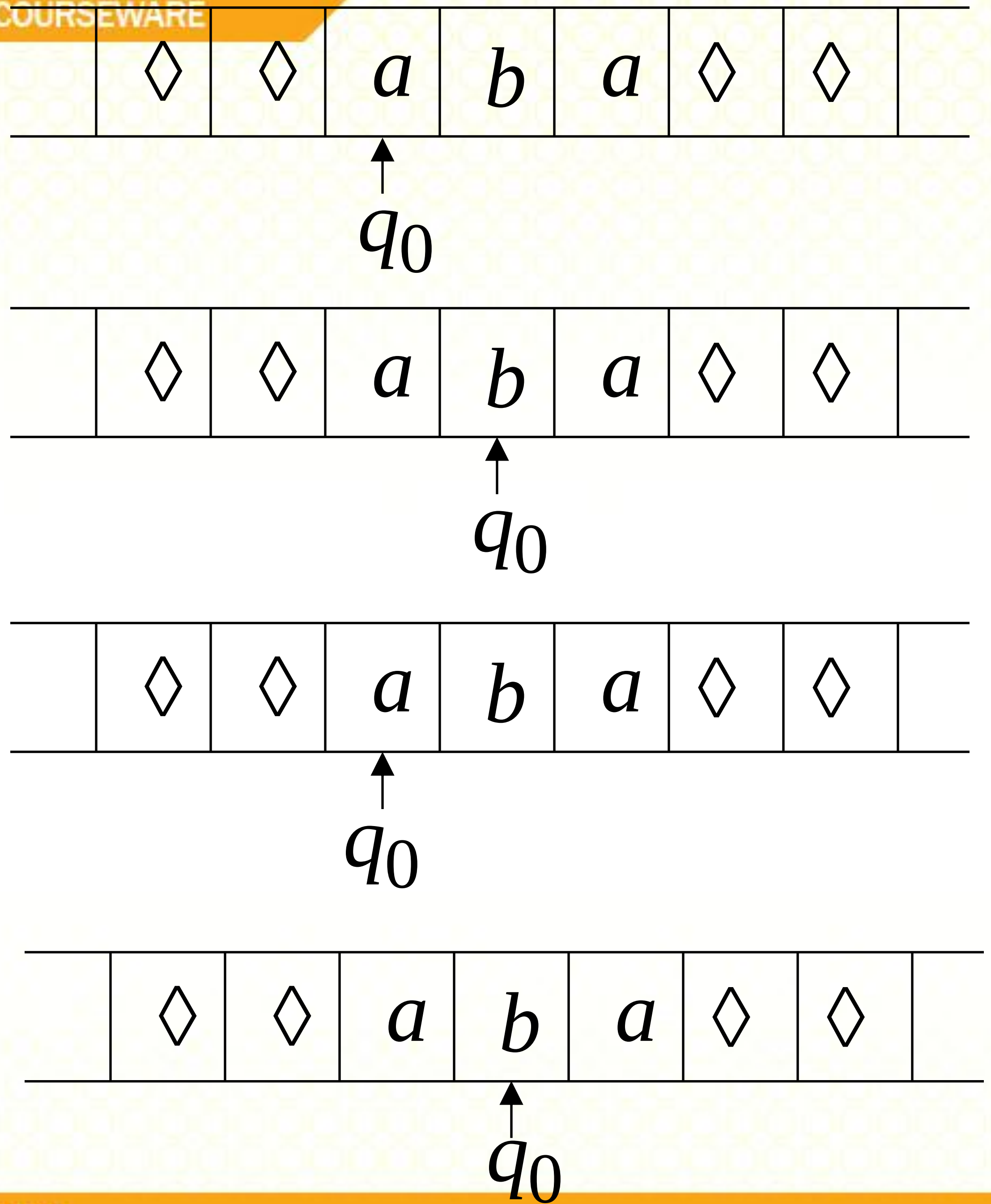


Time 2

Time 3

Time 4

Time 5



Infinite loop

Because of the infinite loop:

- The final state cannot be reached
- The machine never halts
- The input is not accepted

Formal Definitions for Turing Machines

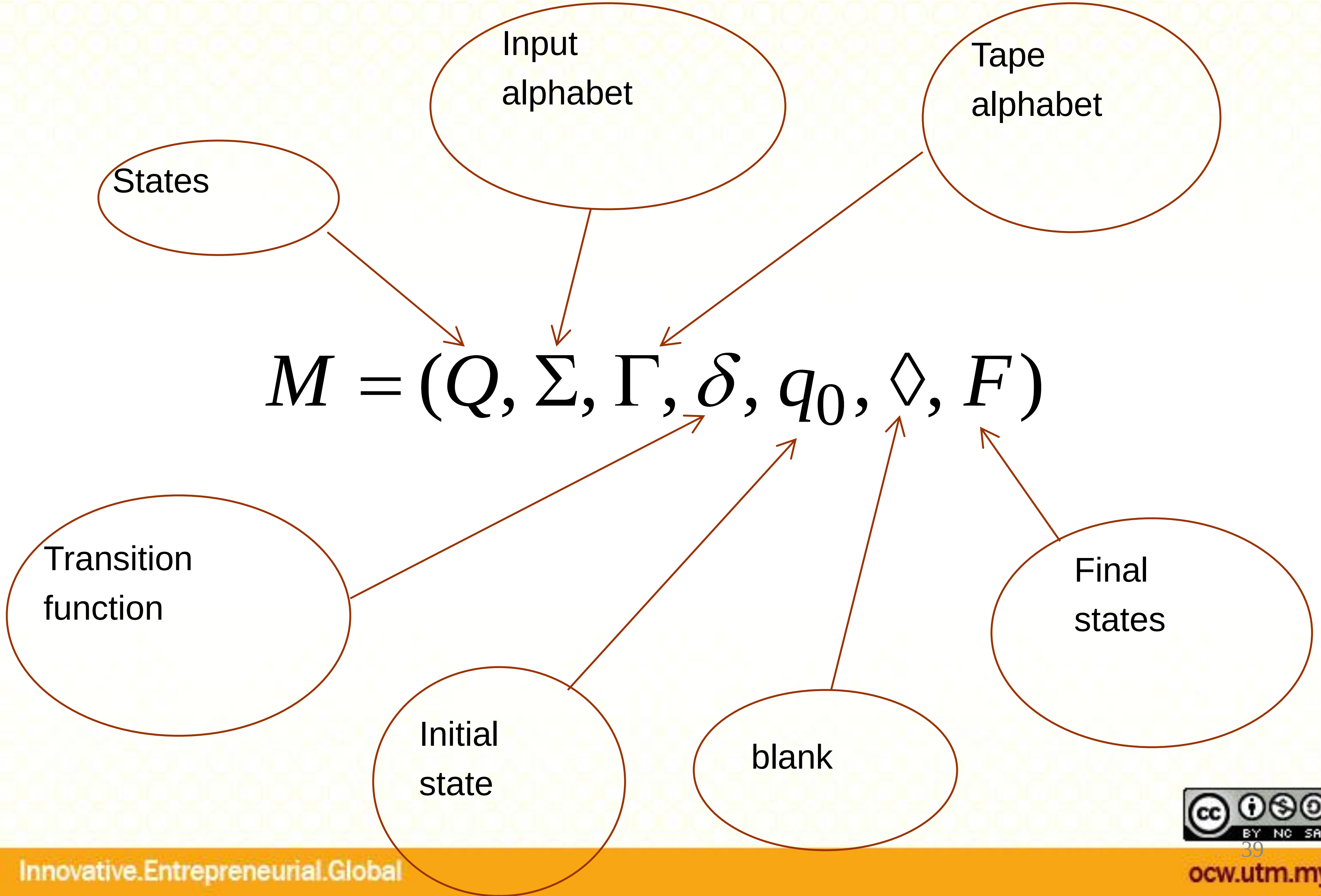
Transition Function

$$\delta(q_1, a) = (q_2, b, R)$$

Transition Function

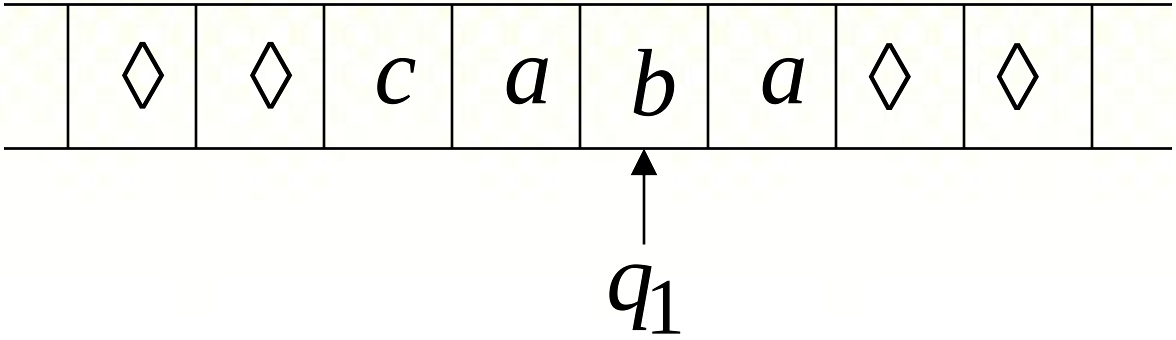
$$\delta(q_1, c) = (q_2, d, L)$$

Turing Machine:



$$M = (Q, \Sigma, \Gamma, \delta, q_0, \Diamond, F)$$

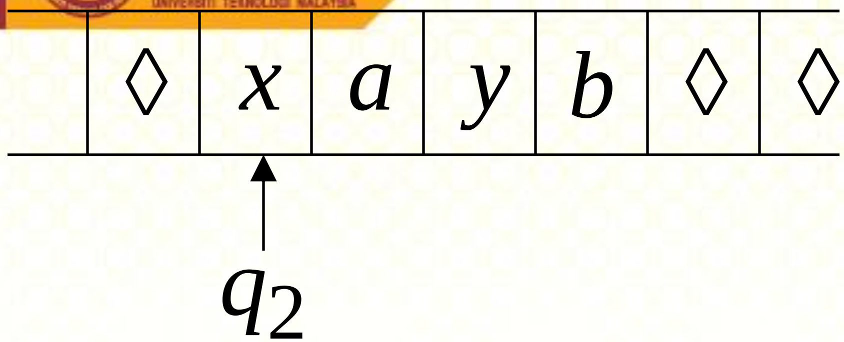
Configuration



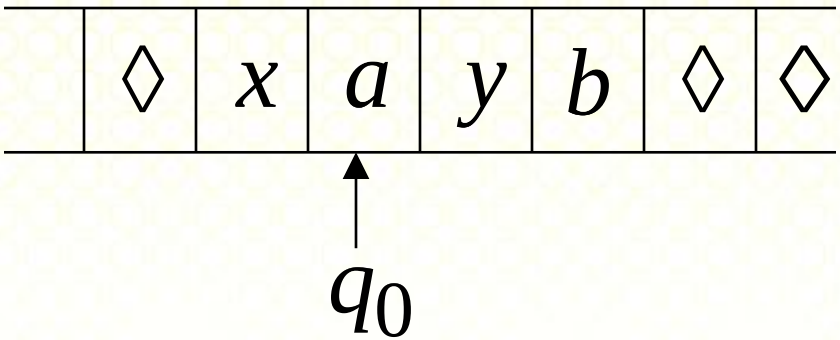
Instantaneous description:

$$ca\ q_1\ ba$$

Time 4



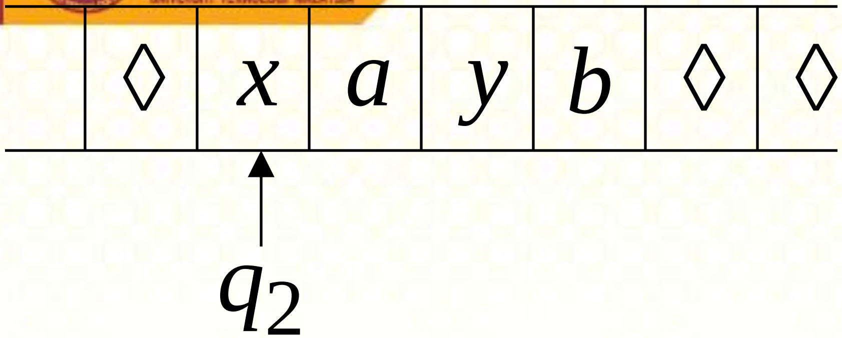
Time 5



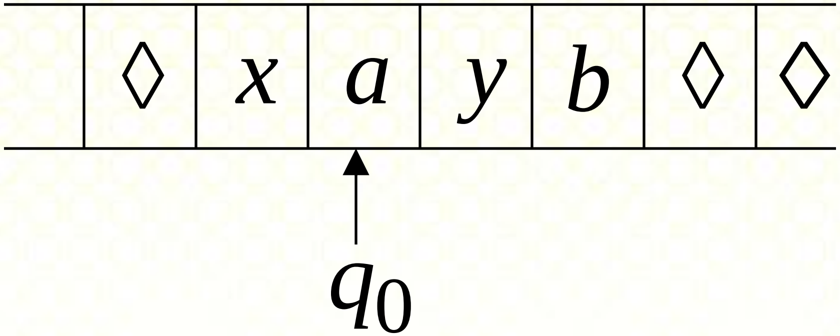
A Move:

$$q_2\ xayb \ \phi\ x\, q_0\ ayb$$

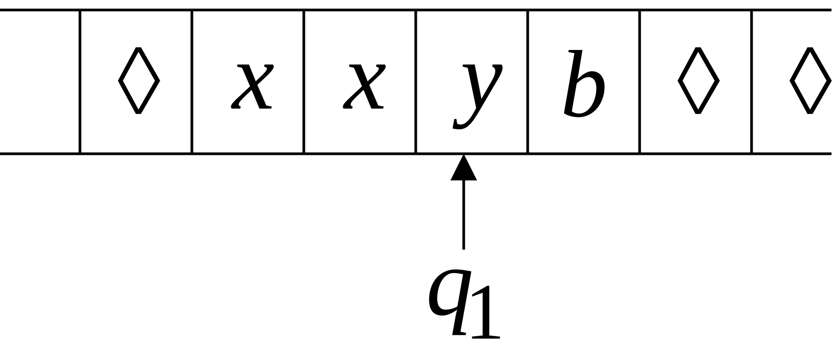
Time 4



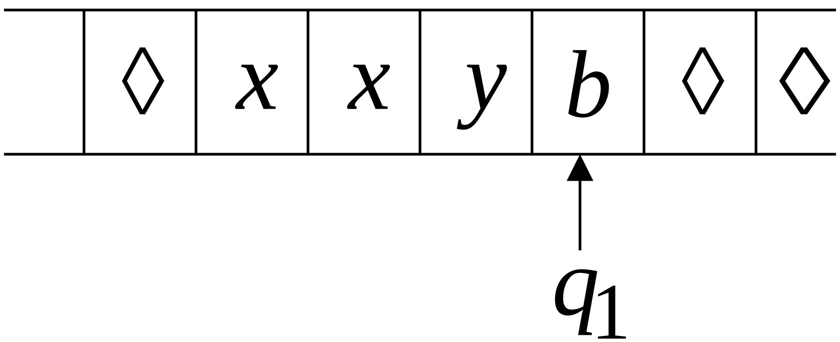
Time 5



Time 6



Time 7



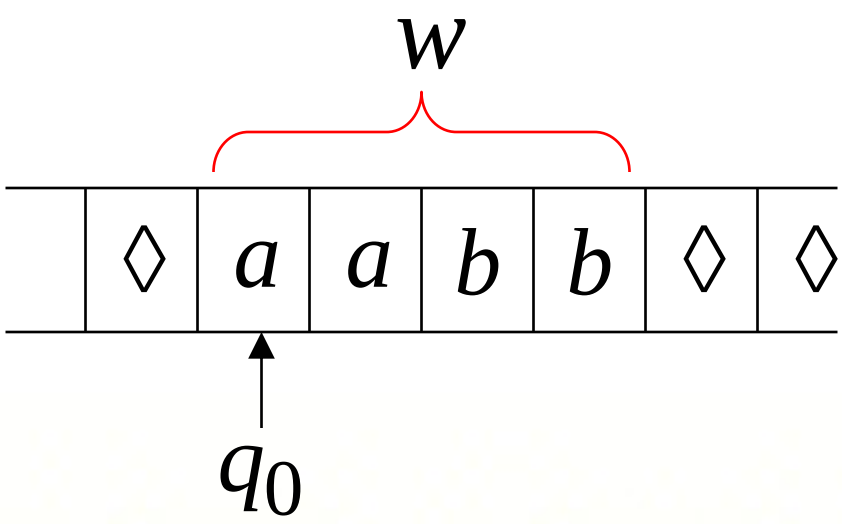
$$q_2\ xayb \ \phi\ x\ q_0\ ayb \ \phi\ xx\ q_1\ yb \ \phi\ xxy\ q_1\ b$$

$$q_2\ xayb\ \phi\ x\, q_0\ ayb\ \phi\ xx\, q_1\ yb\ \phi\ xxy\, q_1\ b$$

Equivalent notation: $$q_2\ xayb\ \overset{*}{\phi}\ xxy\, q_1\ b$$

Initial configuration: $q_0\ w$

Input string



Standard Turing Machine

The machine we described is the standard:

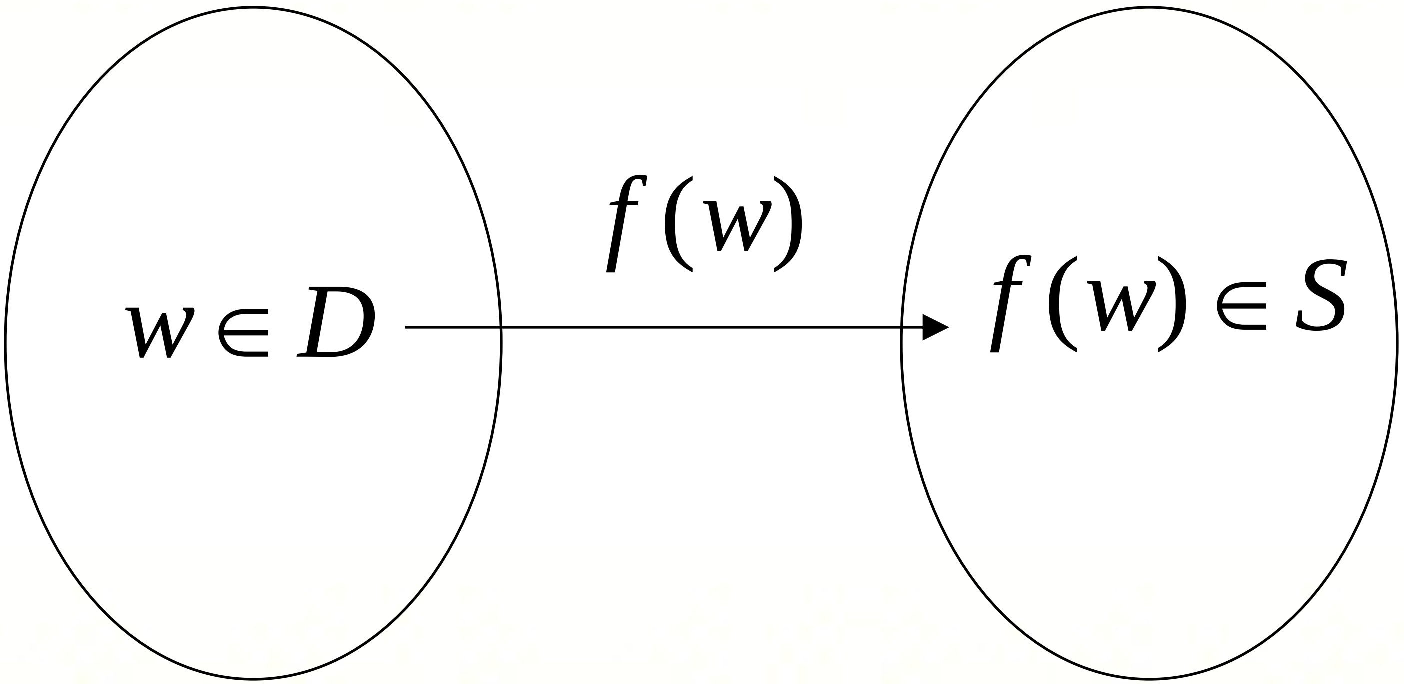
- Deterministic
- Infinite tape in both directions
- Tape is the input/output file

Computing Functions with Turing Machines

A function $f(w)$ has:

Domain: D

Result Region: S



A function may have many parameters:

Example:

Addition function

$$f(x, y) = x + y$$

Integer Domain

Decimal: 5

Binary: 101

Unary: 11111

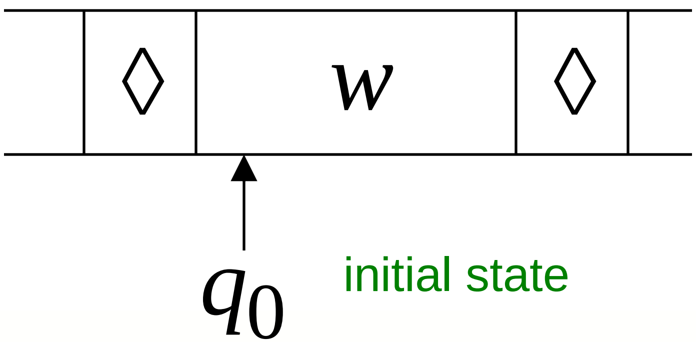
We prefer **unary** representation:

easier to manipulate with Turing machines

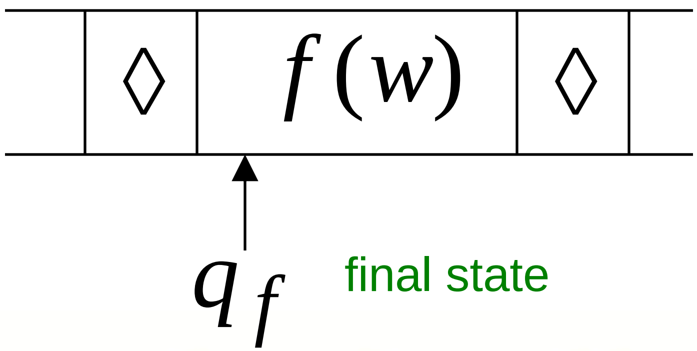
Definition:

A function f is computable if there is a Turing Machine M such that:

Initial configuration



Final configuration



For all $w \in D$ Domain

In other words:

A function f is computable if there is a Turing Machine M such that:

$$q_0 w \overset{*}{\vdash} q_f f(w)$$

Initial Configuration

Final Configuration

For all $w \in D$ Domain

Example

The function $f(x, y) = x + y$ is computable

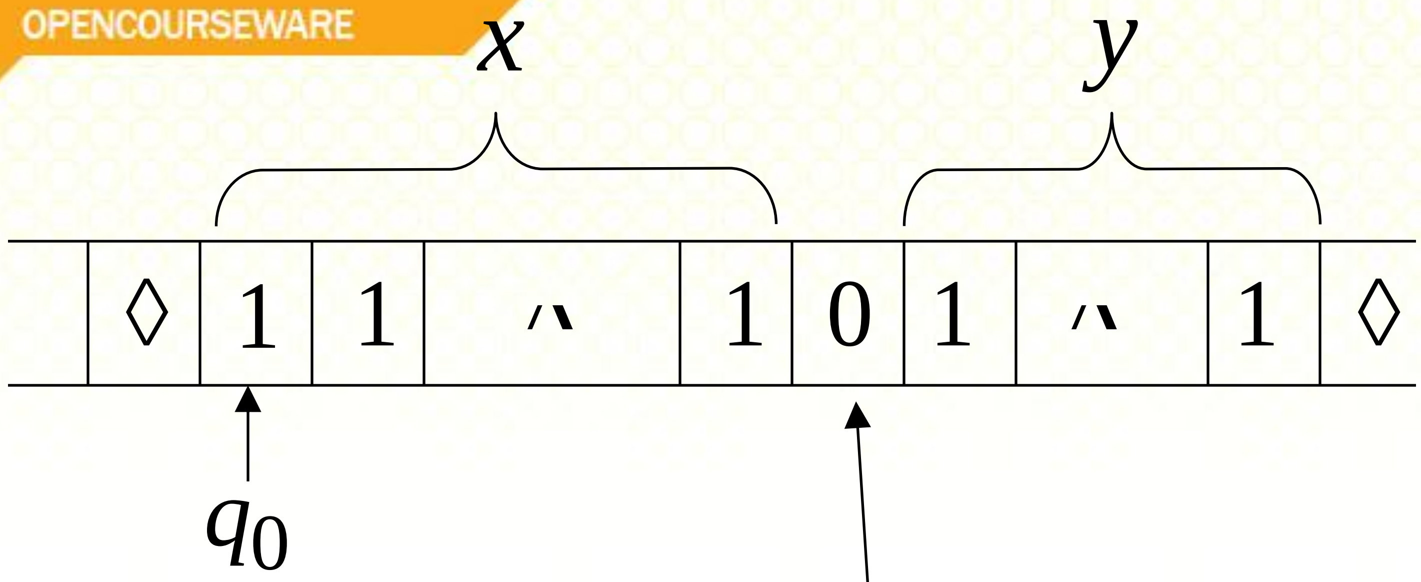
x, y are integers

Turing Machine:

Input string: $x0y$ unary

Output string: $xy0$ unary

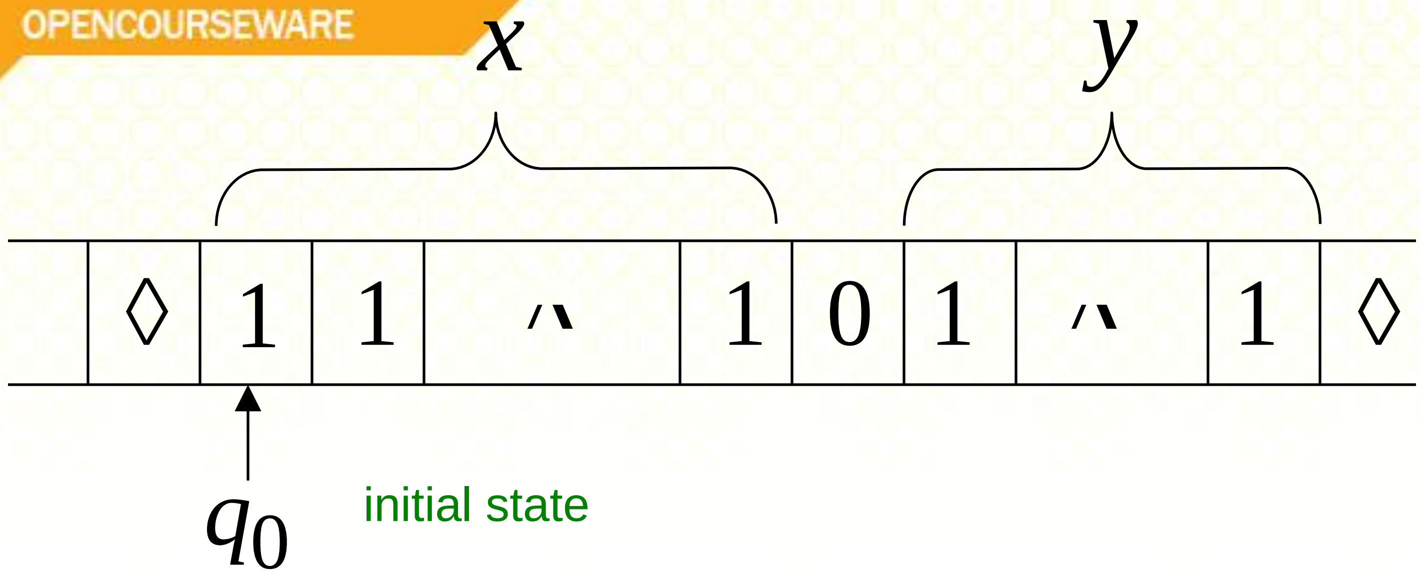
Start



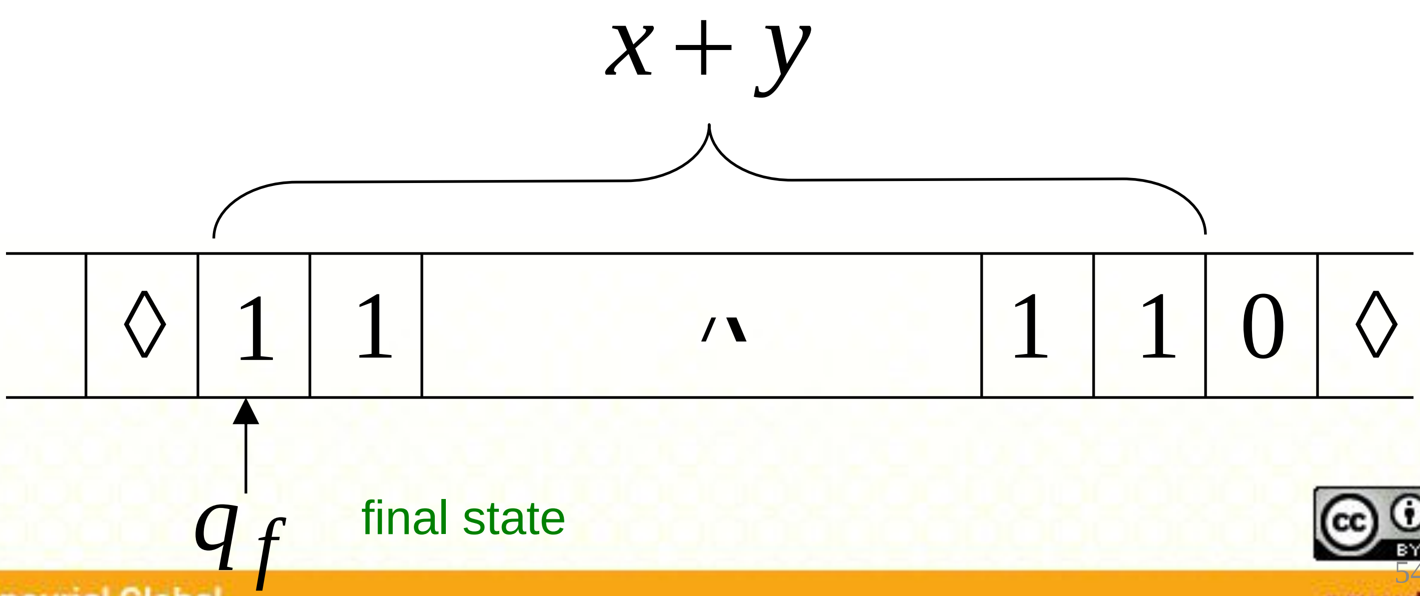
initial state

The 0 is the delimiter that separates the two numbers

Start

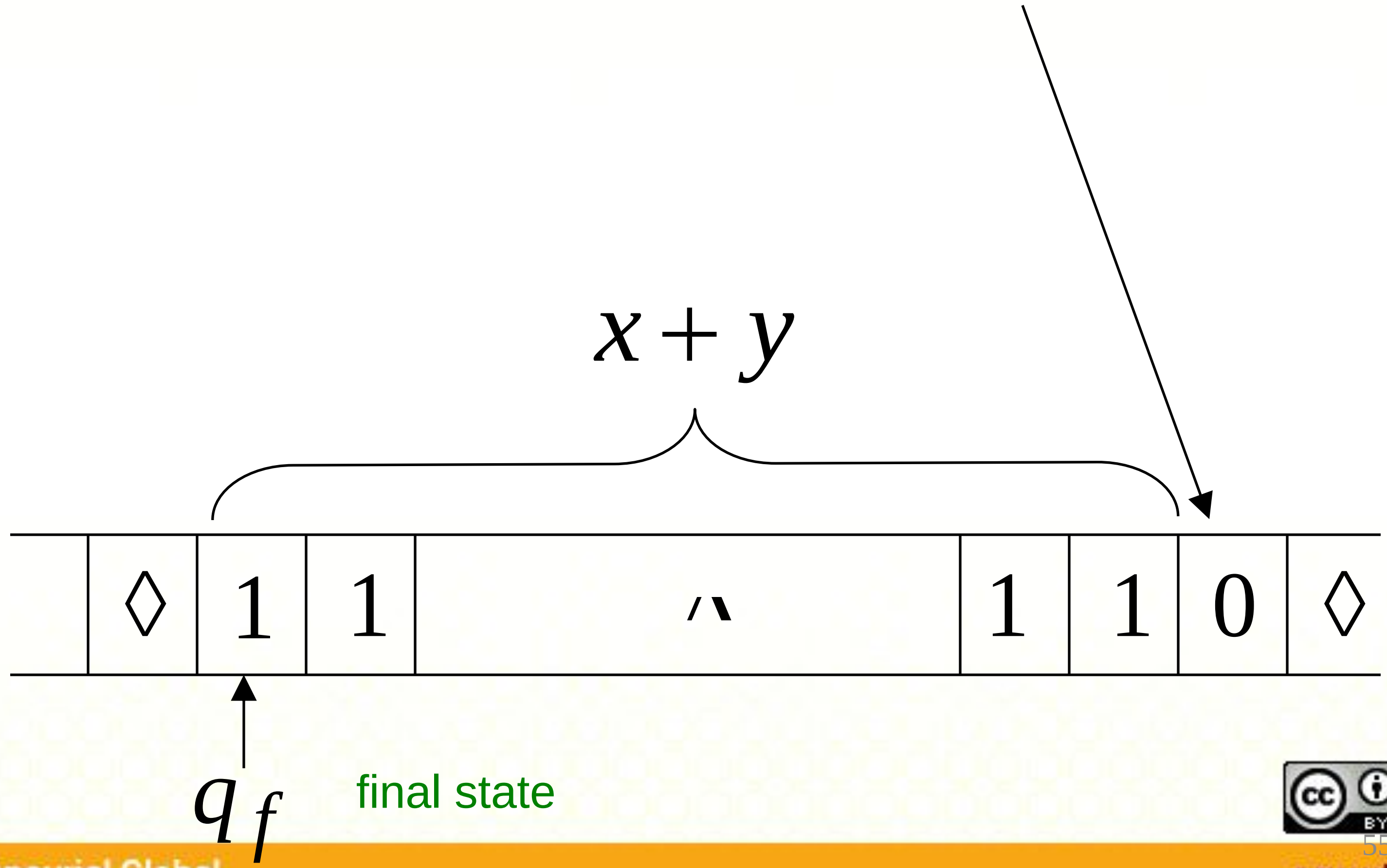


Finish



The 0 helps when we use
the result for other operations

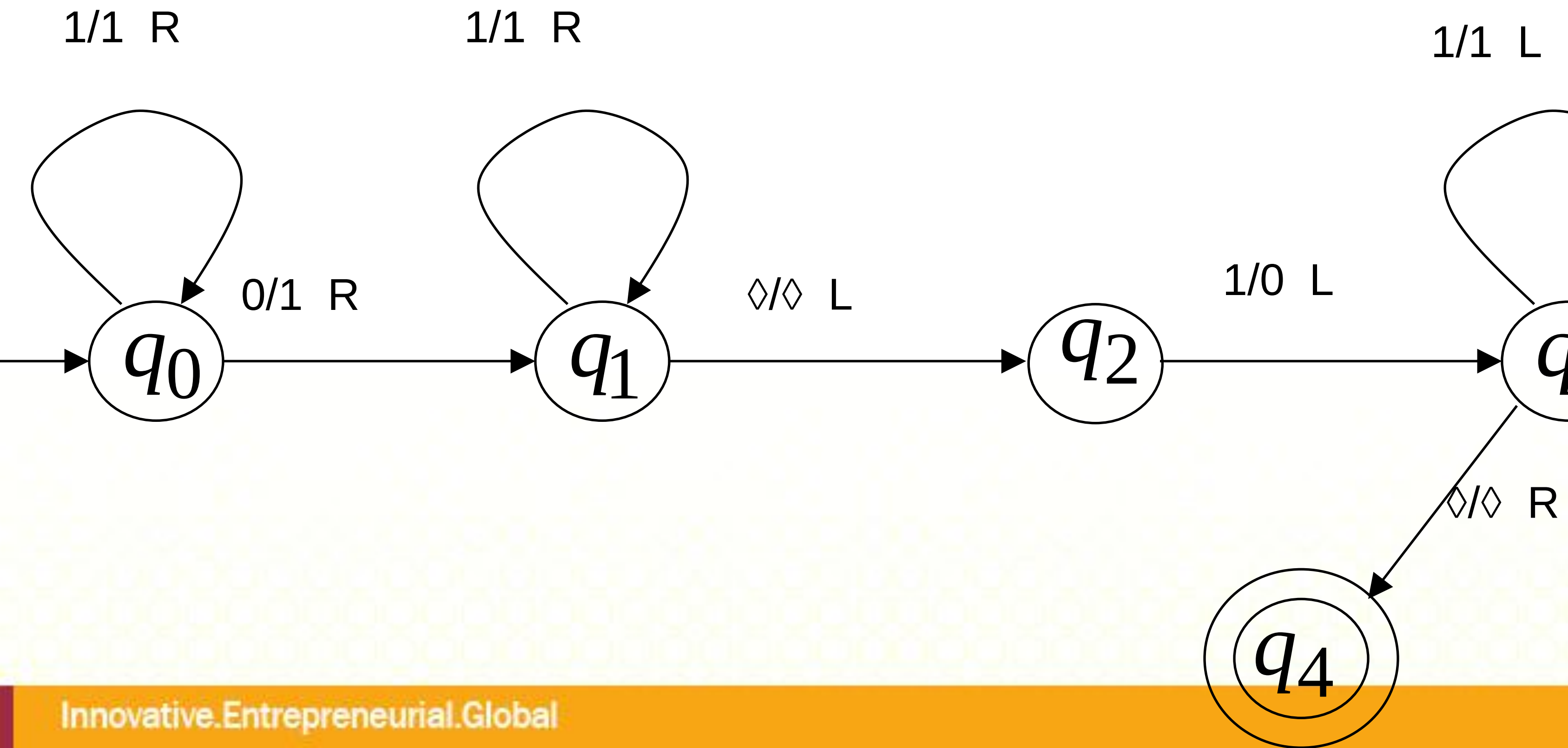
Finish



final state

Turing machine for function

$$f(x, y) = x + y$$

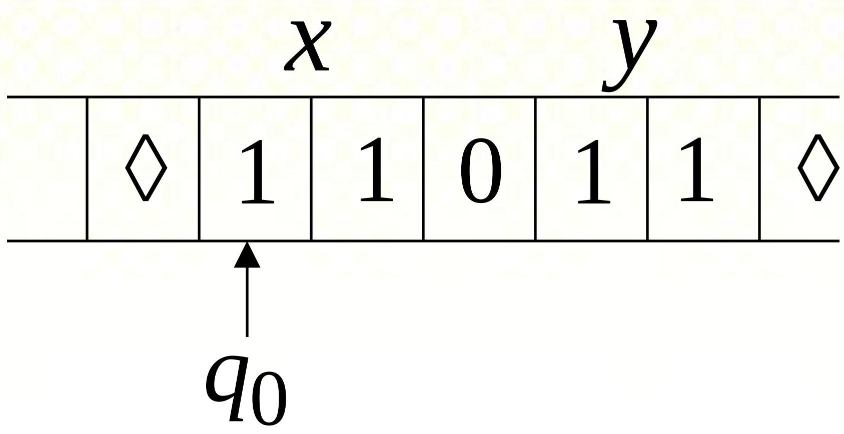


Execution Example:

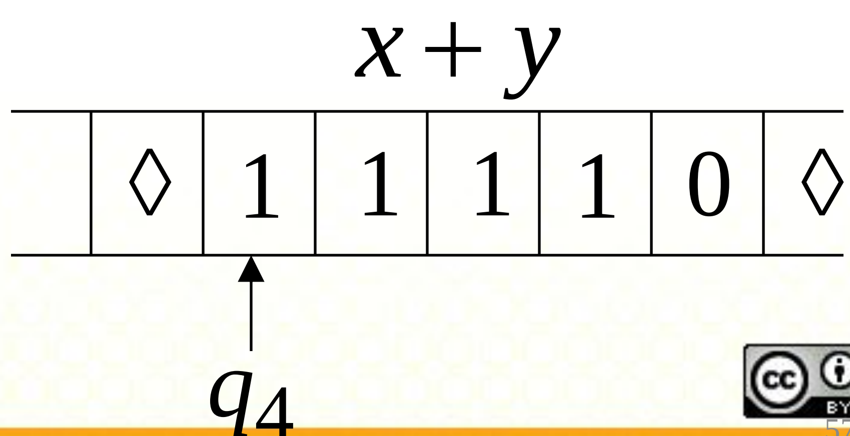
$x = 11$ (2)

$y = 11$ (2)

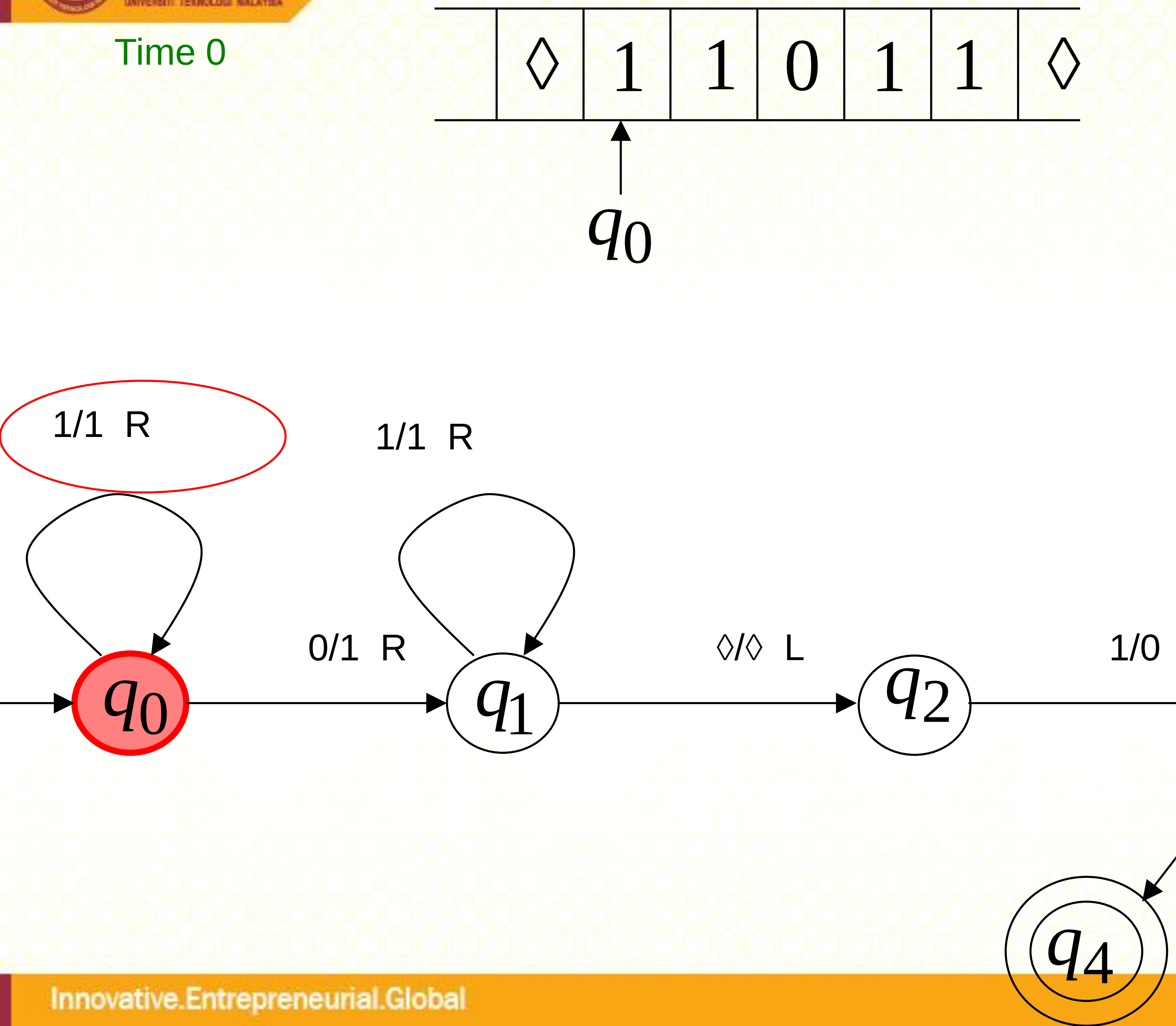
Time 0



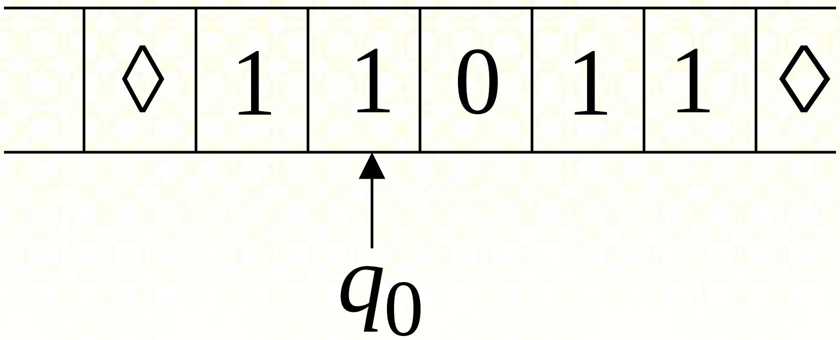
Final Result

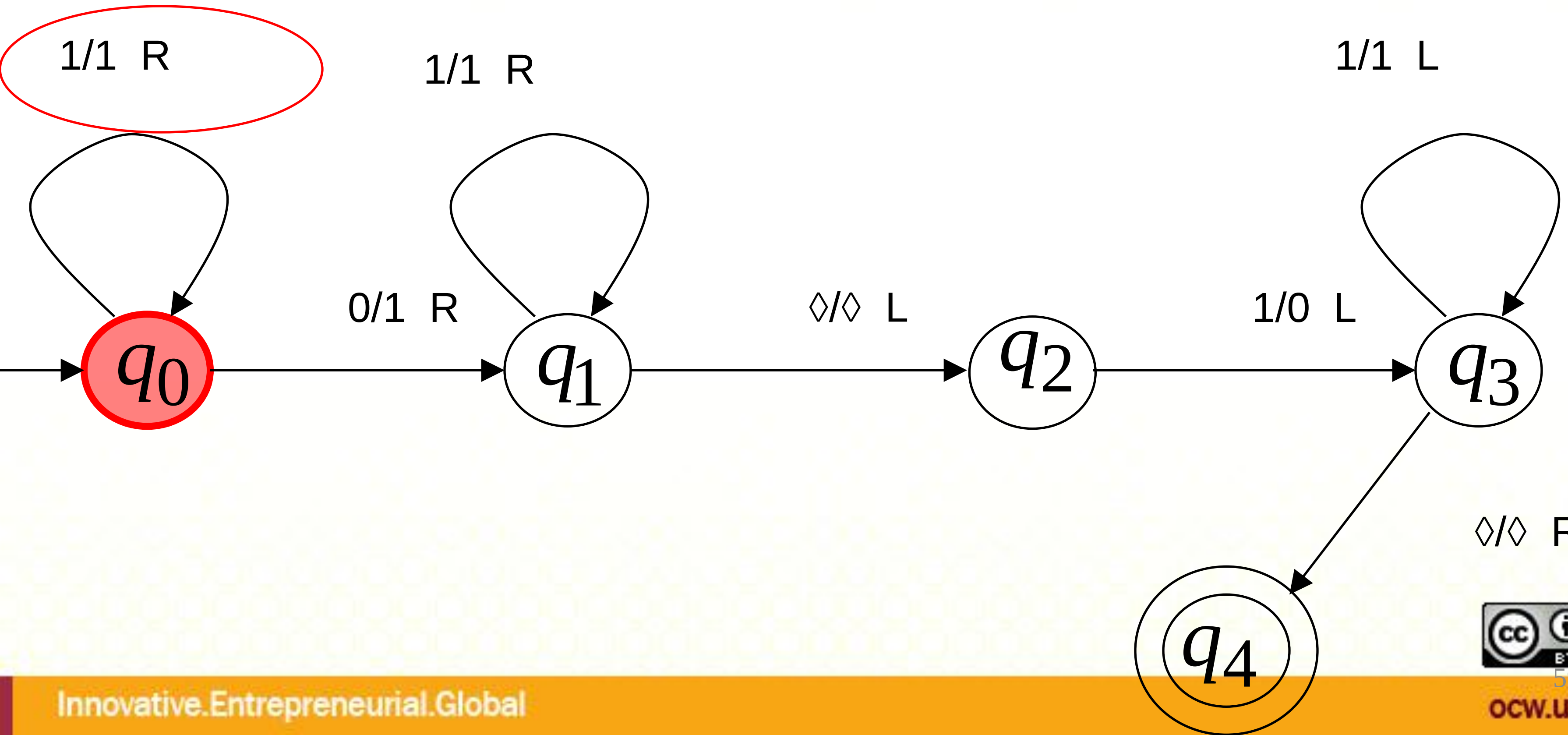


Time 0

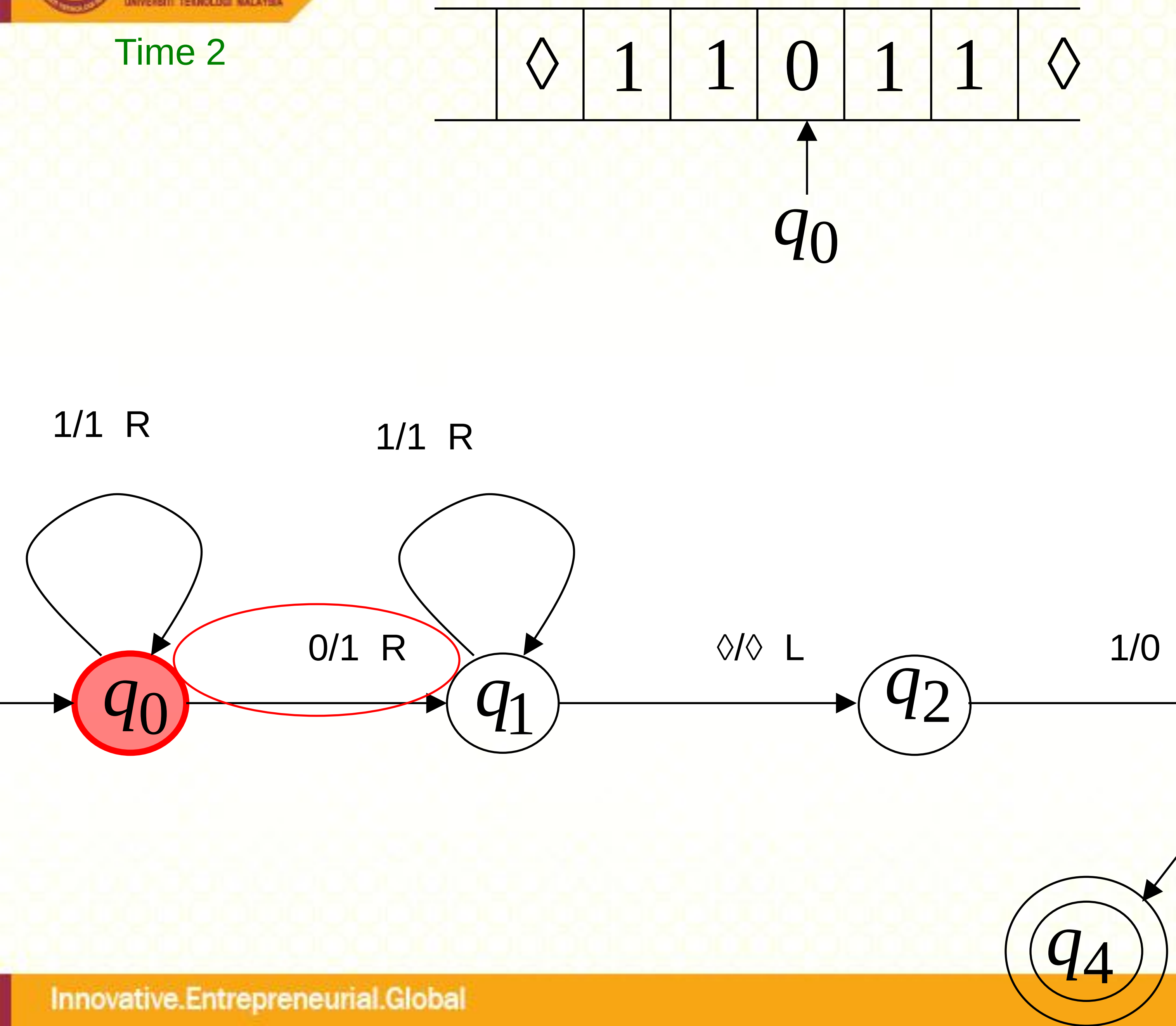


Time 1

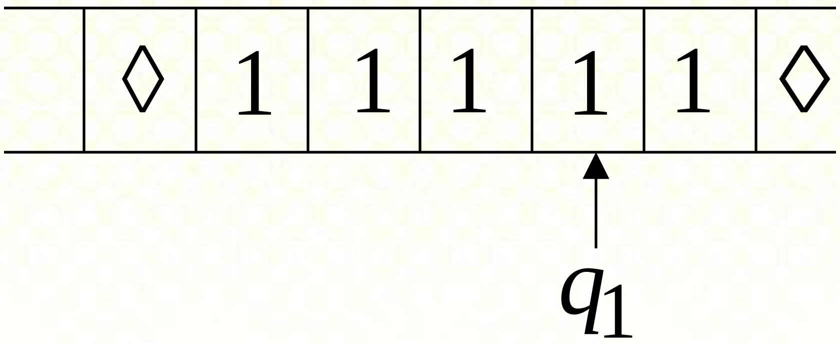


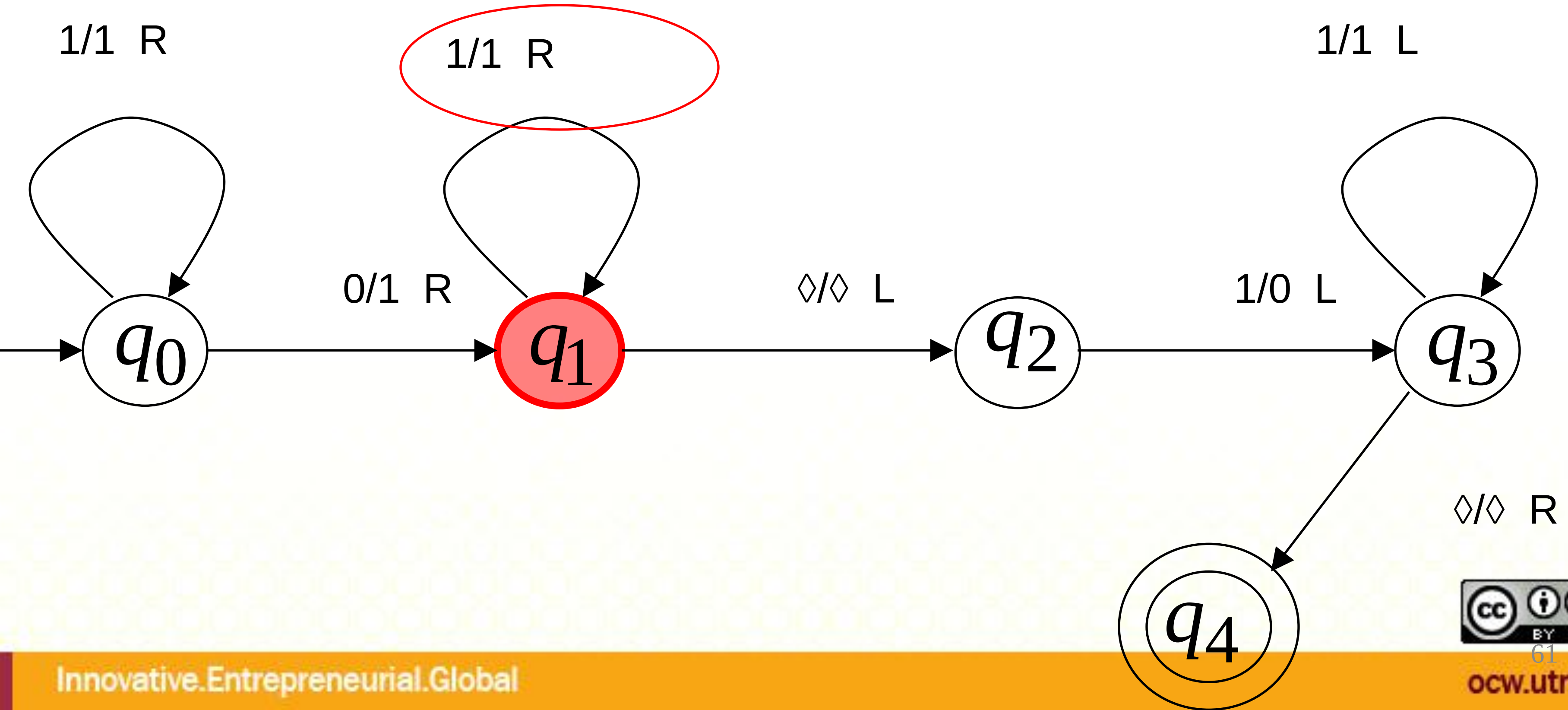


Time 2

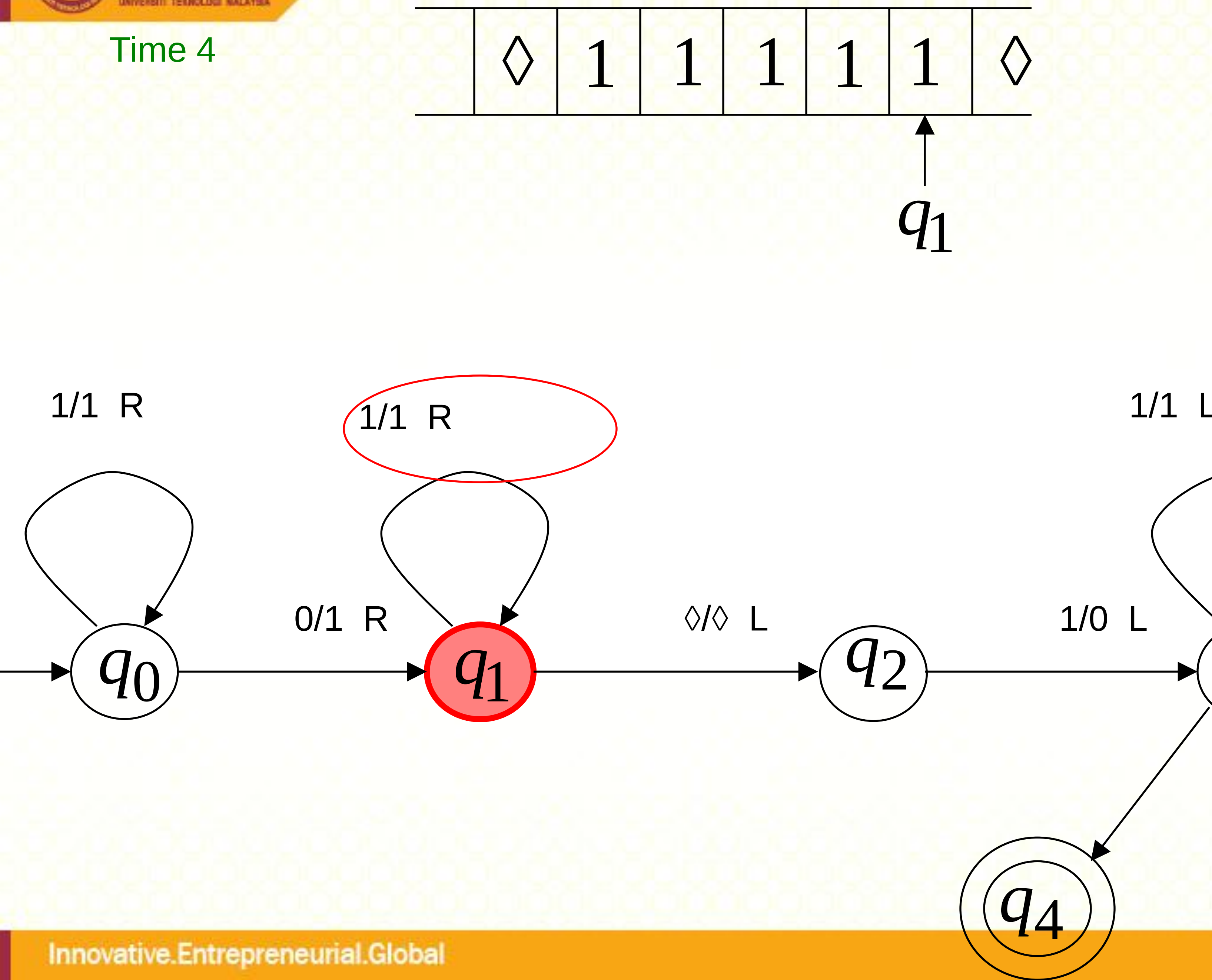


Time 3

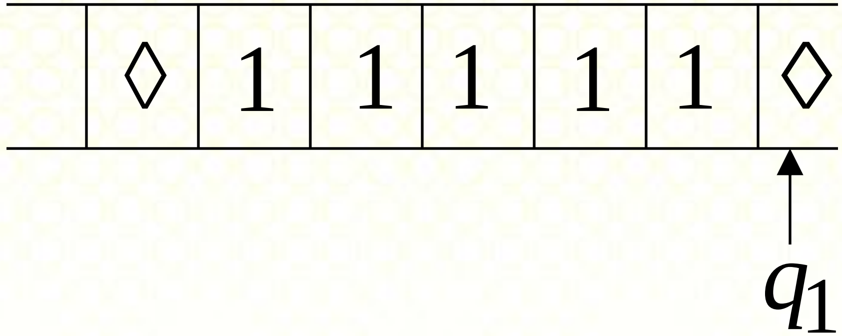


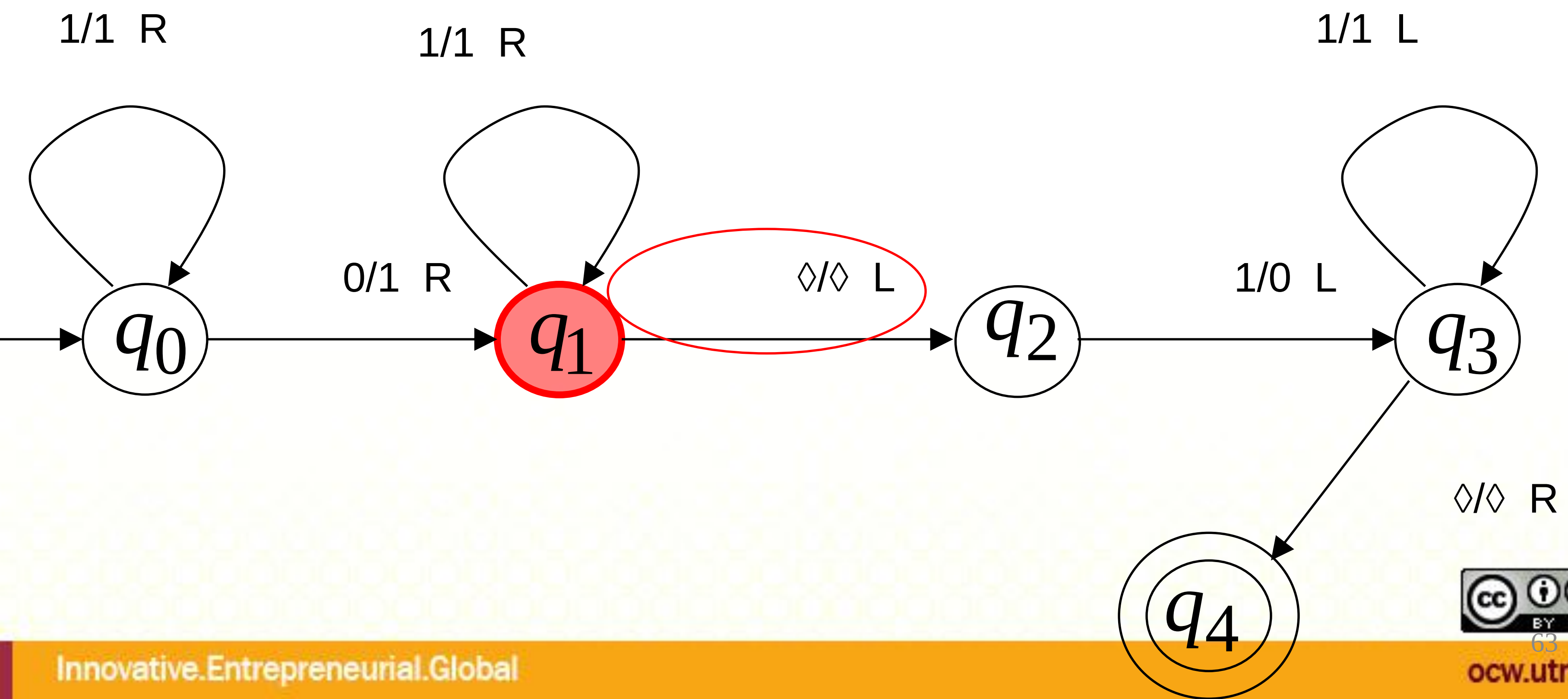


Time 4

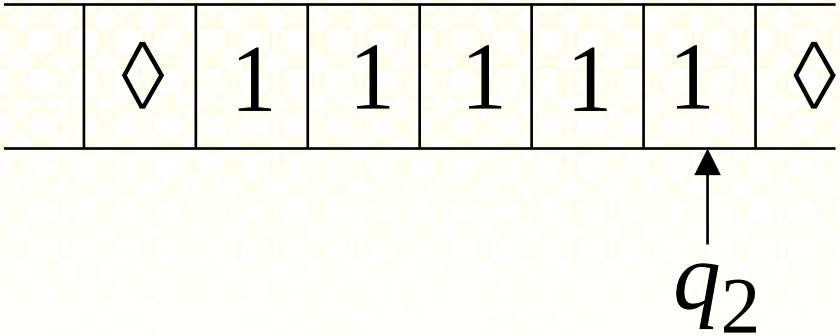


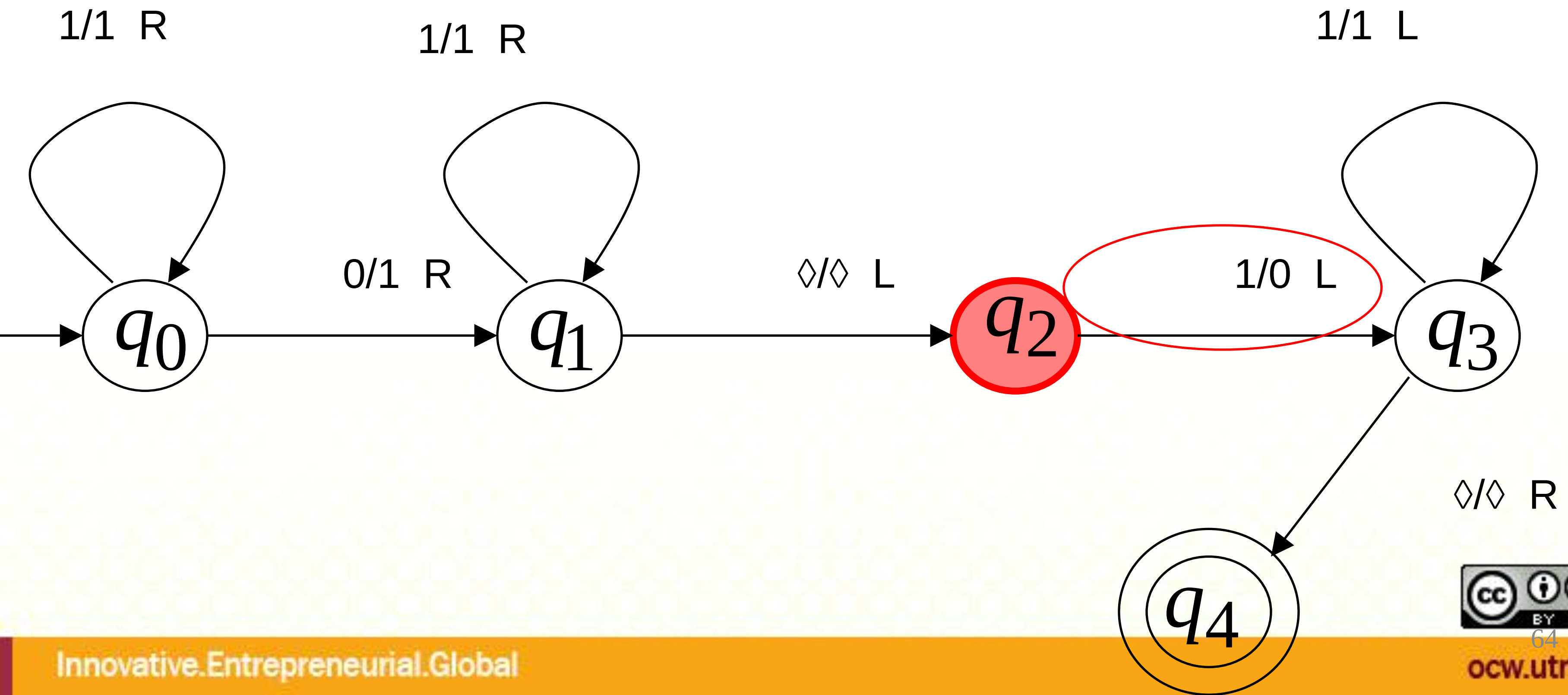
Time 5



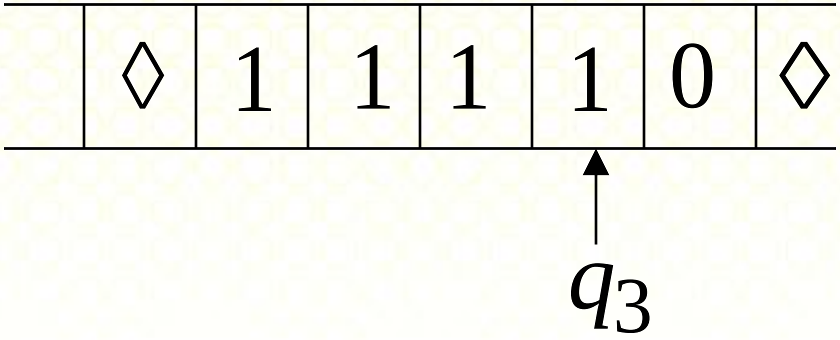


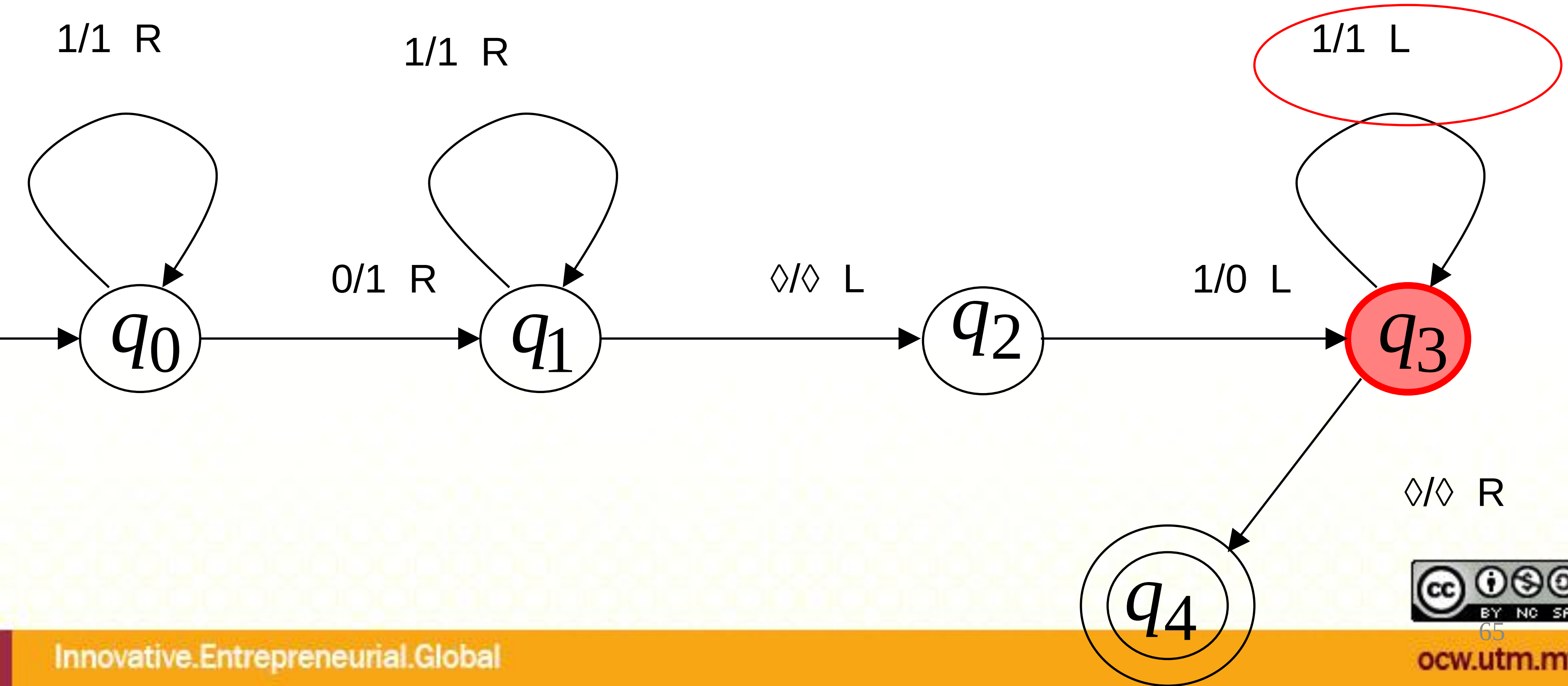
Time 6



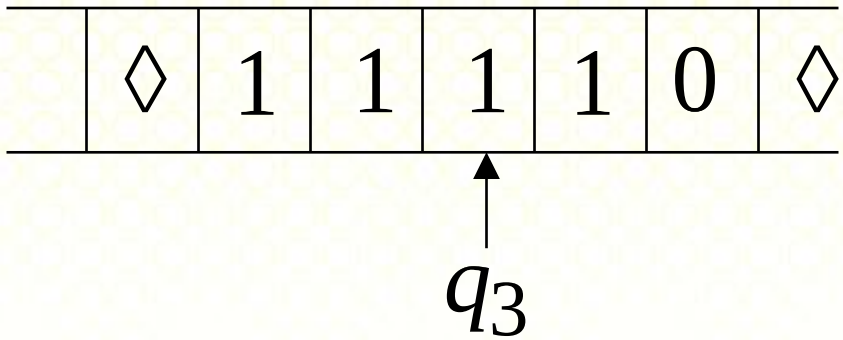


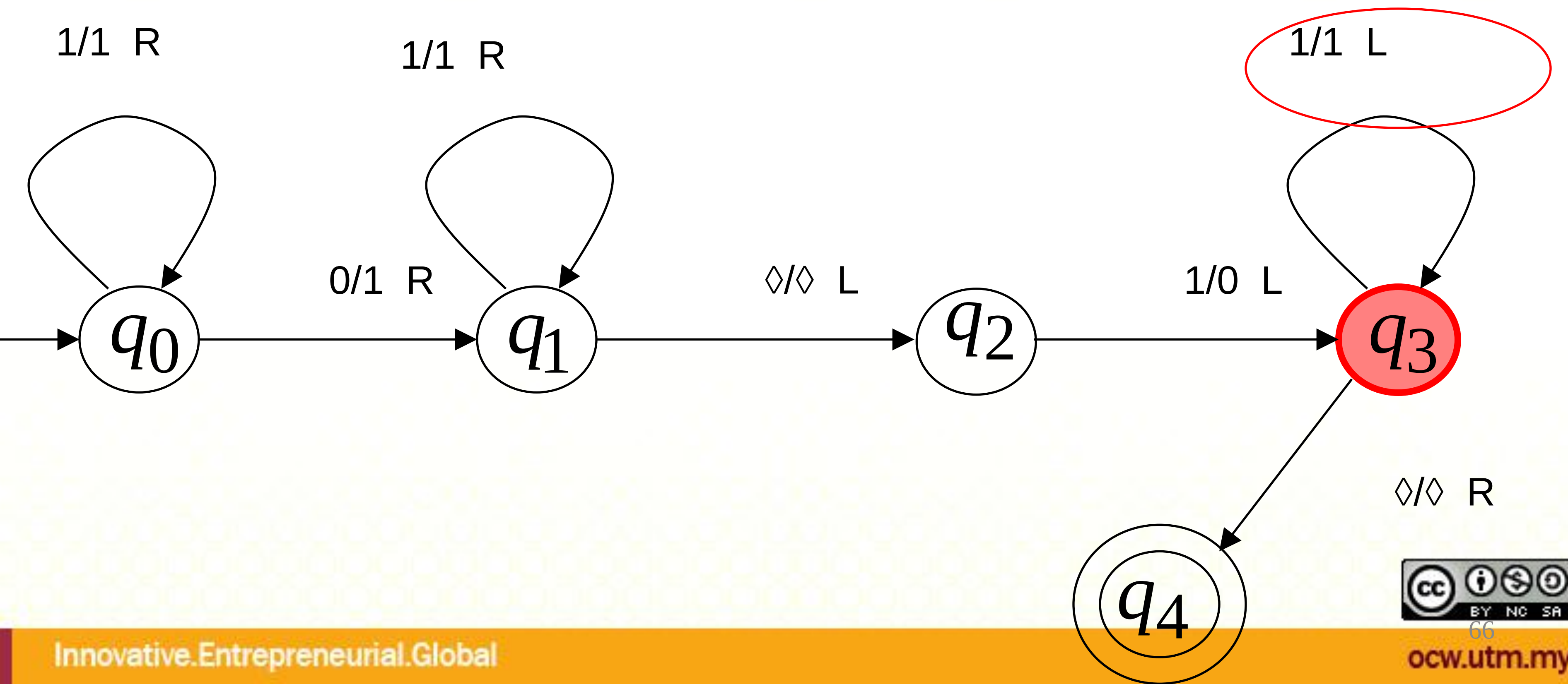
Time 7



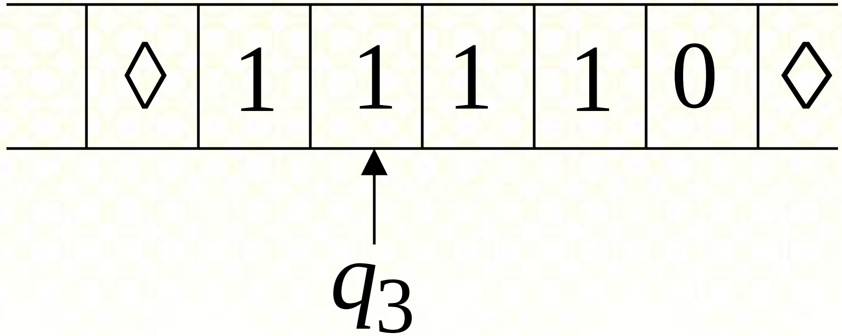


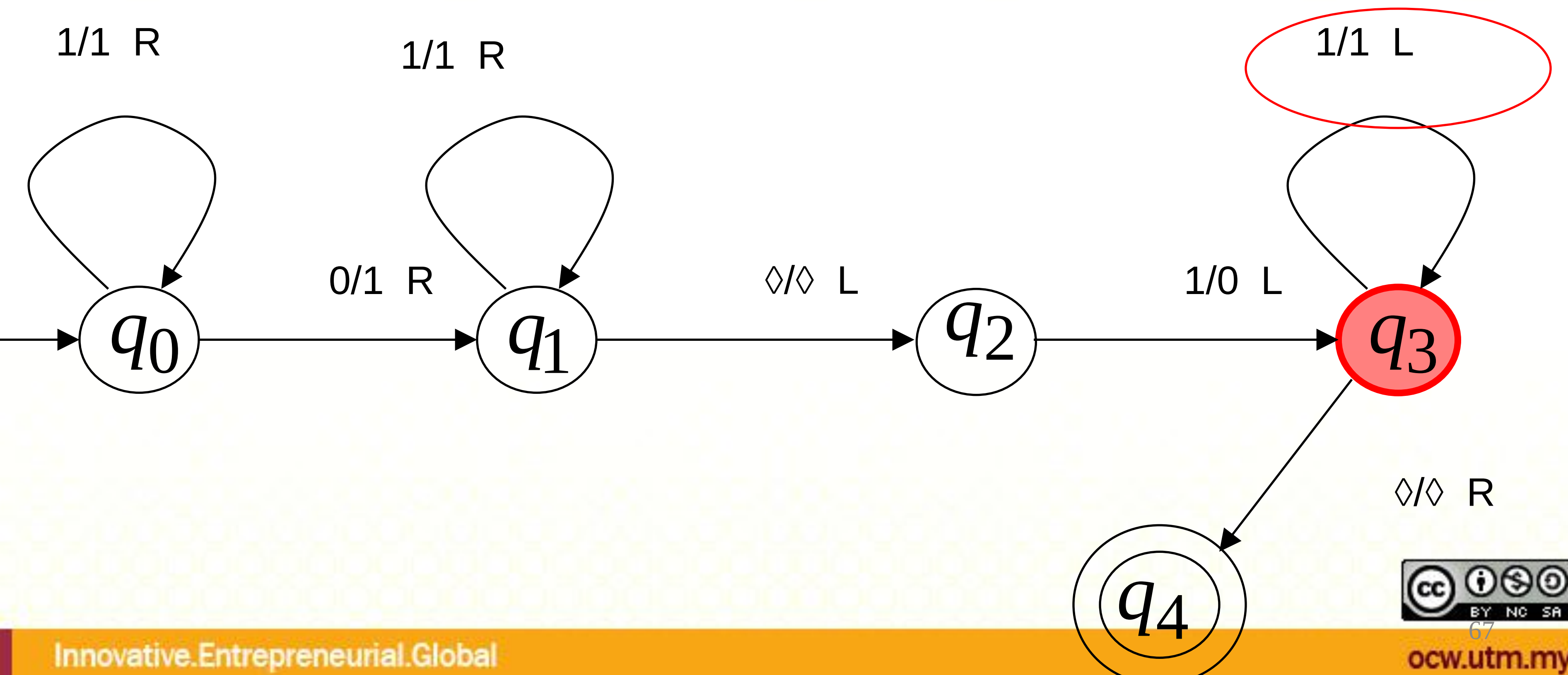
Time 8



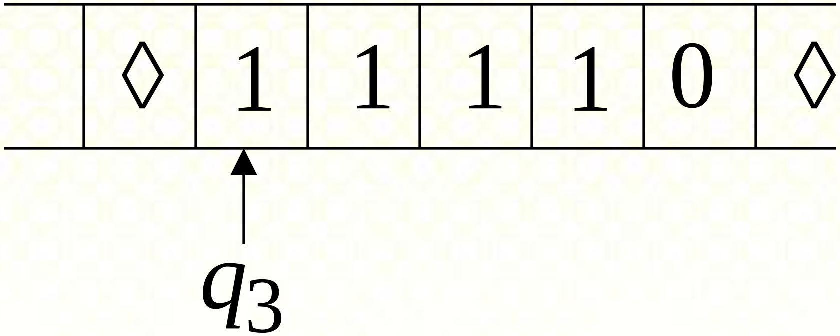


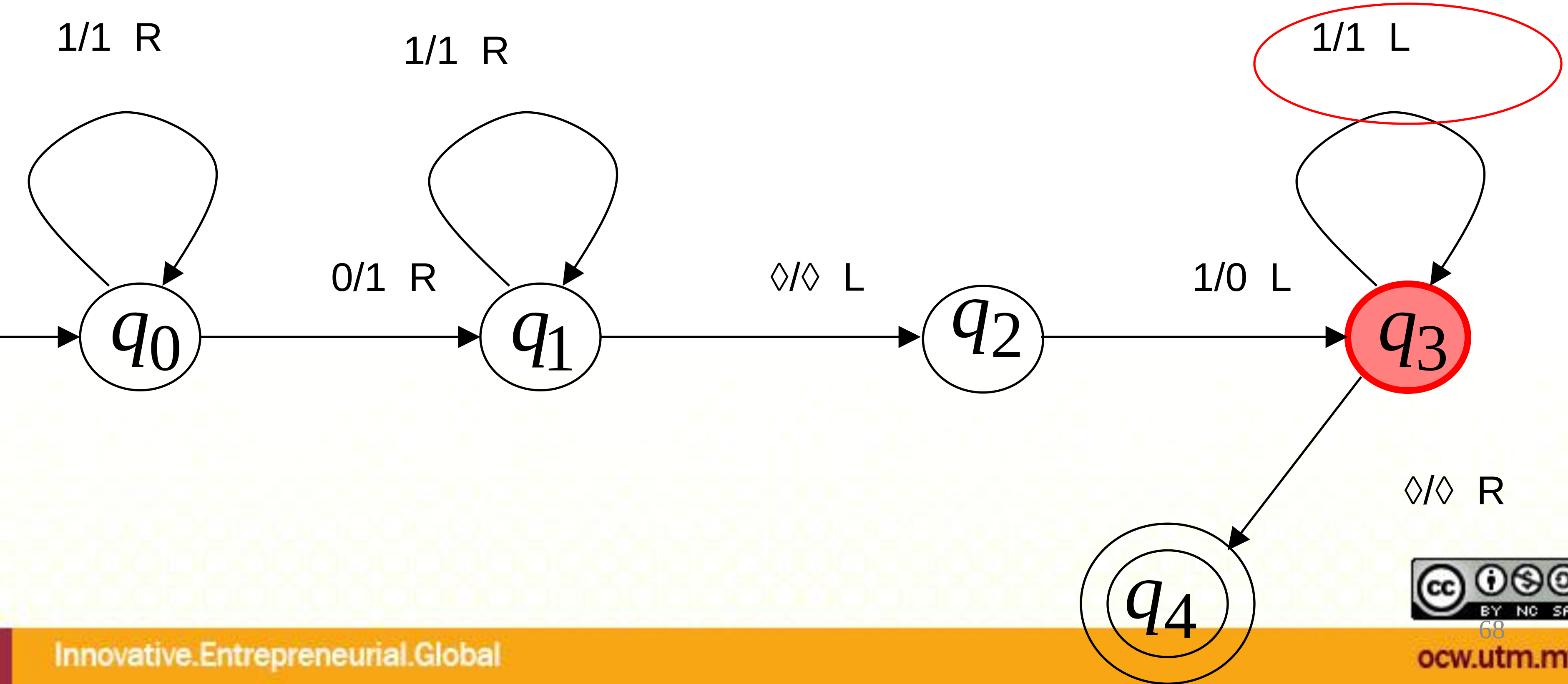
Time 9



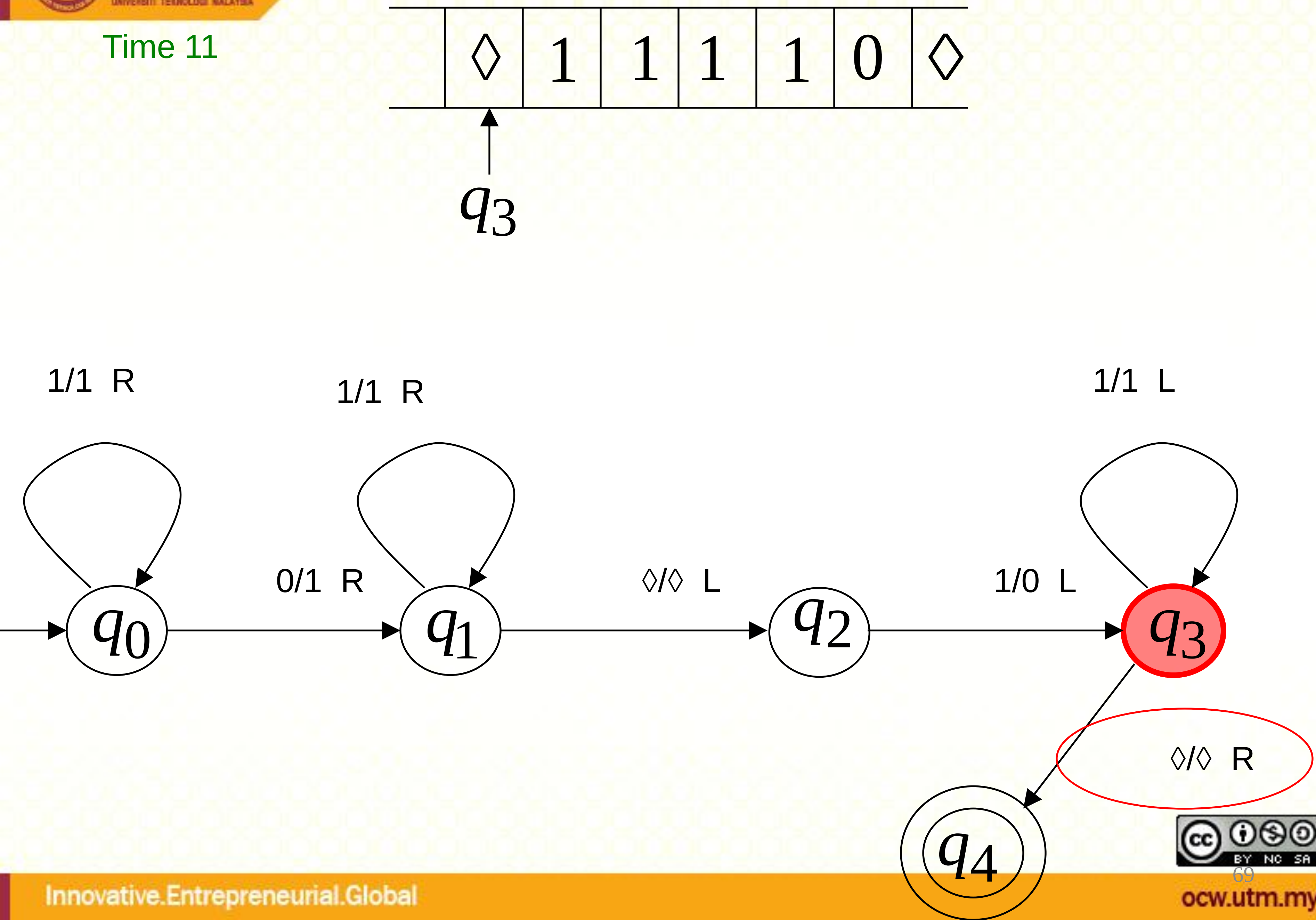


Time 10

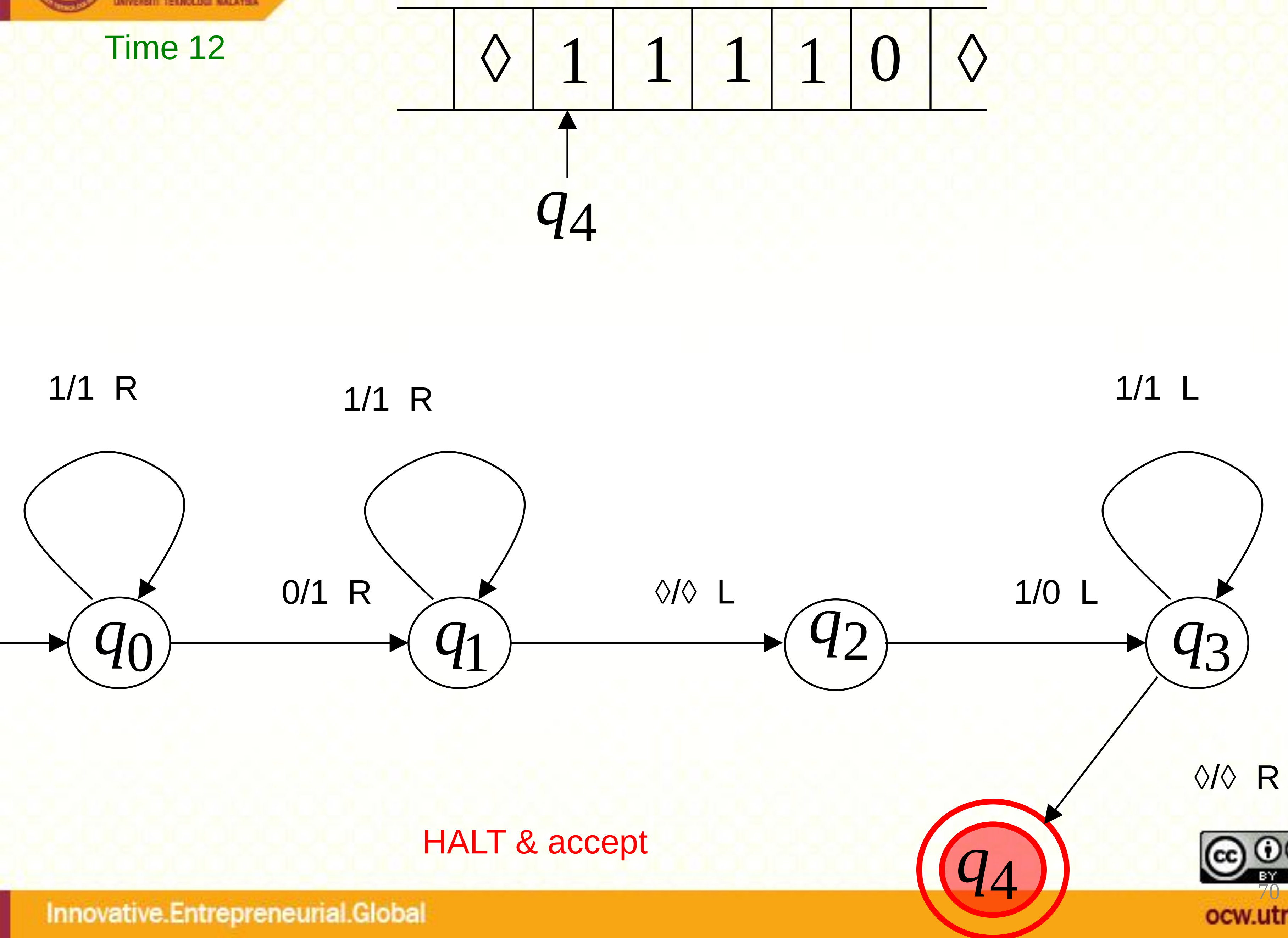




Time 11



Time 12



HALT & accept

Another Example

The function $f(x) = 2x$ is computable

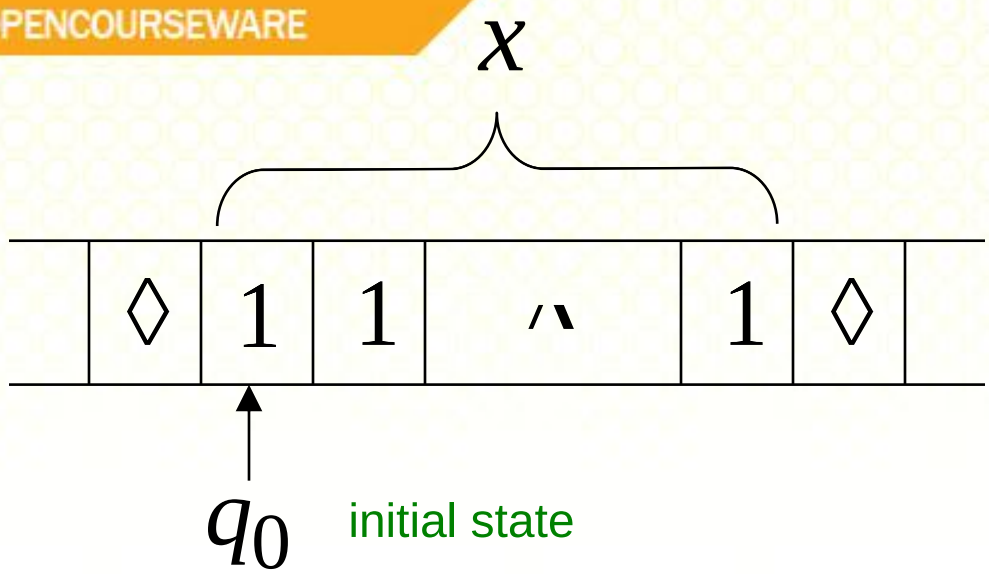
x is integer

Turing Machine:

Input string: x unary

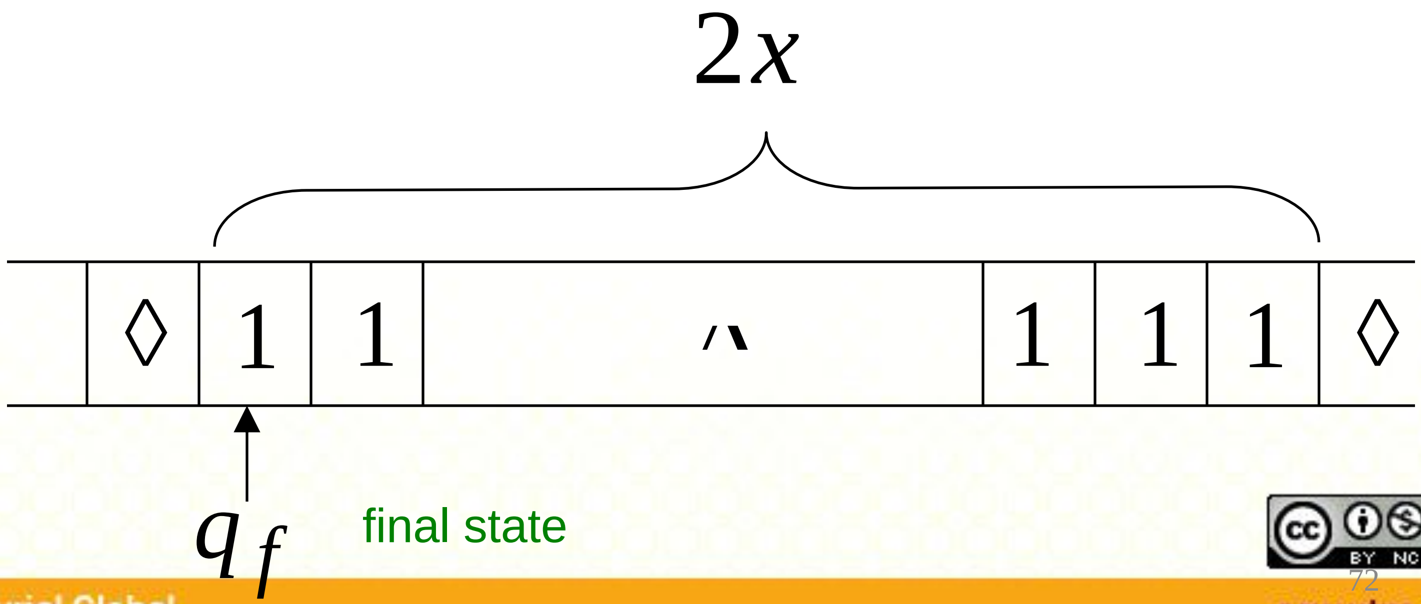
Output string: xx unary

Start



initial state

Finish



final state

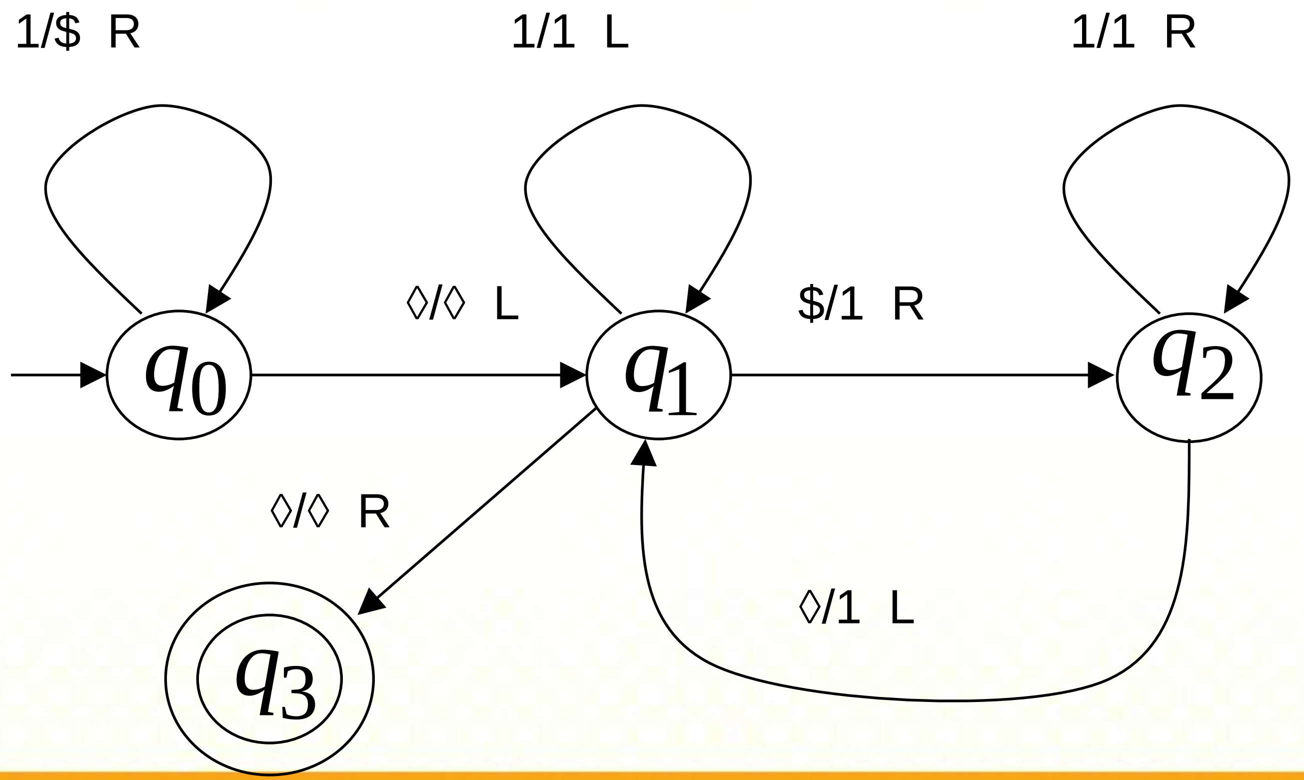
Turing Machine Pseudocode for $f(x) = 2x$

- Replace every 1 with $

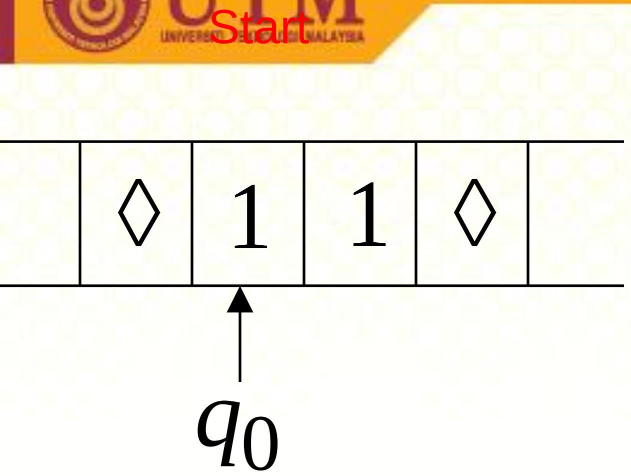
- Repeat:
    - Find rightmost $, replace it with 1
    - Go to right end, insert 1

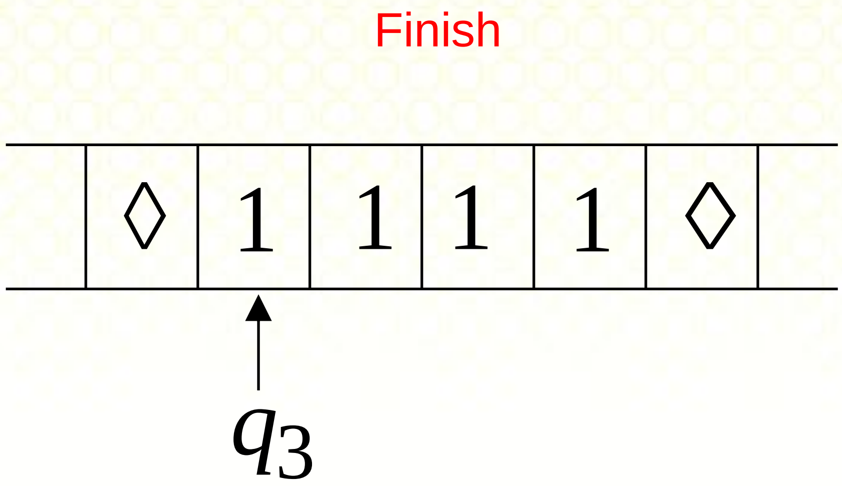
Until no more $ remain

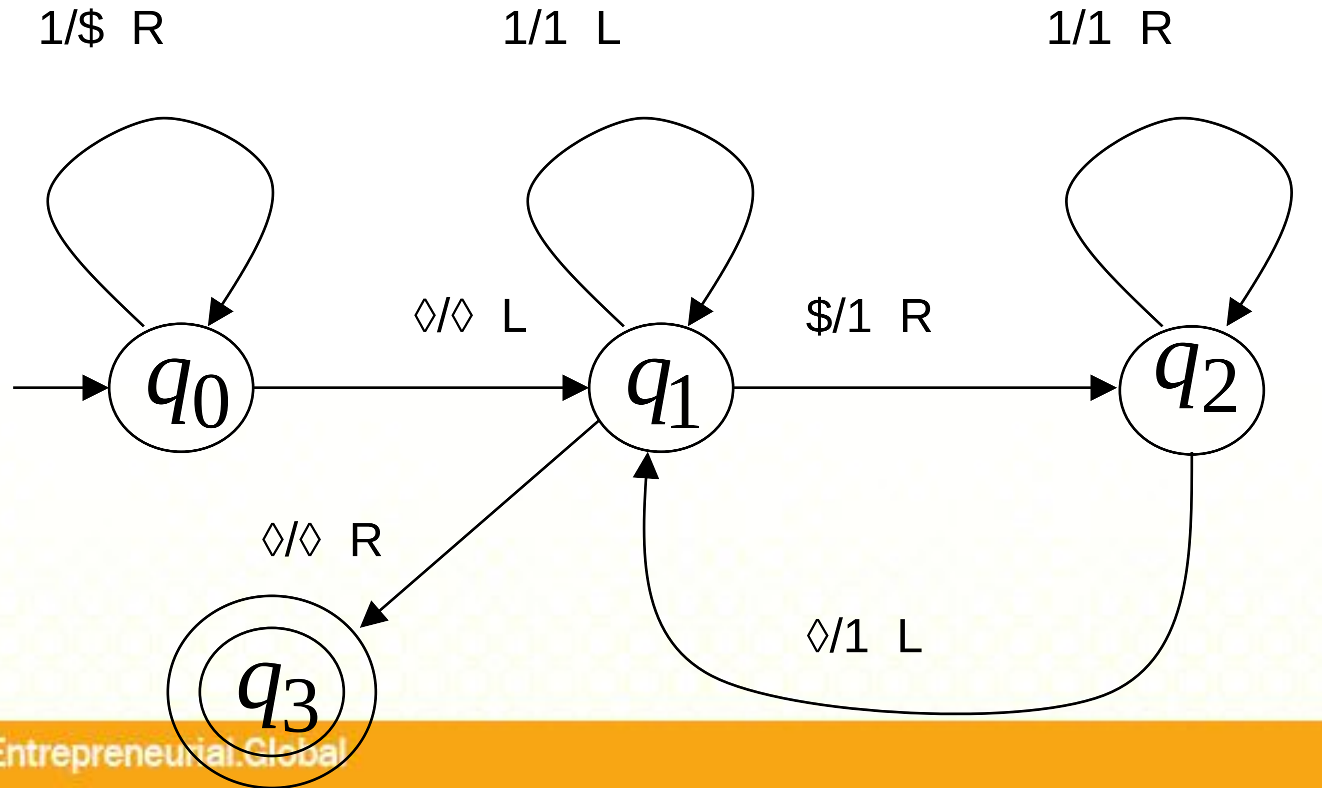
Turing Machine for $f(x) = 2x$



Example







Turing’s Thesis

Turing's thesis:

Any computation carried out
by mechanical means
can be performed by a Turing Machine

(1930)

Computer Science Law:

A computation is mechanical
if and only if
it can be performed by a Turing Machine

There is no known model of computation
more powerful than Turing Machines

Definition of Algorithm:

An algorithm for function $f(w)$
is a
Turing Machine which computes $f(w)$

Algorithms are Turing Machines

When we say:

There exists an algorithm

We mean:

There exists a Turing Machine
that executes the algorithm

References