UTM UNIVERSITI TEKNOLOGI MALAYSIA OPENCOURSEWARE

SKF 3143

Process Control and Dynamics:

Dynamic Response Characteristics of More Complicated Systems

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Learning Objectives

When I complete this chapter, I want to be able to do the following:

1. Analyze the response of complex systems

Poles and Zeros; Their Effect on System Response

- The dynamic behavior of a transfer function model can be characterized by the numerical value of its poles and zeros.
- Consider a particular transfer function,

$$G_{(s)} = \frac{K}{s(\tau_1 s+1)(\tau_2^2 s^2 + 2\zeta\tau_2 s+1)} \qquad (8.1)$$

where $0 \le \zeta < 1$. Using the principle of partial fraction expansion followed by the inverse transformation operation, the response of the system to any input will contain the following functions of time:

A constant term resulting from the *s* factor

An e^{-t/τ_1} term resulting from the $(\tau_1 s+1)$ factor

$$\left.\begin{aligned} & e^{-\zeta t/\tau_2} \sin\frac{\sqrt{1-\zeta^2}}{\tau_2}t \\ & \text{and} \\ & e^{-\zeta t/\tau_2} \cos\frac{\sqrt{1-\zeta^2}}{\tau_2}t \end{aligned}\right\}$$ terms resulting from the $(\tau_2^2 s^2 + 2\zeta\tau_2 s+1)$ factor

Poles and Zeros; Their Effect on System Response

- Each of the above response modes is determined from the factors of the denominator polynomial:

$$
\begin{aligned}
s_1 &= 0 \\
s_2 &= -\frac{1}{\tau_1} \\
s_3 &= -\frac{\zeta}{\tau_2} + j\frac{\sqrt{1-\zeta^2}}{\tau_2} \\
s_4 &= -\frac{\zeta}{\tau_2} - j\frac{\sqrt{1-\zeta^2}}{\tau_2}
\end{aligned}
\qquad (8.2)
$$

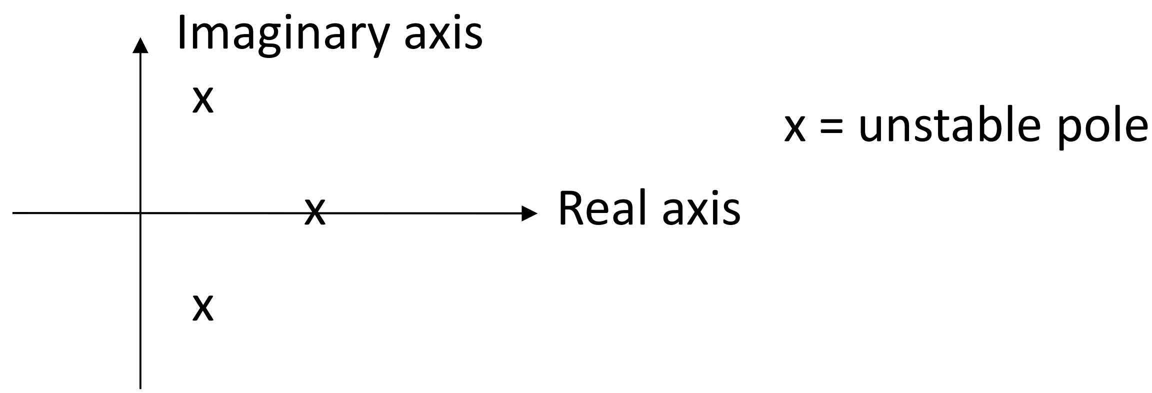
- Control engineers refer to the values of *s* that are roots of the characteristic equation as *poles* of transfer function $G_{(s)}$.
- Sometimes it is useful to plot the roots (poles) and to discuss process response characteristics in terms of root locations in the complex *s* plane.
- Pole at the origin (1/s term in TF model): results in an "integrating process"

Summary: Effect of Poles and Zeros Locations

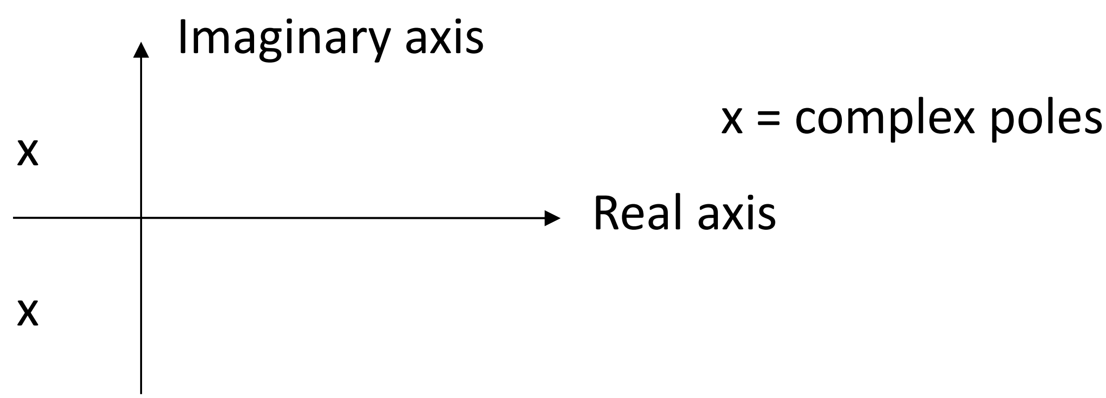
1. Poles

- *Pole in "Right Hand Plane (RHP)"* : results in unstable system (i.e., unstable step responses)

$$p = a + bj$$
$$\left(j = \sqrt{-1} \right)$$



- *Complex pole*: results in oscillatory responses

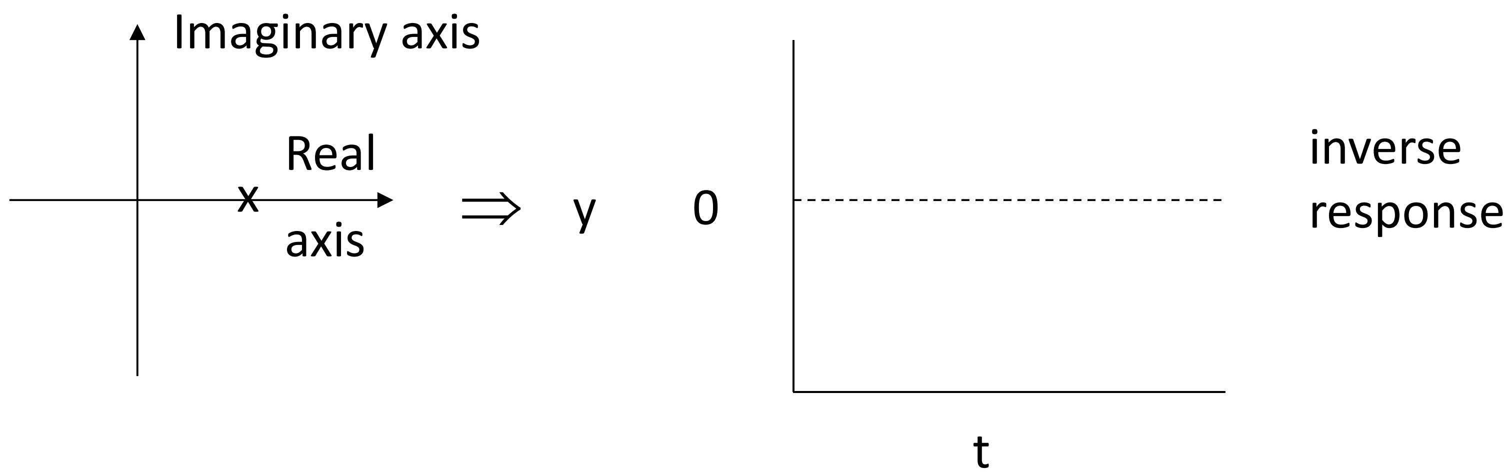


Summary: Effect of Poles and Zeros Locations

2. Zeros

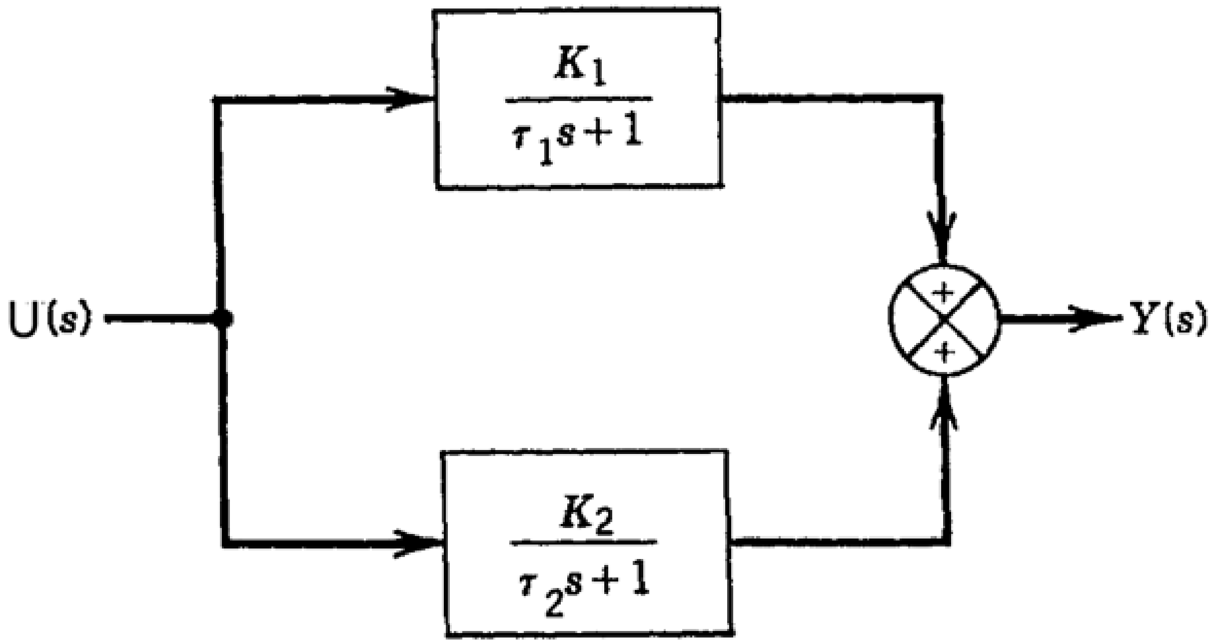
Note: Zeros have no effect on system stability.

- *Zero in RHP:* results in an inverse response to a step change in the input



- *Zero in left half plane:* <u>may</u> result in "overshoot" during a step response.

Inverse Response Due to Two Competing Effects



Two first order process

- The transfer function can be expressed as

$$\frac{Y_{(s)}}{X_{(s)}} = \frac{K_1}{\tau_1 s+1} + \frac{K_2}{\tau_2 s+1}$$

Inverse Response Due to Two Competing Effects

- After rearranging the numerator into standard gain/time constant form, we have

$$\frac{Y_{(s)}}{X_{(s)}} = \frac{(K_1 + K_2)\left(\dfrac{K_1\tau_2 + K_2\tau_1}{K_1 + K_2} s + 1\right)}{(\tau_1 s + 1)(\tau_2 s + 1)} \qquad (8.3)$$

- Equation 8.3 can be put into the form of

$$G_{(s)} = \frac{K(\tau_a s + 1)}{(\tau_1 s + 1)(\tau_2 s + 1)} \qquad (8.4)$$

if

$$K = K_1 + K_2$$

$$\tau_a = \frac{K_1\tau_2 + K_2\tau_1}{K_1 + K_2} = \frac{K_1\tau_2 + K_2\tau_1}{K}$$

Inverse Response Due to Two Competing Effects

- The condition for an inverse response to exist is $\tau_a < 0$ or

$$\frac{K_1\tau_2 + K_2\tau_1}{K} < 0$$

- For either positive or negative *K*, expression above can be arranged to convenient form

$$-\frac{K_2}{K_1} > \frac{\tau_2}{\tau_1} \qquad (8.5)$$

Time Delays

Time delays occur due to:

1. Fluid flow in pipe
2. Transport of solid material (e.g., conveyer belt)
3. Chemical analysis
   - Sampling line delay
   - Time required to do the analysis (e.g., on-line gas chromatograph)

Mathematical description:

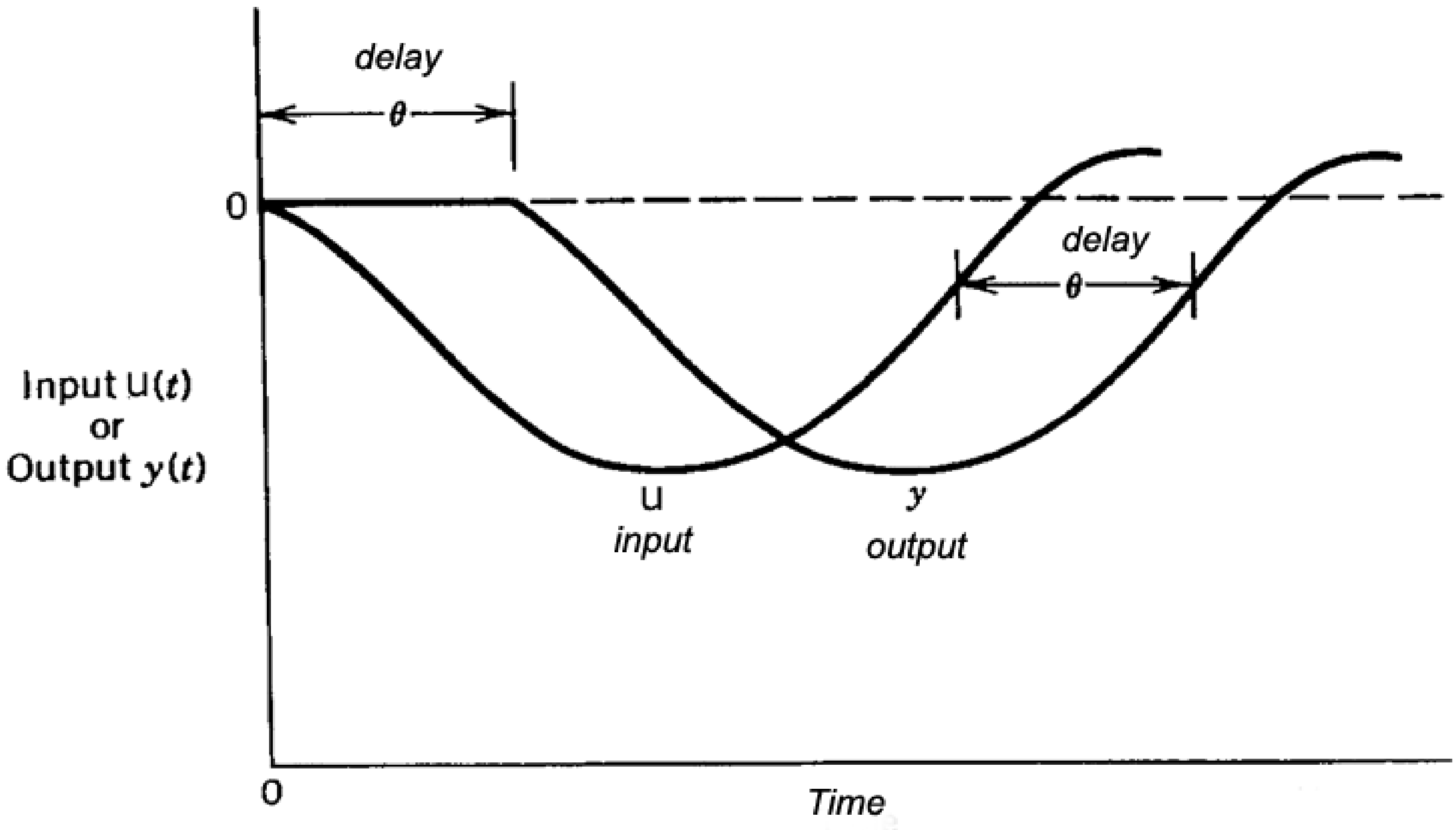
A time delay, θ, between an input *u* and an output *y* results in the following expression:

$$y_{(t)} = \begin{cases} 0 & for \quad t < \theta \\ x(t-\theta) & for \quad t \geq \theta \end{cases}$$

The transfer function of a time delay of θ units is given by

$$\frac{Y_{(s)}}{X_{(s)}} = G_{(s)} = e^{-\theta s} \qquad (8.6)$$

Time Delays: effect of a pure time delay



Time Delays: Polynomial Approximations to $e^{-\theta s}$:

For purposes of analysis using analytical solutions to transfer functions, polynomial approximations for $e^{-\theta s}$ are commonly used. Example: simulation software such as MATLAB and MatrixX.

Two widely used approximations are:

1. Taylor Series Expansion:

$$e^{-\theta s} = 1 - \theta s + \frac{\theta^2 s^2}{2!} - \frac{\theta^3 s^3}{3!} + \frac{\theta^4 s^4}{4!} - \frac{\theta^5 s^5}{5!} + \cdots \qquad (8.7)$$

The approximation is obtained by truncating after only a few terms.

2. Padé Approximations:

Many are available. For example, the 1/1 approximation is,

$$e^{-\theta s} \approx \frac{1 - \frac{\theta}{2} s}{1 + \frac{\theta}{2} s} \qquad (8.8)$$

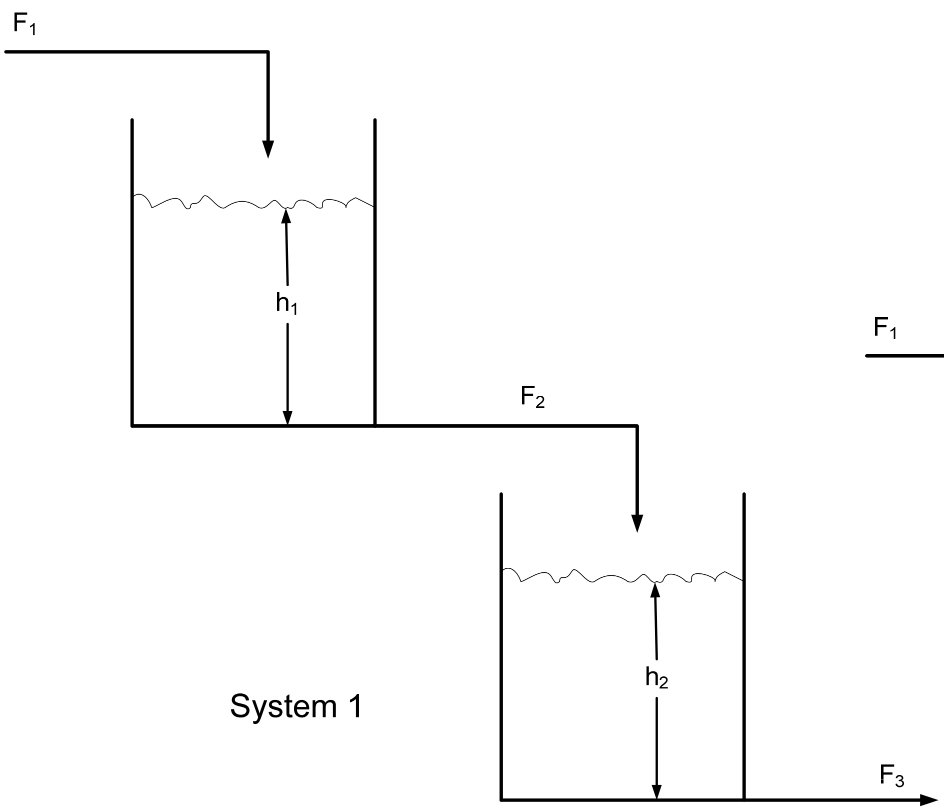
Interacting vs Noninteracting Processes

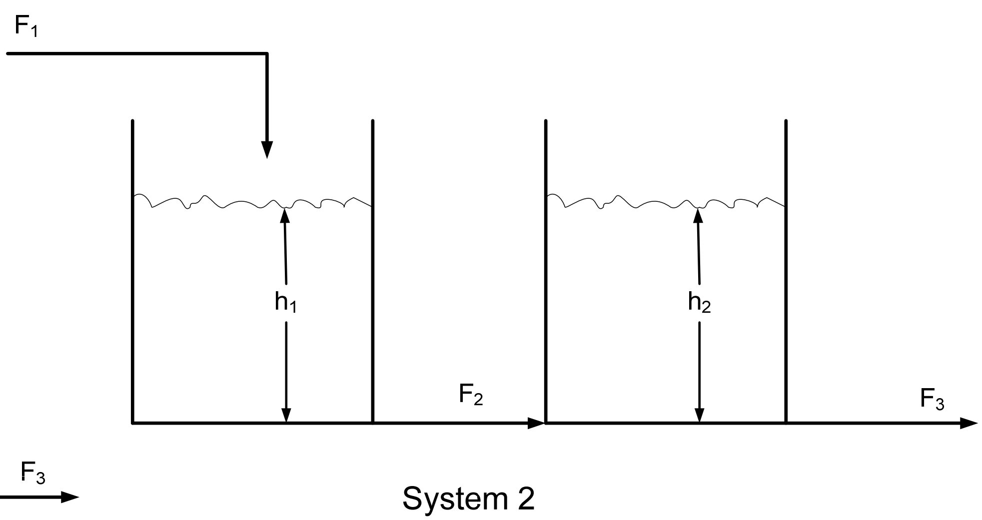
- Consider a process with several invariables and several output variables. The process is said to be *interacting* if:
  - Each input affects more than one output.
- or
  - A change in one output affects the other outputs.

  - Otherwise, the process is called *noninteracting*.

- As an example, we will consider the two liquid-level storage systems.
- In general, transfer functions for interacting processes are more complicated than those for noninteracting processes.

Interacting vs Noninteracting Processes

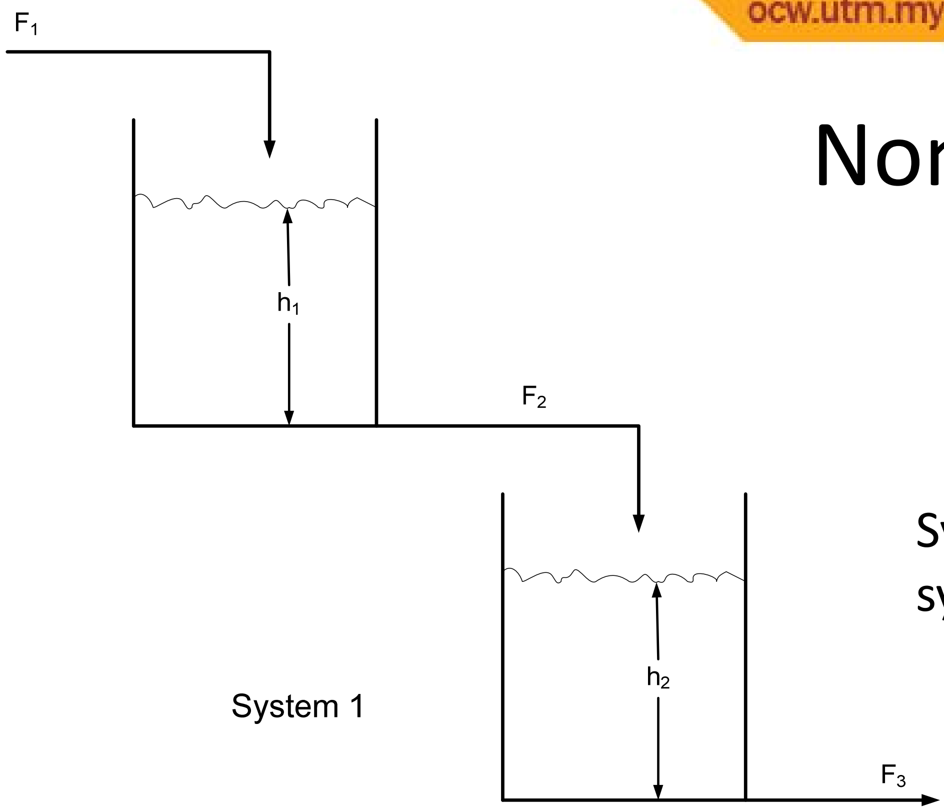




System 1. A noninteracting system: two surge tanks in series.

System 2. Two tanks in series whose liquid levels interact.

Noninteracting Process



System 1. A noninteracting system: two surge tanks in series.

$$\frac{H'_{1(s)}}{Q'_{i(s)}} = \frac{R_1}{A_1 R_1 + 1} = \frac{K_1}{\tau_1 s + 1}$$

$$\frac{H'_{2(s)}}{Q'_{1(s)}} = \frac{R_2}{A_2 R_2 + 1} = \frac{K_2}{\tau_2 s + 1}$$

$$\frac{Q'_{1(s)}}{H'_{1(s)}} = \frac{1}{R_1} = \frac{1}{K_1}$$

$$\frac{Q'_{2(s)}}{H'_{2(s)}} = \frac{1}{R_2} = \frac{1}{K_2}$$

Noninteracting Process

The transfer function relating q_2 and q_i can be derived by:

$$\frac{Q'_{2(s)}}{Q'_{i(s)}} = \frac{Q'_{2(s)}}{H_{2(s)}} \frac{H_{2(s)}}{Q'_{1(s)}} \frac{Q'_{1(s)}}{H_{1(s)}} \frac{H_{1(s)}}{Q'_{i(s)}}$$

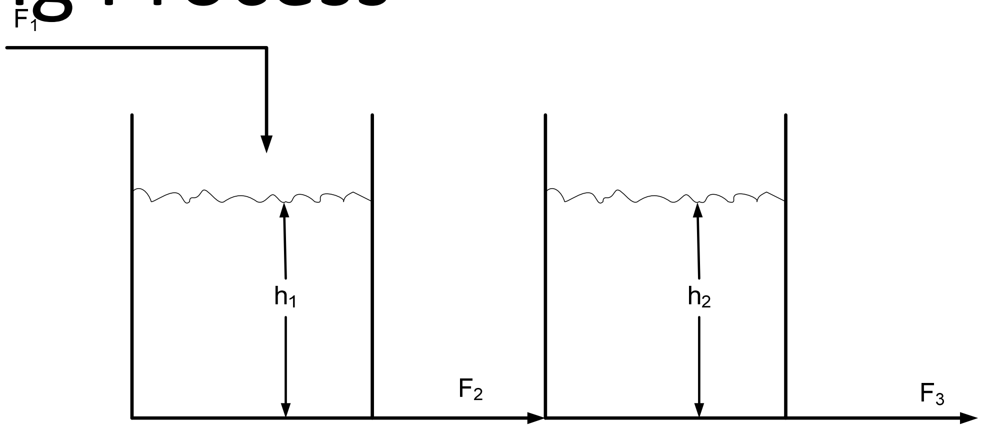
or

$$\frac{Q'_{2(s)}}{Q'_{i(s)}} = \frac{1}{K_2} \frac{K_2}{\tau_2 s + 1} \frac{1}{K_1} \frac{K_1}{\tau_1 s + 1}$$

Which can be simplified to yield

$$\frac{Q'_{2(s)}}{Q'_{i(s)}} = \frac{1}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

Interacting Process



System 2

The transfer functions for the interacting system are:

$$\frac{H'_{2(s)}}{Q'_{i(s)}} = \frac{R_2}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

$$\frac{Q'_{2(s)}}{Q'_{i(s)}} = \frac{1}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

$$\frac{H'_{1(s)}}{Q'_{i(s)}} = \frac{K'_1(\tau_a s + 1)}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

where

$$\tau = \sqrt{\tau_1 \tau_2}$$

$$\zeta = \frac{\tau_1 + \tau_2 + R_2 A_1}{2\sqrt{\tau_1 \tau_2}}$$

$$\tau_a = \frac{R_1 R_2 A_2}{R_1 + R_2}$$

Model Comparison

- Noninteracting system

$$\frac{Q'_{2(s)}}{Q'_{i(s)}} = \frac{1}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

where $\tau_1 = A_1 R_1$ and $\tau_2 = A_2 R_2$

- Interacting system

$$\frac{Q'_{2(s)}}{Q'_{i(s)}} = \frac{1}{\tau^2 s^2 + 2\zeta \tau s + 1}$$

where $\zeta > 1$ and $\tau = \sqrt{\tau_1 \tau_2}$

- General Conclusions

1. The interacting system has a slower response.
   (Example: consider the special case where $t = t_1 = t_2$.)
2. Which two-tank system provides the best damping of inlet flow disturbances?

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