SKAA 1213 - Engineering Mechanics

TOPIC 9

Moment of Inertia

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Moment of Inertia for Areas

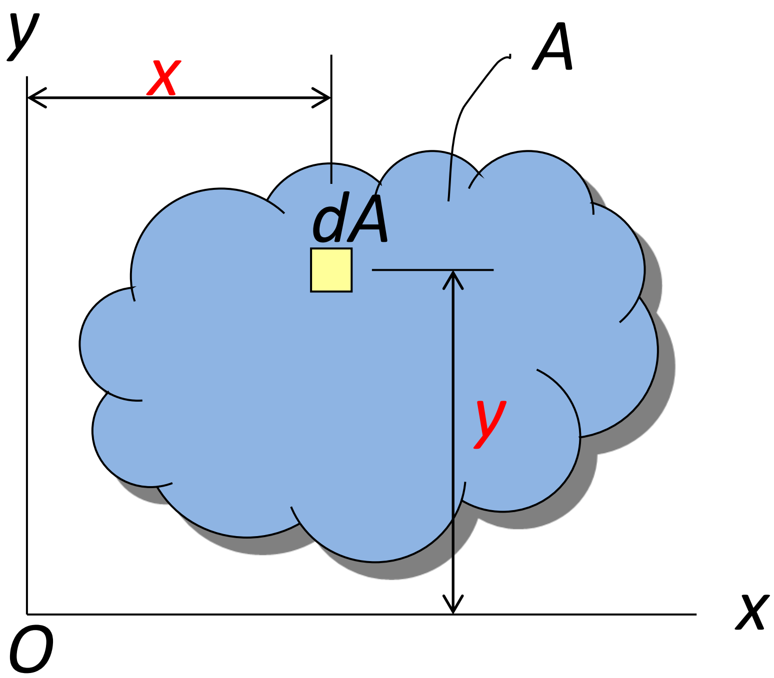
Definition : $\int x^2 \, dA$

Moment of inertia of a differential area dA about x and y axes are :

$$dI_x = y^2 dA \qquad dI_y = x^2 dA$$

For the entire area, the moment of inertia;

$$\boxed{I_x = \int_A y^2 dA \qquad I_y = \int_x x^2 dA}$$



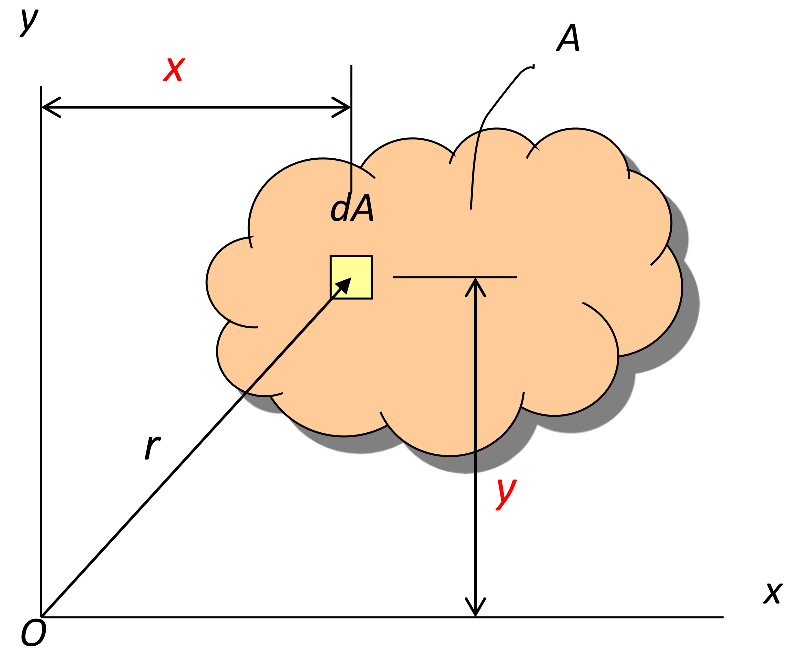
Polar Moment of Area

The polar moment of inertia of *dA* (about z-axis),

$$dJ_o = r^2\, dA\ .$$

For the entire area;

$$\boxed{J_o = \int_A r^2\, dA = I_x + I_y}$$



Radius of Gyration of an Area

$$K_x = \sqrt{\frac{I_x}{A}}$$

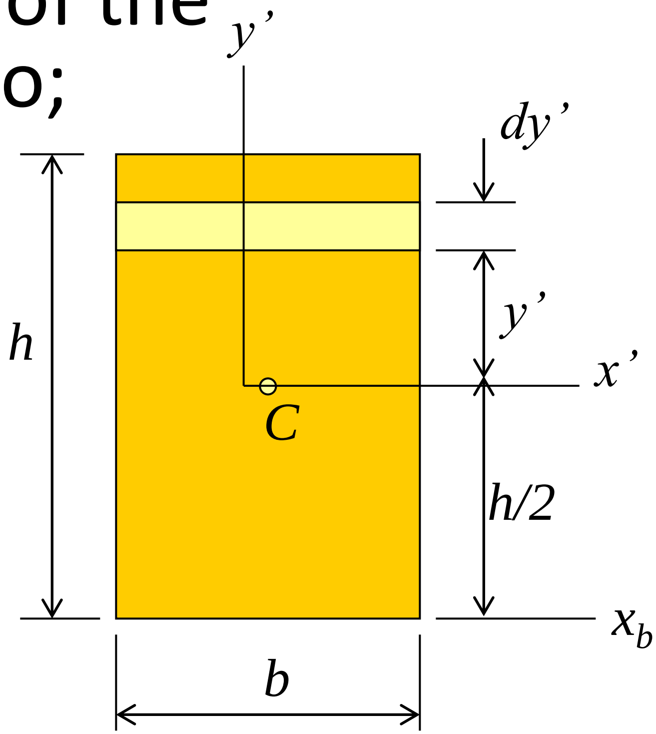
$$K_y = \sqrt{\frac{I_y}{A}}$$

$$K_z = \sqrt{\frac{I_z}{A}}$$

Example 1

Determine the moment of inertia of the rectangular section with respect to;

(a) centroidal axis x',
(b) axis x_b
(c) z axis passing through C, and
(d) the radius of gyration x'.



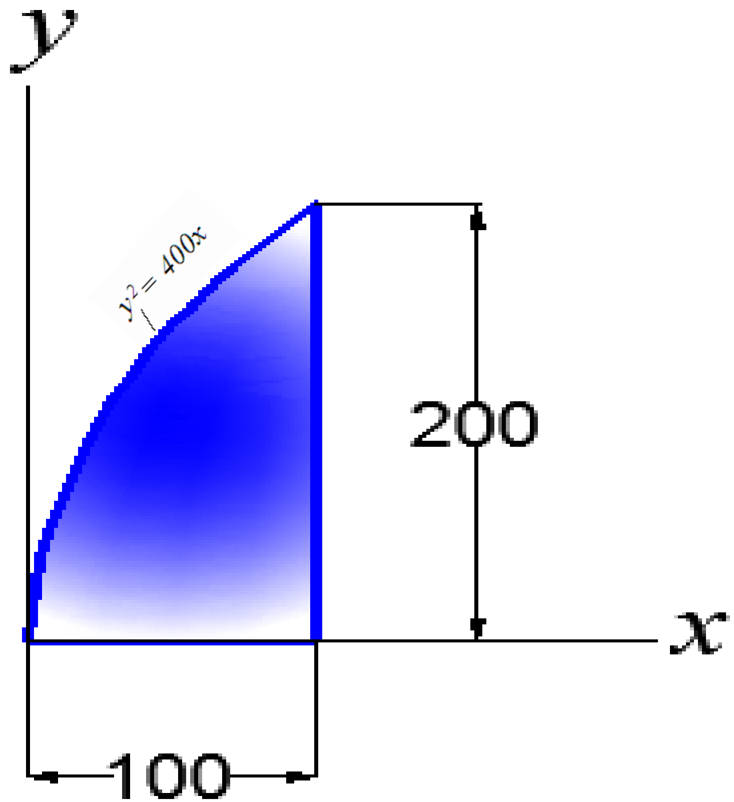
[Answer]

(a) $I_{x'} = \underline{bh^3/12}$
(b) $I_{xb} = \underline{bh^3/3}$
(c) $J_c = \frac{1}{12} bh\,(h^2 + b^2)$

(d) $K_x = \sqrt{h^2/12}$

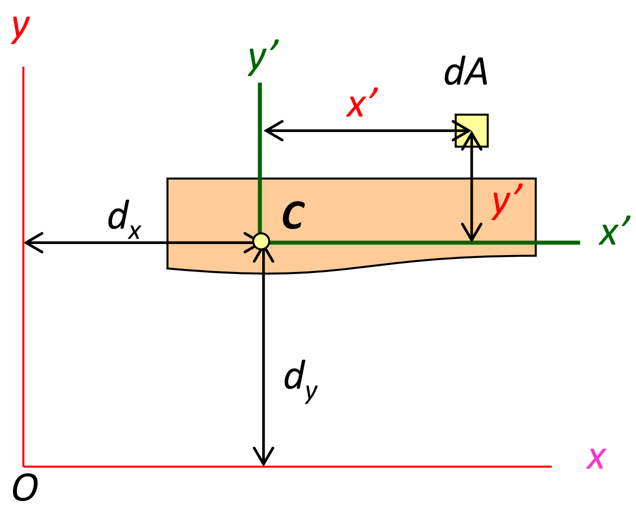
Example 2

Compute the moment of inertia of the shaded area about the x axis. *[Answer : $I_x = 106x10^6\ mm^4$]*



Parallel-Axis Theorem for an Area

Definition & Usage : Provided that the moment of inertia about an axis which pass through the centroid is known, the moment of inertia about a corresponding **parallel** axis can be determined easily by using *parallel axis theorem.*



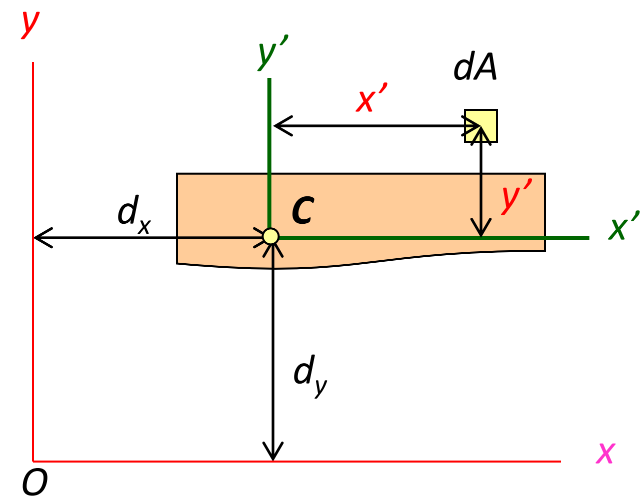
Prove

Consider a differential area *dA* about the *x* axis;

$dI_x = (y' + d_y)^2\, dA$

$I_x = \int_A (y' + d_y)^2\, dA$

$= \int_A y'^2 dA + 2d_y \int_A y' dA + d_y^2 \int_A dA$



The 1st intergal is the MOI about the centroid.

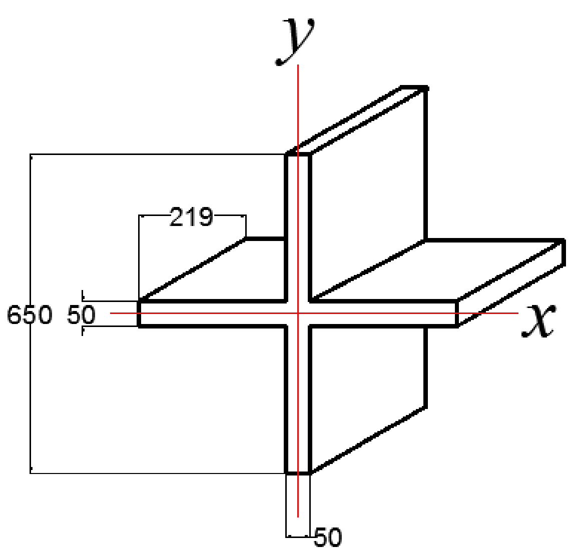
The 2nd integral is zero since the moment of area about the centroidal axis is 0.

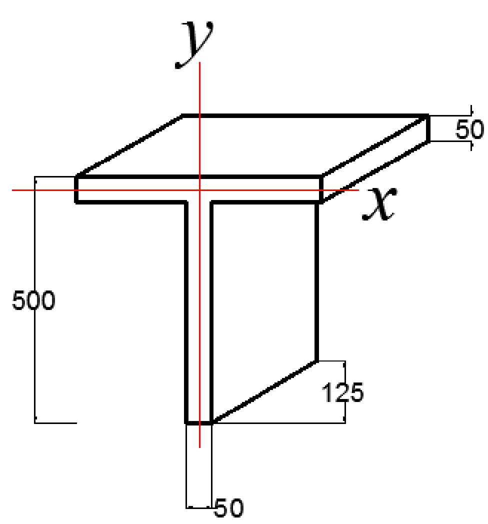
The 3rd integral is the total area.

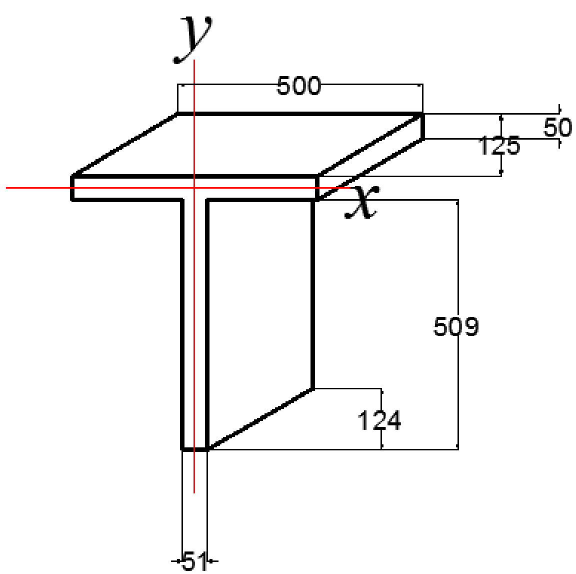
Conclusion : $\boxed{I_x = I_{x'} + Ad_y^2}$ $\boxed{I_y = I_{y'} + Ad_x^2}$ $\boxed{J_o = I_C + Ad^2}$

Moments of Inertia for Composite Areas

Provided the moment of inertia of each parts is known , then the moment of inertia of the composite area equals the algebraic summation of the moments of inertia of each individual part.





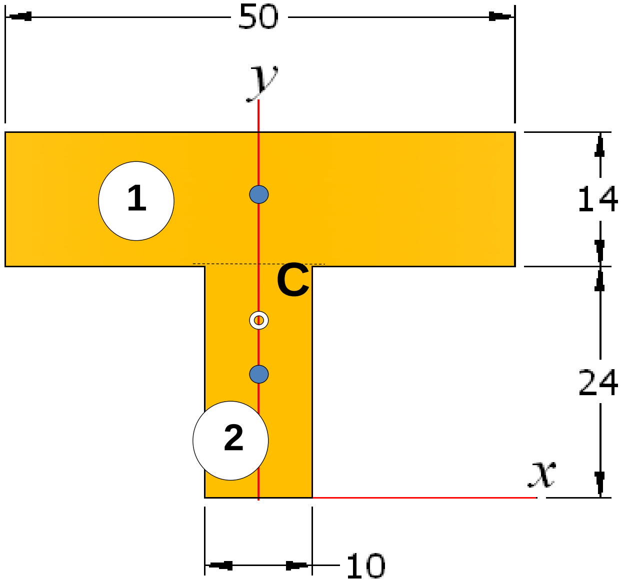


Example 3

Determine the moment of inertia of the T-section about the centroidal *x'* and *y'* axes.

[Answer : , $x = 0$, $y = 26.1$cm, $I_x = 87474$ cm^4 , $I_y = 147833$ cm^4]

Dimensions cm



Example 4

Compute the moment of initial of a composite area about the ***x'*** centroidal axis. *[Answer : $I_{x'} = 14905\ cm^4$]*