SKAA 1213 - Engineering Mechanics

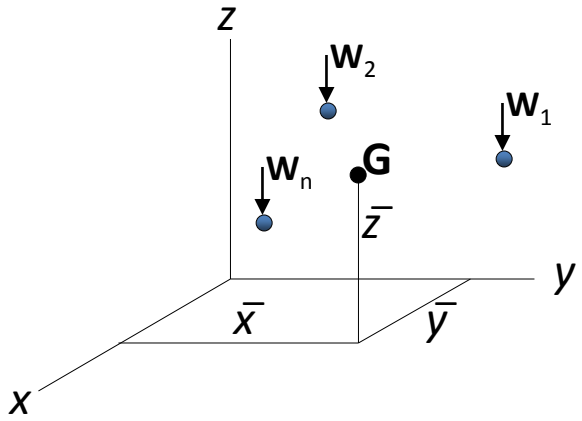
TOPIC 8

Centre of Gravity & Centroid

Lecturers:
Rosli Anang
Dr. Mohd Yunus Ishak
Dr. Tan Cher Siang

Centre of Gravity & Centre of Mass

Centre of Gravity: a point where the resultant weight of the system of particles act.



$\bar{x}, \bar{y}, \bar{z}$ Coordinates of the centre of gravity G of the system of particles

$\tilde{x}, \tilde{y}, \tilde{z}$ Coordinates of each particle in the system of particles

$\sum W$ The resultant sum of the weights of all the particles in the system

Formulas :

Centre of Gravity

$$\bar{x} = \frac{\sum \tilde{x}W}{\sum W} \qquad \bar{y} = \frac{\sum \tilde{y}W}{\sum W} \qquad \bar{z} = \frac{\sum \tilde{z}W}{\sum W}$$

Centre of Mass

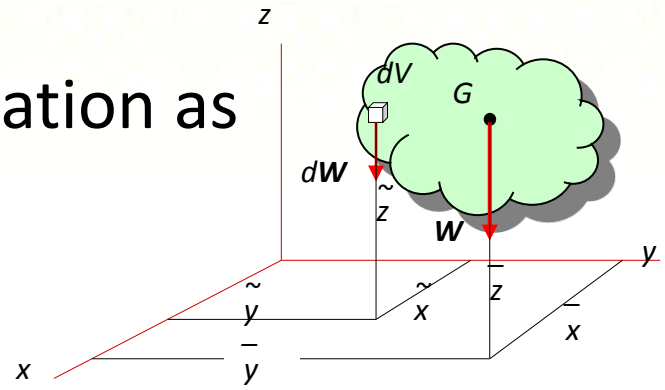
$$\bar{x} = \frac{\sum \tilde{x}m}{\sum m} \qquad \bar{y} = \frac{\sum \tilde{y}m}{\sum m} \qquad \bar{z} = \frac{\sum \tilde{z}m}{\sum m}$$

Rigid Body - Centre of Gravity and Centre of Mass & Centroid

- composed of an infinite number of particles.

Centre of Gravity

Apply the integration instead of summation as similar to the system of particles.



For an arbitrary particle at $(\tilde{x},\tilde{y},\tilde{z})$

$$\bar{x} = \frac{\int \tilde{x}\,dW}{\int W} \qquad \bar{y} = \frac{\int \tilde{y}\,dW}{\int W} \qquad \bar{z} = \frac{\int \tilde{z}\,dW}{\int W}$$

Replacing dW with γdV

$$\bar{x} = \frac{\int_v \tilde{x}\gamma\,dV}{\int_v \gamma\,dV} \qquad \bar{y} = \frac{\int_v \tilde{y}\gamma\,dV}{\int_v \gamma\,dV} \qquad \bar{z} = \frac{\int_v \tilde{z}\gamma\,dV}{\int_v \gamma\,dV}$$

where γ = specific weight

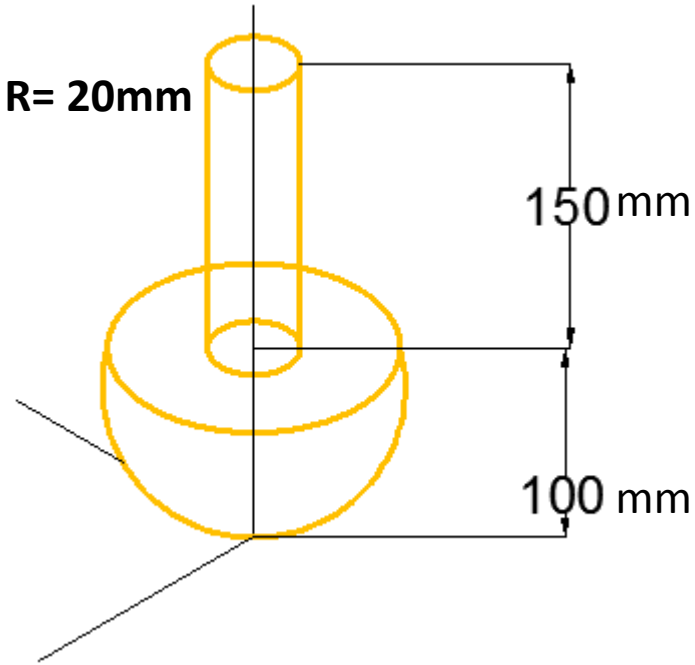
Centre of Mass

Replace γ with ρ in the above equation, since $\gamma = \rho g$.

Example 1

Locate the centre of mass of the composite assembly shown. The cylinder has a density of ρ_C=8000 kg/m^3, and the hemisphere ρ_H=5000 kg/m^3.

[Answer : $\bar{x} = \bar{y} = 0$, $\bar{z}$= 0.0766 m]



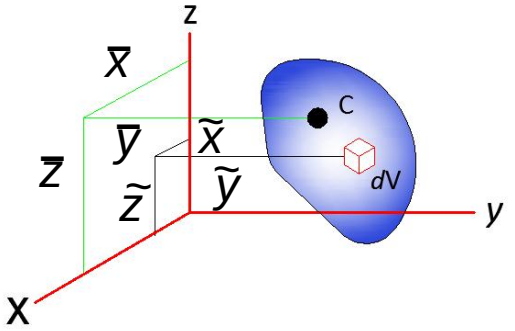
Centroid

Definition: a point which defines the **geometric centre** of an object.

Equations: Similar to those of the centre of gravity/mass if the material is homogeneous where the specific weight / density is constant.

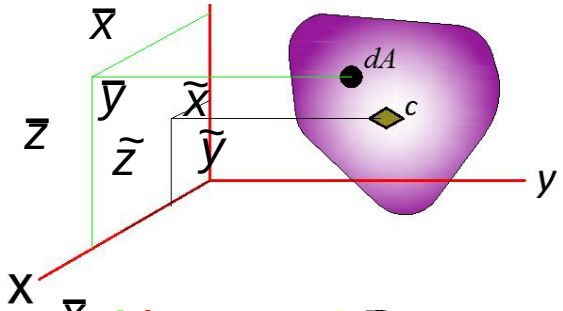
Volume

$$\bar{x} = \frac{\int_V \tilde{x}\,dV}{\int_V dV} \qquad \bar{y} = \frac{\int_V \tilde{y}\,dV}{\int_V dV} \qquad \bar{z} = \frac{\int_V \tilde{z}\,dV}{\int_V dV}$$



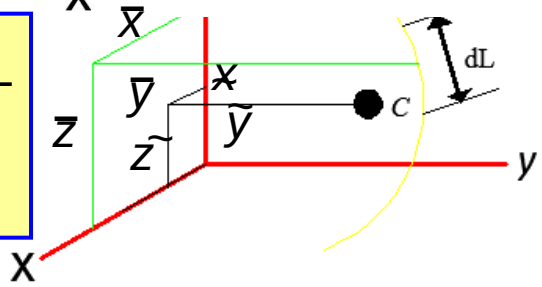
Area

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} \qquad \bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} \qquad \bar{z} = \frac{\int_A \tilde{z}\,dA}{\int_A dA}$$



Line

$$\bar{x} = \frac{\int_L \tilde{x}\,dL}{\int_L dL} \qquad \bar{y} = \frac{\int_L \tilde{y}\,dL}{\int_L dL} \qquad \bar{z} = \frac{\int_L \tilde{z}\,dL}{\int_L dL}$$



Composite Bodies

- A composite body is made up of a series of connected simpler shaped bodies
- Each of the composite part is treated as a particle
- Useful when weight and the location of the centre of gravity of each of these parts is given to avoid long process of integration.

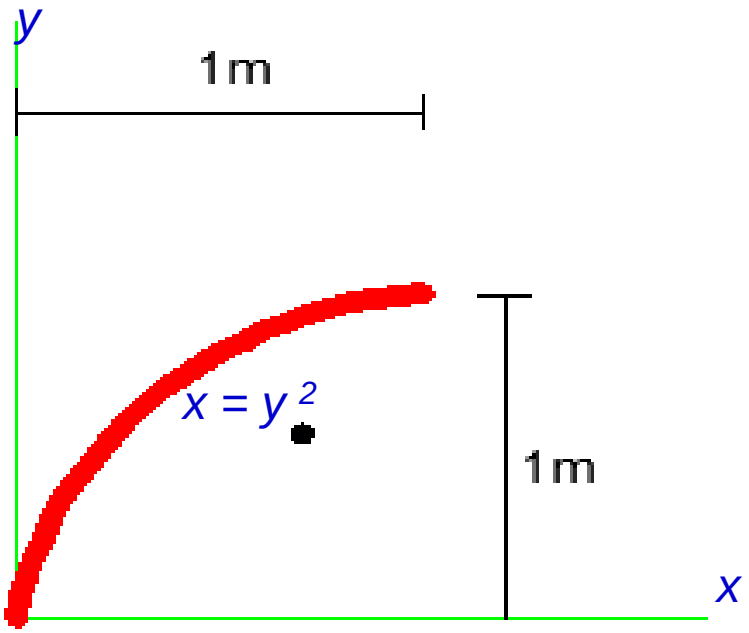
Formula :

$$\bar{x} = \frac{\sum \tilde{x}W}{\sum W} \qquad \bar{y} = \frac{\sum \tilde{y}W}{\sum W} \qquad \bar{z} = \frac{\sum \tilde{z}W}{\sum W}$$

$\bar{x}, \bar{y}, \bar{z}$	Coordinates of the centre of gravity G of the system of the composite body
$\tilde{x}, \tilde{y}, \tilde{z}$	Coordinates of each particle in the system of each of the composite part of the body
$\sum W$	The resultant sum of the weights of all the composite parts in the body/the total weight of the body

Example 2

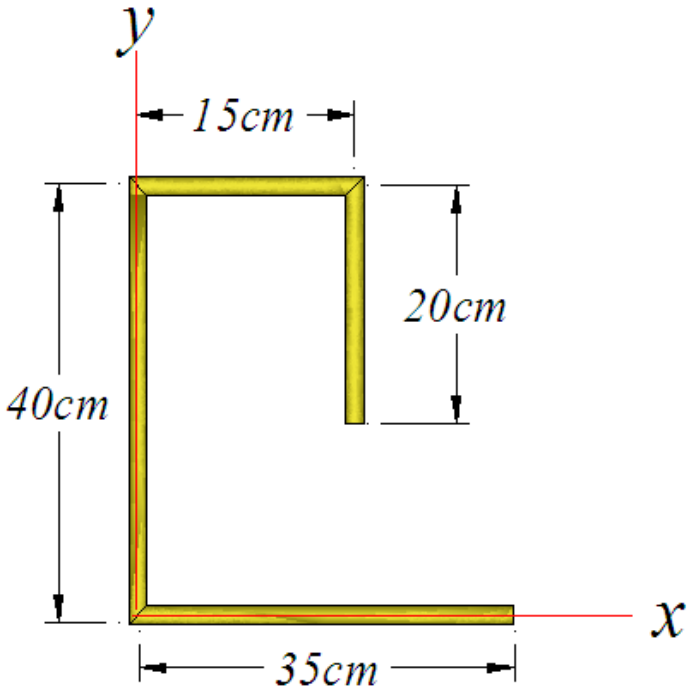
Locate the centroid of the rod bent into a parabolic arc as shown.



[Answer : y =0.574m , x =0.410m]

Example 3

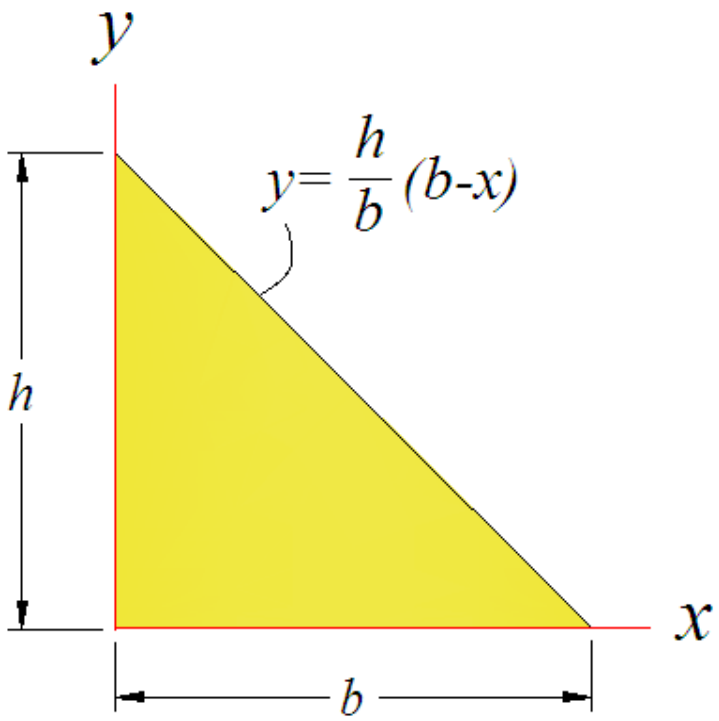
Determine the location of the centroid of rod.



[Answer : $\bar{X}$ = 9.32cm , $\bar{y}$ = 18.18 cm]

Example 4

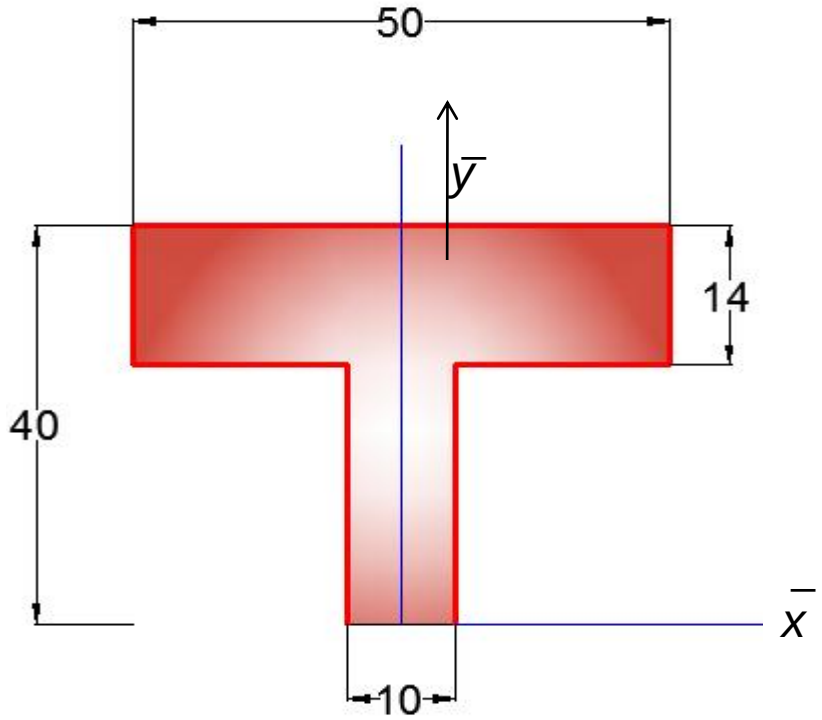
Determine the distance $\tilde{y}$, the distance from the centroid to the x axis. *[Answer :* $\tilde{y} = \frac{h}{3}$ *]*



Example 5

Determine the location of the centroid of the T-section beam. Dimensions in cm.

[Answer : $\bar{y}$= 27.58 cm , $\bar{x}$ = 0cm]



Example 6

Compute the centroid of the plate as shown below with all the dimensions in m.

[Answer : $\bar{Y}$ = 1.76 m, $\bar{X}$= 1.32 m]