SKAA 1213 - Engineering Mechanics

TOPIC 9

FORCE AND ACCELERATION

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Outline

- *Introduction*
- *Equation of Motion*
- *Normal and Tangential Coordinates*
- *Problems*

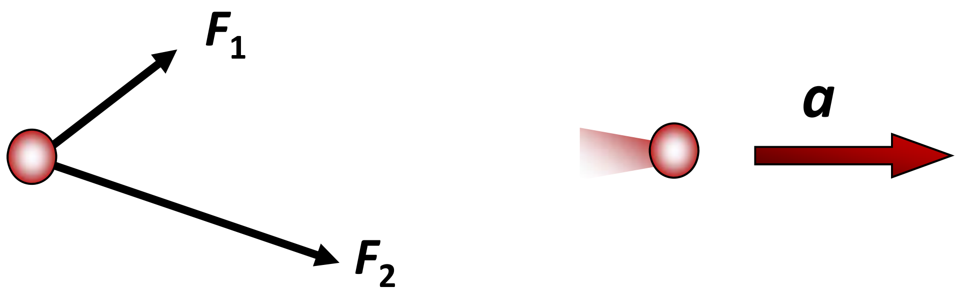
Introduction

- What happen to a particle if unbalance forces acting on it?

- Newton's Second Law of Motion
  – A particle acted upon by an *unbalanced force (F)* will experience an acceleration (*a*) that has the same direction as the force and the magnitude is directly proportional to the force.

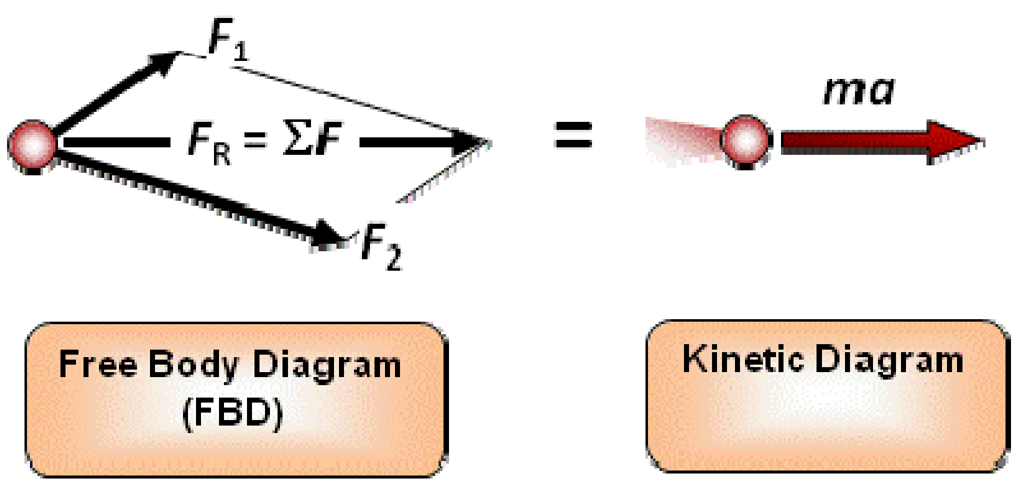
Equation of Motion

- Equation of motion:
  - Consider *a* particle with the mass (m) subjected to two forces, F_1 and F_2
  - From free body diagram, the resultant of these forces produces the vector $m\mathbf{a}$



Equation of Motion

– Represented graphically on the kinetic diagram
– If $F_R = \Sigma F = 0$, acceleration is zero. Such a condition is called static equilibrium, Newton's First Law of Motion that we learned in Static.



Equation of Motion

- Therefore, equation of motion is written as:

$$\sum F = ma$$

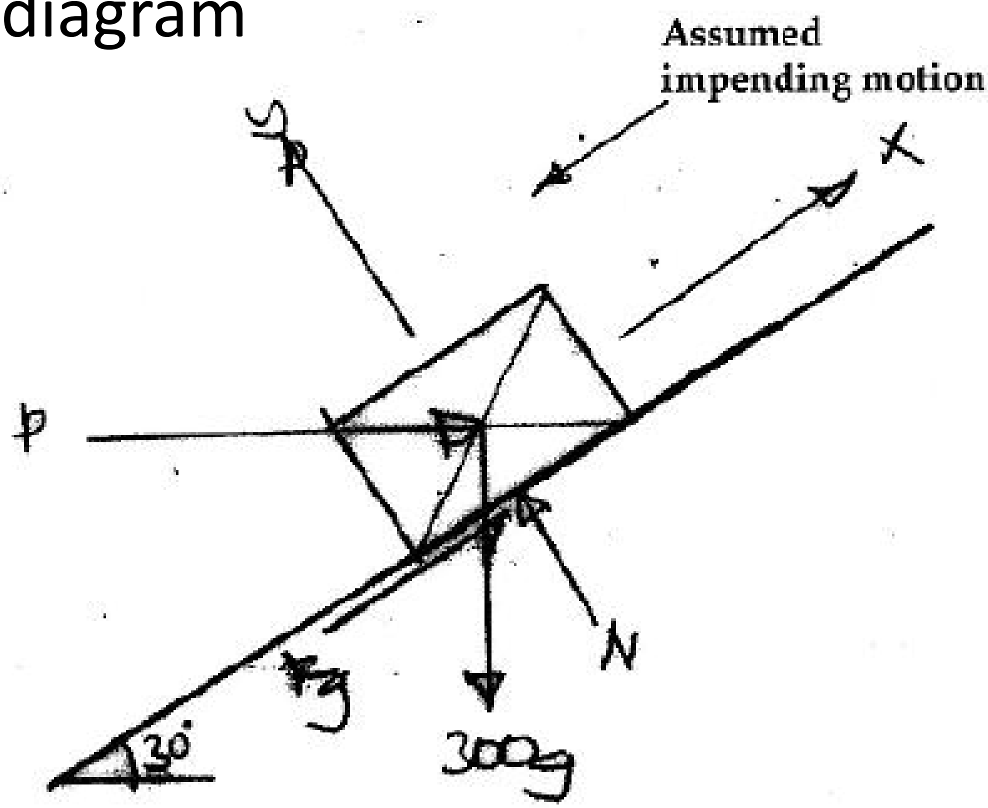
- Definition:
  - *The **sum of the external forces** acting on the system of particles is equal to the **total mass** of the particles times the **acceleration** of its center of mass G*

Equation of Motion

- A horizontal force *P* acts on a 300 kg mass placed on a rough inclined plane as shown. The inclined plane has μ_k =1/4 and μ_s = 5/16. Determine the value of *P* required:
  a) to stop the mass from sliding down the inclined plane
  b) to move the mass <u>up</u> the inclined plane a distance of 5 m in 1 sec
  c) to move the mass <u>down</u> the inclined plane a distance of 5 m in 1 sec.

Equation of Motion

- Solution for (a):
  - Free body diagram



Equation of Motion

- Solution for (a):
  - Equations of Motion

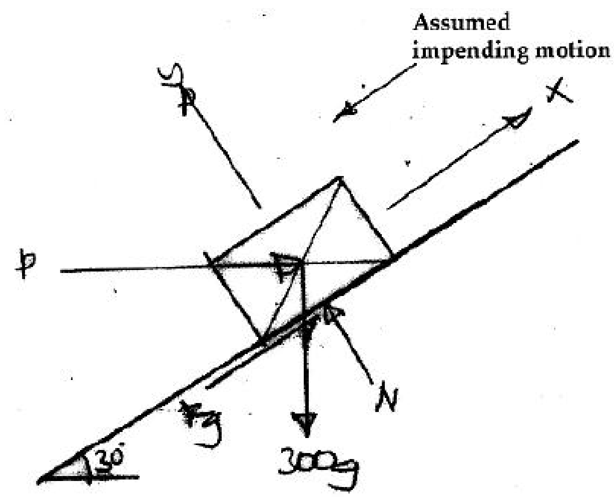
$$\sum F_y = ma_y = 0$$

$$N - P\sin 30^o - 300g\cos 30^o = 0$$

$$\therefore N = 2549 + 0.5P$$

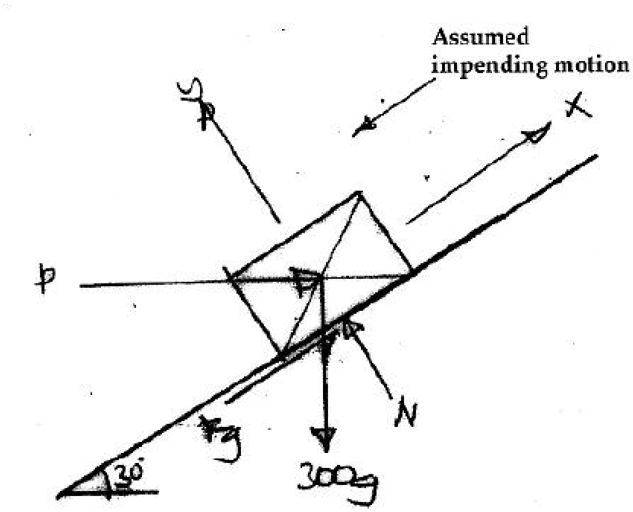
$$\sum F_x = ma_x = 0$$

$$P\cos 30^o + F_s - 300(9.81)\sin 30^o = 0$$



Equation of Motion

- Solution for (a):
  - Equations of Motion
    - Since:



$$F_s = \mu_s N$$

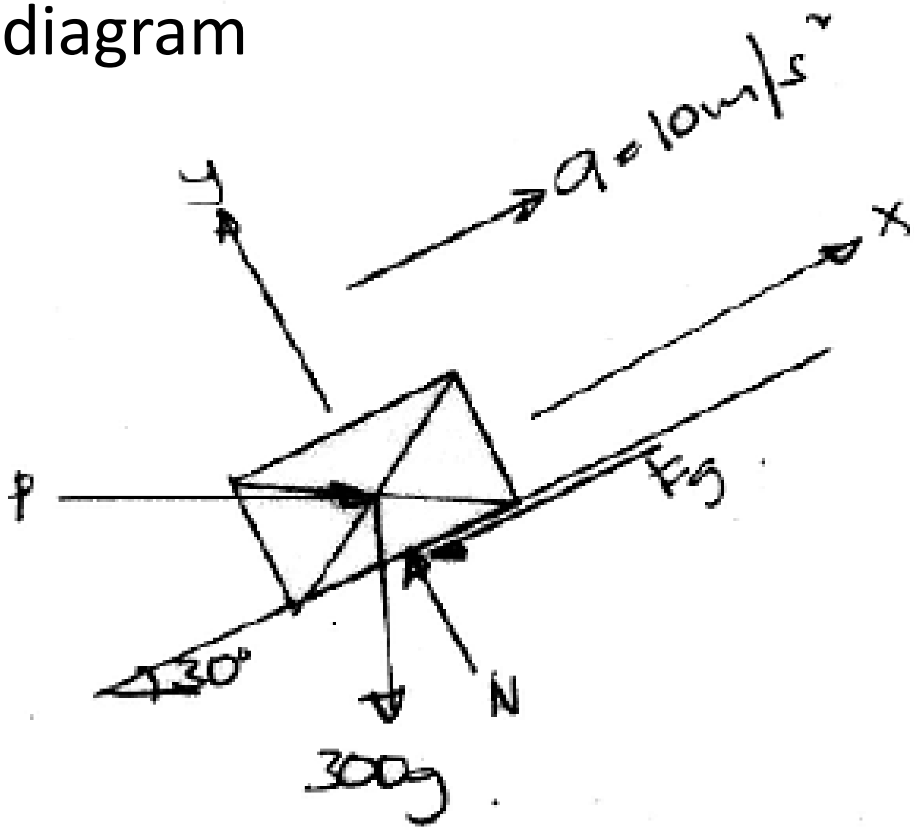
$$F_s = \frac{5}{16}(2549 + 0.5P)$$

$$\sum F_x = P\cos 30^o + \frac{5}{16}(2549 + 0.5P) - 1471.5 = 0$$

$$\therefore P = 661N$$

Equation of Motion

- Solution for (b):
  - Free body diagram



Equation of Motion

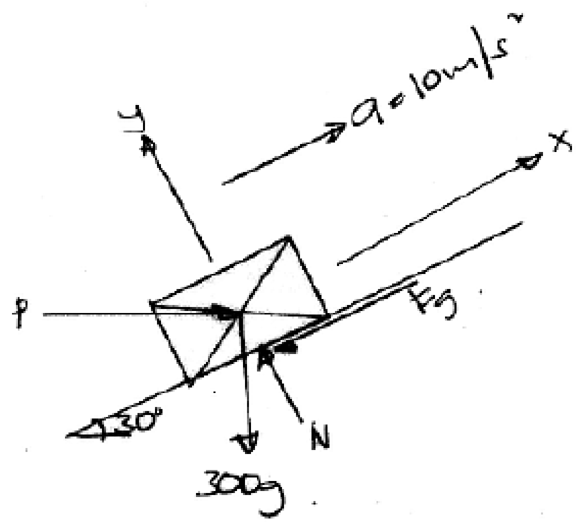
- Solution for (b):
  - *P* to move the mass up the inclined plane a distance of 5 m in 1 sec. (→ *a* = constant)
  - From Kinematics:

$$s = 5m; t = 1s; \quad u = 0$$

$$s = ut + \tfrac{1}{2}at^2$$

$$5 = 0 + \tfrac{1}{2}a(1)^2$$

$$\therefore a = 10ms^{-2}$$

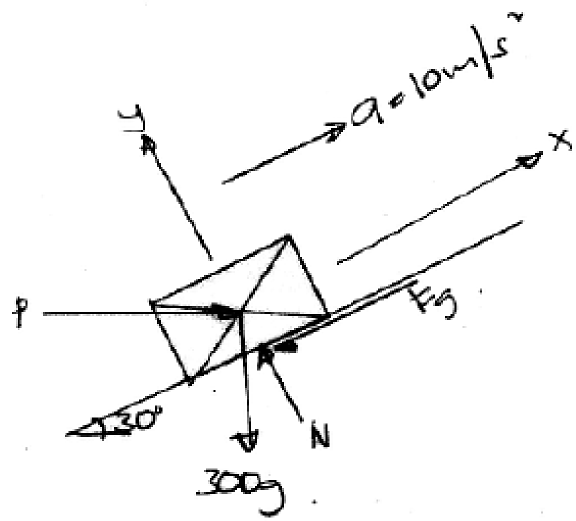


Equation of Motion

- Solution for (b):
  - Equations of Motion

$$\sum F_x = ma_x :$$

$$P\cos 30^o - F_k - 300g\sin 30^o = 300(10)$$



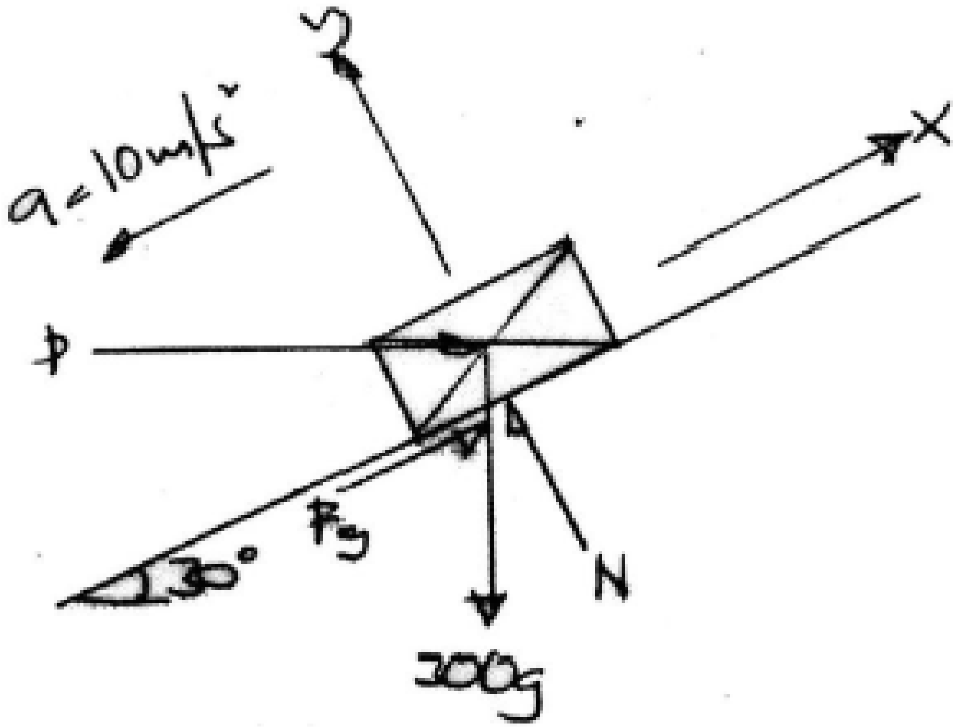
$$F_k = \mu_k N = \frac{1}{4}\left(2549 + 0.5P\right)$$

$$P\cos 30^o - \frac{1}{4}\left(2549 + 0.5P\right) - 300g\sin 30^o = 3000$$

$$P = 6894N$$

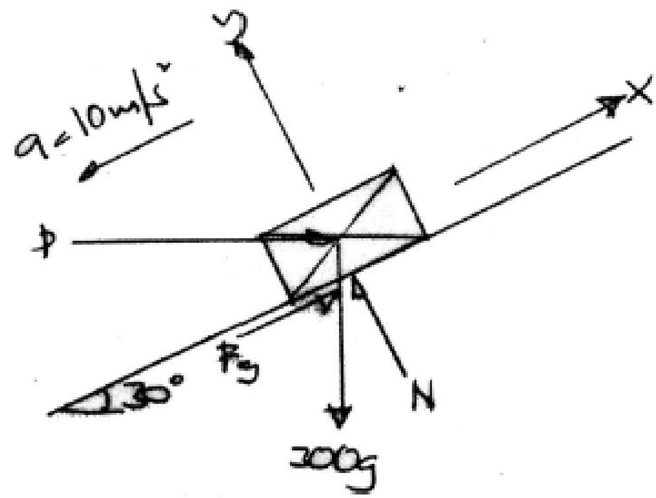
Equation of Motion

- Solution for (c):
  - Free body diagram



Equation of Motion

- Solution for (c):
  - Equation of Motion:



$$N = 2549 + 0.5P$$

$$\sum F_x = ma_x :$$

$$P\cos 30^o + F_k - 300g\sin 30^o = 300(-10)$$

Equation of Motion

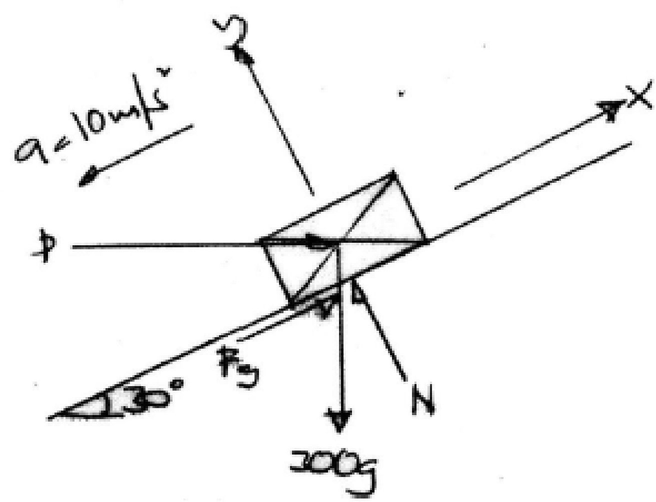
- Solution for (c):
  - Equations of Motion

$$F_k = \mu_k N = \frac{1}{4}(2549 + 0.5P)$$

$$\sum F_x = ma_x :$$

$$P\cos 30^o + \frac{1}{4}(2549 + 0.5P) - 300g\sin 30^o = -3000$$

$$\therefore P = -2185.2N = 2185.2N(\leftarrow)$$



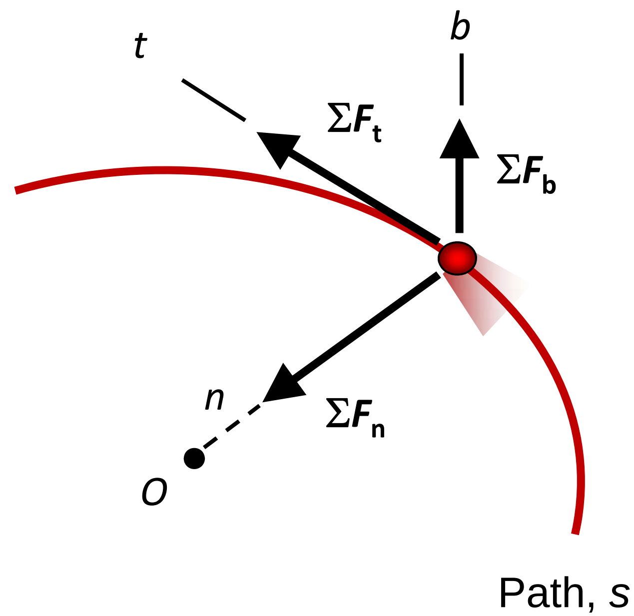
Normal and Tangential Coordinates

- Equation of motion for the particle may be written in the tangential, normal and bi-normal directions.
- There is no motion in the bi-normal direction since the particle is constrained to move along the path.

$$\sum F_t = ma_t$$

$$\sum F_n = ma_n$$

$$\sum F_b = 0$$



Normal and Tangential Coordinates

- $a_t = dv/dt$
  - represents the time rate of change in the magnitude of velocity
- $a_n = v^2/r$
  - represents the time rate of change in the velocity's direction

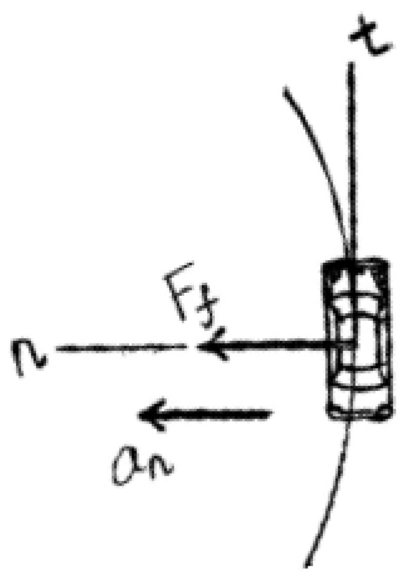
- For path $y = f(x)$: $$r = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{d^2y/dx^2}$$

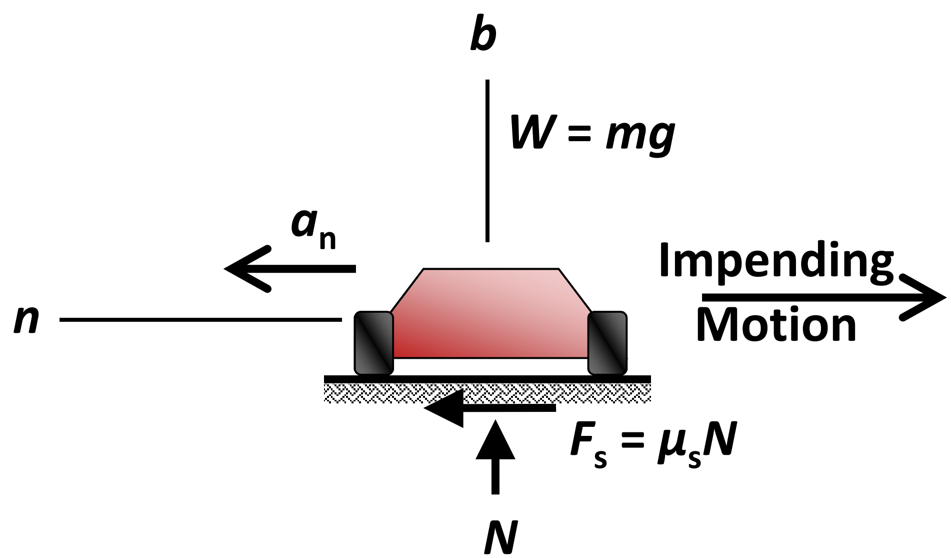
Normal and Tangential Coordinates

- A car having a mass of 1500 kg, travels horizontally along a track which is circular and has a radius of 80 m. If the coefficient of static friction (μ_s) between the tires and the road surface is 0.25, determine:
  a) the maximum speed of the car without causing it to slide when it travels on the flat curve.
  b) the maximum speed of the car without causing it to slide up when it travels on the curve if the curve is banked 15°
  c) the minimum speed of the car without causing it to slide down when it travels on the curve if the curve is banked 15°.
  d) For part a) determine the total force acting on the car if the car was increasing its speed at a rate of $3 m/s^2$

Normal and Tangential Coordinates

- Solution for (a)
  - FBD and Kinetic Diagram:





Normal and Tangential Coordinates

- Solution for (a)
  - Equation of Motion:

$$+\uparrow \sum F_b : \qquad N - mg = N - 1500(9.81) = 0$$

$$N = 14715N$$

$$+\leftarrow \sum F_n : \qquad F_g = ma_n$$

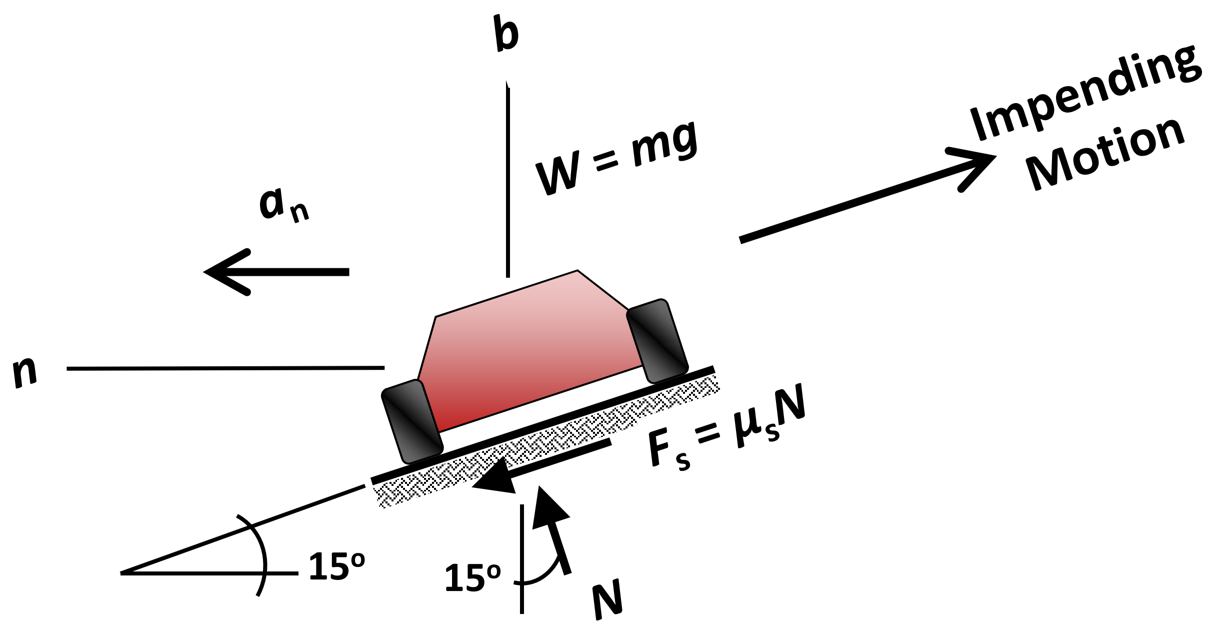
$$\mu_s N = m\frac{v^2}{R}$$

$$(0.25)(14715) = 1500\frac{v^2}{80}$$

$$v = 14.01m/s$$

Normal and Tangential Coordinates

- Solution for (b)
  - FBD:



Normal and Tangential Coordinates

- Solution for (b)
  - Equation of Motion:

$$+\uparrow \sum F_b = 0: \quad N\cos 15° - F_s \sin 15° - mg = 0$$

$$N\cos 15° - 0.25N\sin 15° - 1500(9.81) = 0$$

$$N = 16328N$$

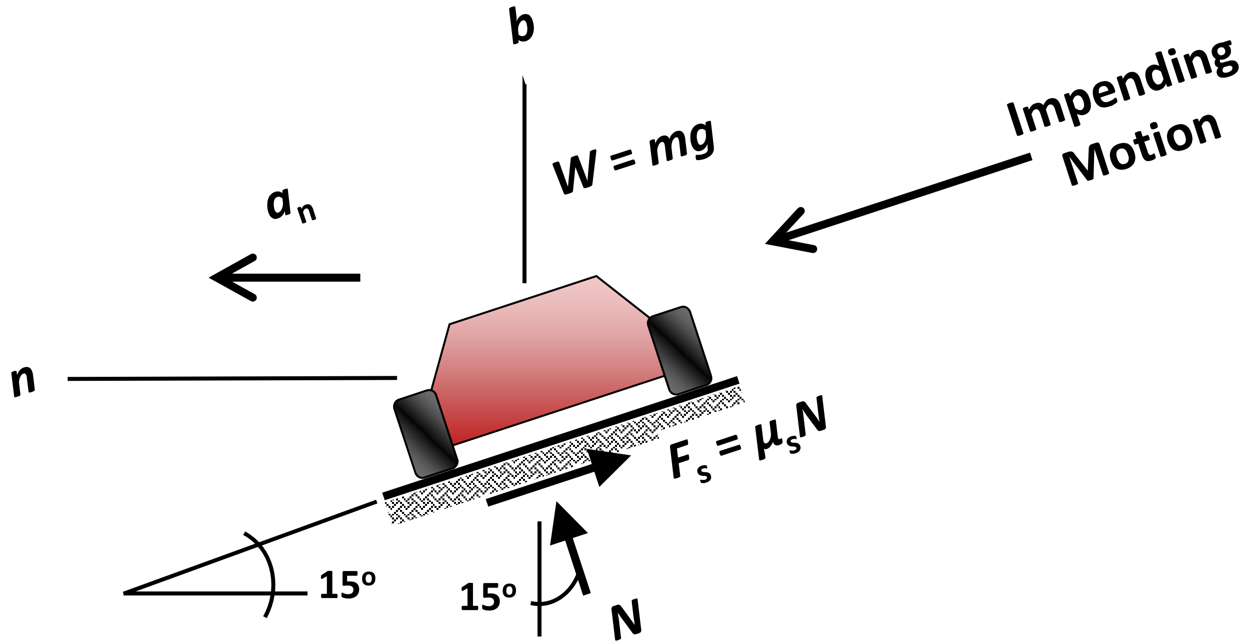
$$+\leftarrow \sum F_n = ma_n: \quad N\sin 15° + F_s \cos 15° = ma_n$$

$$16328\sin 15° + 0.25(16328)\cos 15° = 1500\frac{v^2}{80}$$

$$v = 20.87m/s$$

Normal and Tangential Coordinates

- Solution for (c)
  - FBD:



Normal and Tangential Coordinates

- Solution for (c)
  - Equation of motion:

$$+\uparrow \sum F_b = 0: \quad N\cos 15° + F_s \sin 15° - mg = 0$$

$$N\cos 15° + 0.25N \sin 15° - 1500(9.81) = 0$$

$$N = 14278N$$

$$+\leftarrow \sum F_n = ma_n: \quad N\sin 15° - F_s \cos 15° = ma_n$$

$$14278\sin 15° - 0.25(14278)\cos 15° = 1500\frac{v^2}{80}$$

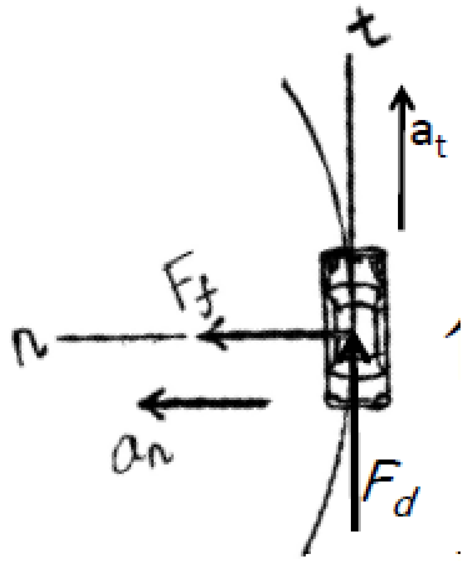
$$v = 13.20m/s$$

Normal and Tangential Coordinates

- Solution for (d):

$$+\uparrow\sum F_t: \qquad F_d = ma_t = 1500(3) = 4500N$$

$$F_s = \mu_s N = 0.25(14715) = 3678.75N$$

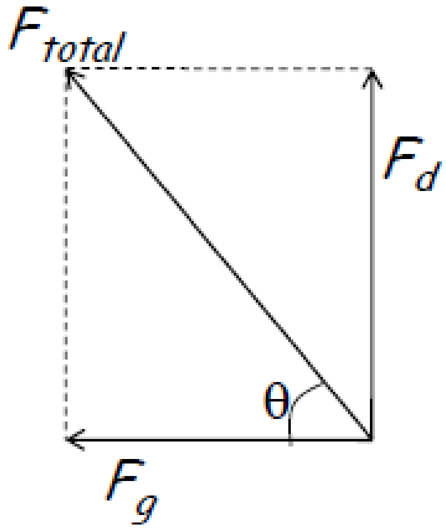


Normal and Tangential Coordinates

- Solution for (d):

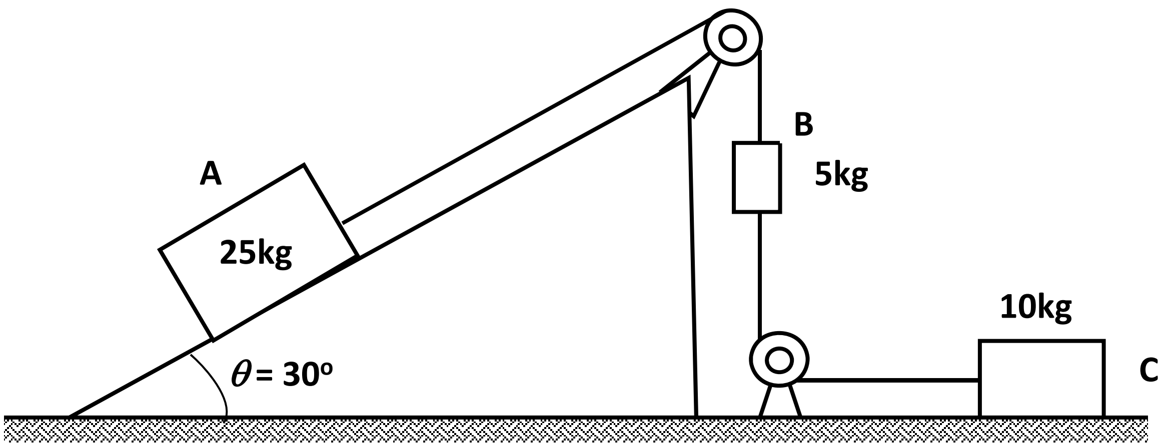
$$F_{total} = \sqrt{F_d^2 + F_s^2} = \sqrt{4500^2 + 3678.75^2} = 5812.33N$$

$$\theta = \tan^{-1}\left(\frac{4500}{3678.75}\right) = 50.73°$$



Problem P1

- Figure P1 shows a box A of mass = 25 kg is moving downward at a smooth inclined plane with slope θ = 30º. The box A will move the 5-kg cylinder B and the 10-kg block C. The coefficient of kinetic friction between the horizontal surface and block C is μ_k = 0.2.

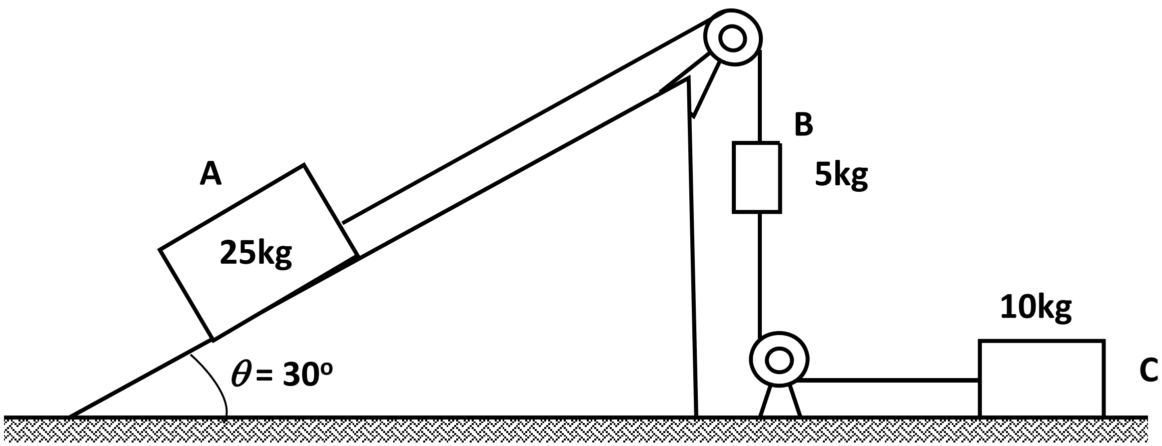


Problem P1 (cont.)

a) Draw the free body diagram for the Box A, cylinder B and block C.

b) Determine the acceleration a caused by the system.

c) Calculate the tension force at the rope between AB (T_{AB}) and BC (T_{BC}).



Problem P2

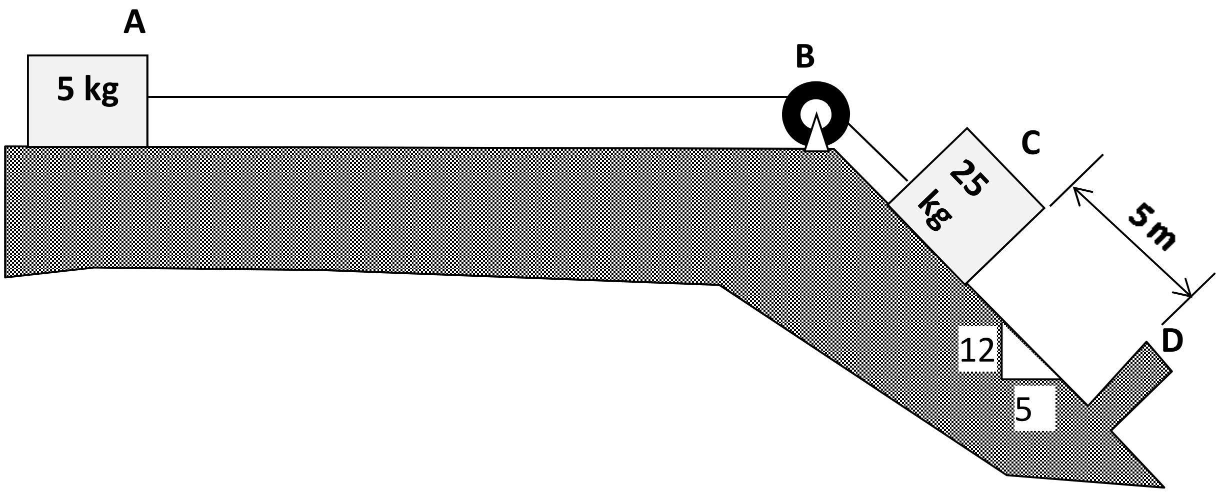
- a 5 kg mass placed on a rough surface at position A and is connected to a 25 kg mass located on an inclined rough surface at C. Both masses are connected by an inelastic weightless rope passing through a frictionless pulley at B. The 25 kg mass is initially held in position at C and is then released to slide down the inclined surface. The coefficient of static, μ_s and kinetic friction, μ_k between both the masses and the rough surface is 0.25 and 0.20 respectively.

Problem P2 (cont.)

a) Show that both masses will move as soon as the 25 kg mass is released.

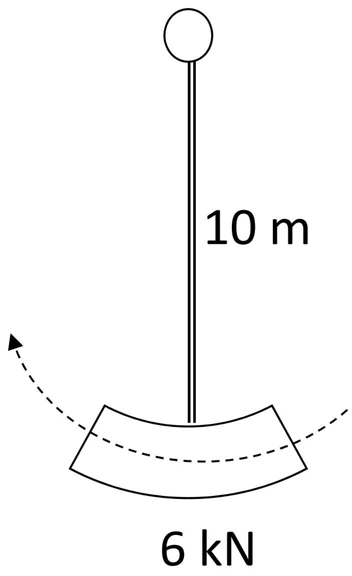
b) Determine the acceleration of both masses.

c) Using Newton's Second Law and kinematics, determine the distance travelled by the 5 kg mass before coming to a stop after the 25 kg mass comes to a stop at D.



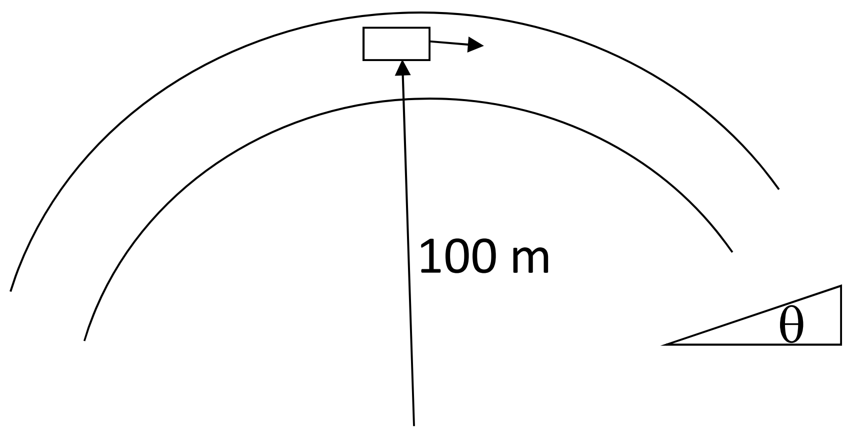
Problem P3

- A 6 kN weight is swung by a 10 m steel arm. If the velocity of the weight at the bottom of the circle is 6 m/s, determine the tension force on the arm at this instance. It is given that the centrifugal acceleration normal to the radius is $a_n = v^2/r$, where r is the radius of the curve.



Problem P4

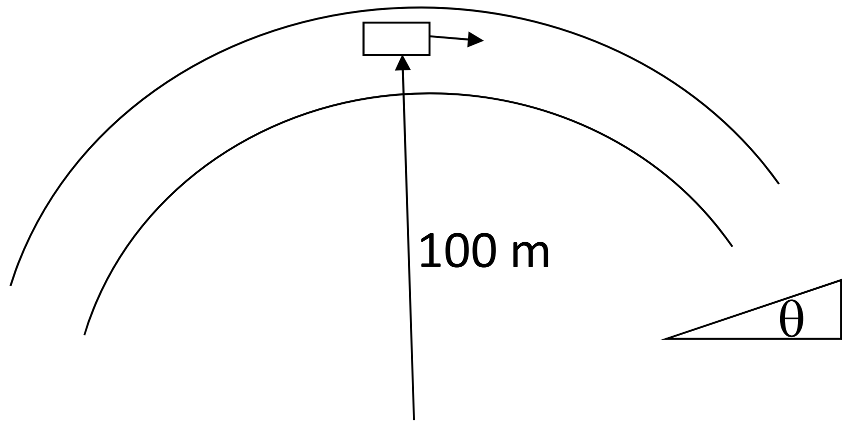
- A 900 kg car going round a corner on a road surface which has the slope θ. The radius of the corner is 100 m.



Problem P4 (cont.)

a) If $\theta = 0$, and if the velocity of the car is 30 m/s, what is the friction force required to be exerted by the surface to avoid the car from sliding.

b) If the surface is frictionless, what is the angle θ so that car can move without sliding?



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