SKAA 1213 - Engineering Mechanics

TOPIC 6

FRICTION

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Lesson 7 Outline

- *Introduction*
- *Equilibrium on a horizontal plane*
- *Equilibrium on an inclined plane*
- *Problems*

Introduction

- Definition:
  - Friction: Force that resists the movement of two contacting surfaces that slide relative to one another.
  - Friction force acts tangent to the surfaces of contact, and opposing direction of the slide motion.
  - Two types of friction – Fluid Friction and Dry Friction (Coulomb Friction)

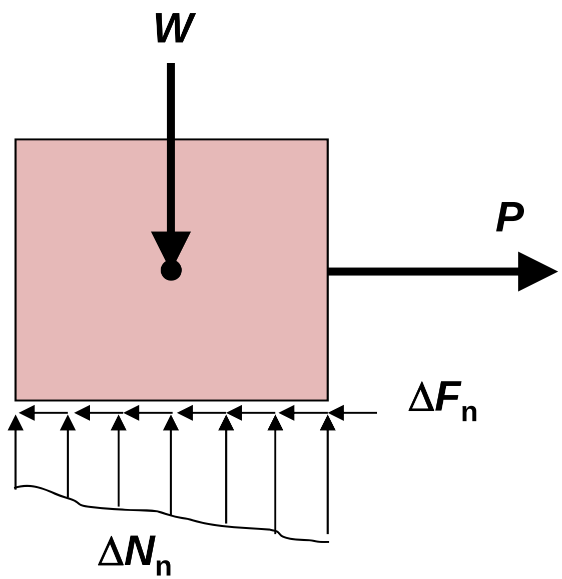
Introduction

- **Fluid friction:**
  - Occurs when the contacting surface are separated by a layer of fluid (liquid or gas).

- **Dry friction (Coulomb friction) :**
  - Exists between contacting surfaces of bodies with the absence of lubricating fluid.

Dry Friction

– Refer to the figure, consider the effects caused by horizontally pulling force (***P***) on a block of uniform weight (***W***) which is resting on a rough horizontal surface.

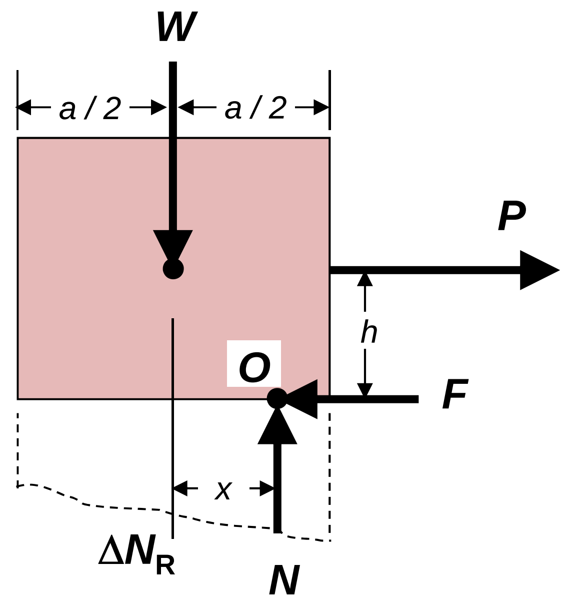


Dry Friction

- Normal force $\Delta \boldsymbol{N}_n$ and frictional force $\Delta \boldsymbol{F}_n$ will act along the contact surface.
- For equilibrium, normal forces ($\Delta \boldsymbol{N}_n$) act upward to balance the block's weight $\boldsymbol{W}$, while frictional forces ($\Delta \boldsymbol{F}_n$) act to the left to prevent force $\boldsymbol{P}$ from moving the block to the right.

Equilibrium on a Horizontal Plane

- Equilibrium
  - Effect of normal and frictional loadings are indicated by their resultant ***N*** and ***F***
  - Distribution of ΔF_n indicates that ***F*** is tangent to the contacting surface, opposite to the direction of ***P***
  - Normal force ***N*** is determined from the distribution of ΔN_n



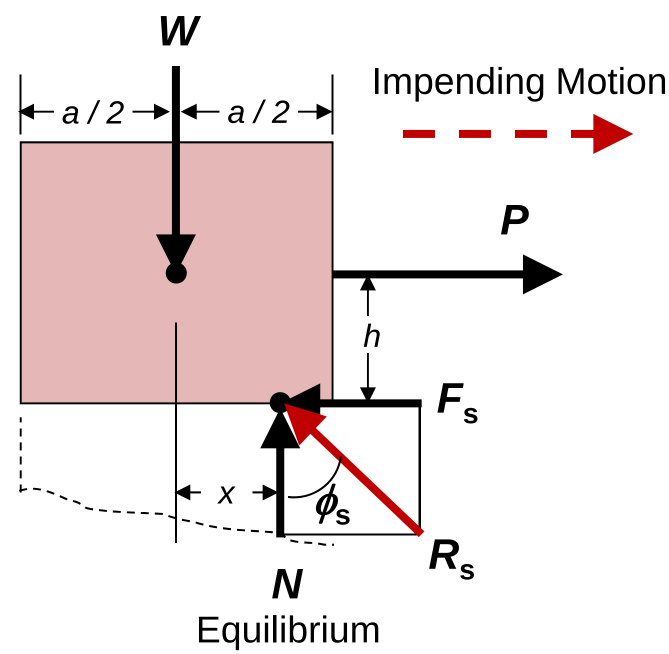
Resultant Normal and Frictional Forces

Equilibrium on a Horizontal Plane

- Equilibrium
  - ***N*** is directed upward to balance ***W***
  - ***N*** acts at a distance ***x*** to the right of the line of action of ***W***
  - This location coincides with the centroid of the loading diagram in order to balance the “tipping effect” caused by ***P***

Equilibrium on a Horizontal Plane

- Impending Motion
  - As ***P*** is slowly increased, ***F*** correspondingly increase until it achieve maximum value called the limiting static frictional force, F_s
  - F_s is directly proportional to the resultant normal force ***N***



$$F_s = \mu_s N$$

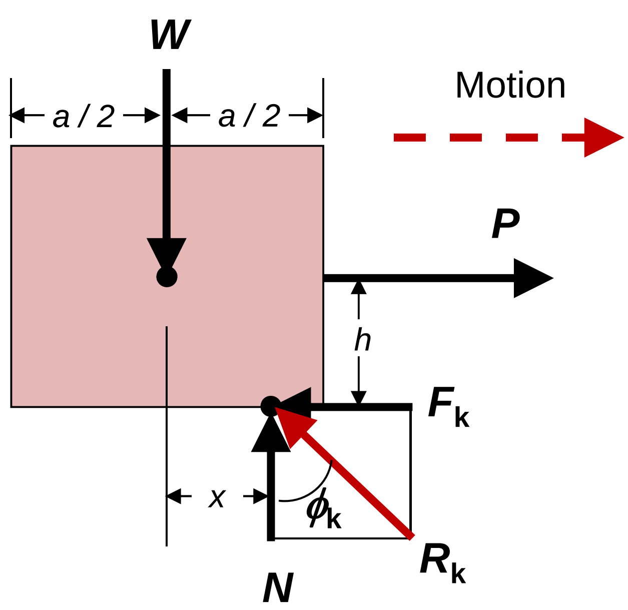
Equilibrium on a Horizontal Plane

- Impending Motion
  - μ_s is known as the coefficient of static friction
  - Angle Φ_s is called the angle of static friction.

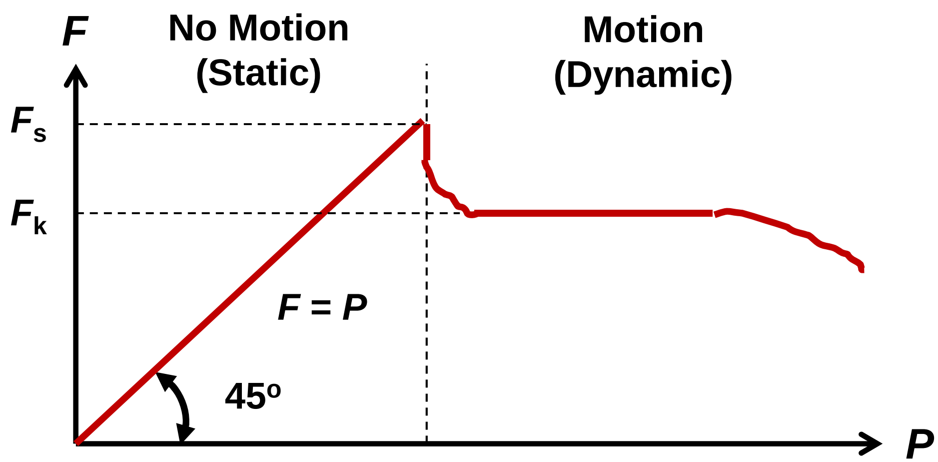
$$\phi_s = \tan^{-1}\left(\frac{F_s}{N}\right) = \tan^{-1}\left(\frac{\mu_s N}{N}\right) = \tan^{-1}\mu_s$$

Equilibrium on a Horizontal Plane

- Motion
  - When P is greater than F_s, the frictional force is slightly smaller value than F_s, called kinetic frictional force, F_k
  - The block will not be held in equilibrium ($P > F_s$) but slide with increasing speed



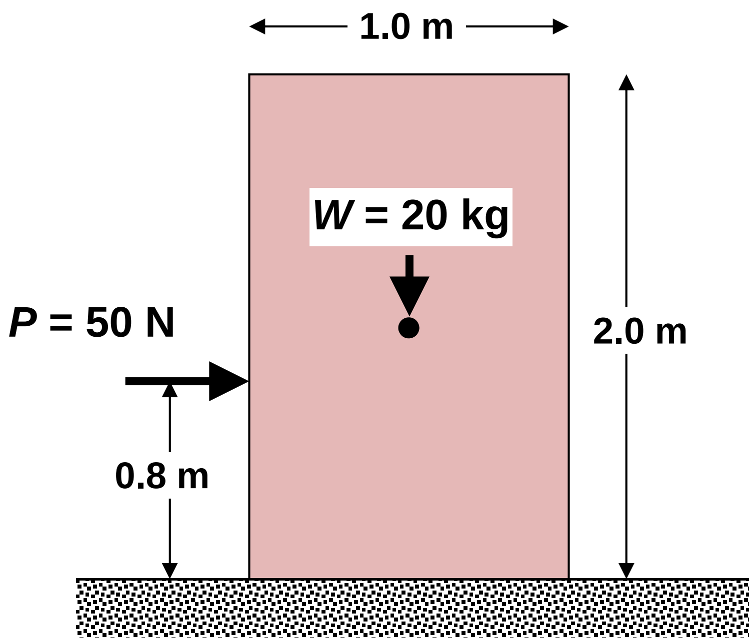
Equilibrium on a Horizontal Plane



- ***F*** - *static frictional force* if equilibrium is maintained
- F_s - *limiting static frictional force* when *F* reaches a maximum value in equilibrium
- F_k - *kinetic frictional force* when sliding occurs at the contacting surface

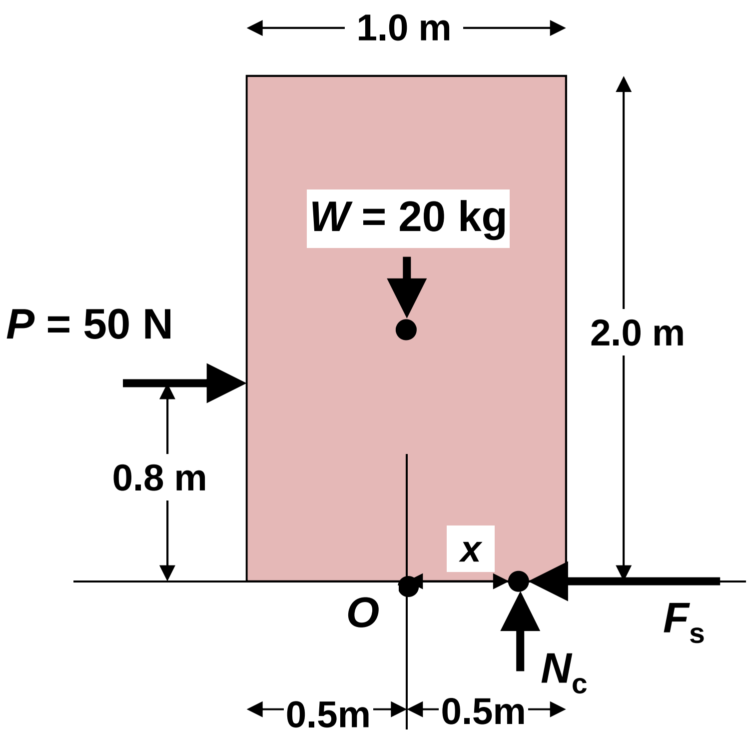
Equilibrium on a Horizontal Plane

- Consider a 20 kg box is subjected to a pushing force (P) of 50 N. The coefficient of static friction (μ_s) is equal to 0.3. Find out if the box would tip or slip, and would it remain in equilibrium in the system.



Equilibrium on a Horizontal Plane

- Free body diagram (FBD):



$$W = 25(9.81) = 245.25N$$

Equilibrium on a Horizontal Plane

- Equilibrium Equation:

$$+\uparrow \sum F_y = 0 \qquad W - N_c = 0$$

$$N_c = 245.25N$$

$$\sum M_O = 0 \qquad N_c(x) - 50(0.8) = 0$$

$$x = \frac{40}{245.25} = 0.16m$$

- Since *x* = 0.16 < 0.5m, no tipping will occur.

Equilibrium on a Horizontal Plane

- Equilibrium Equation:

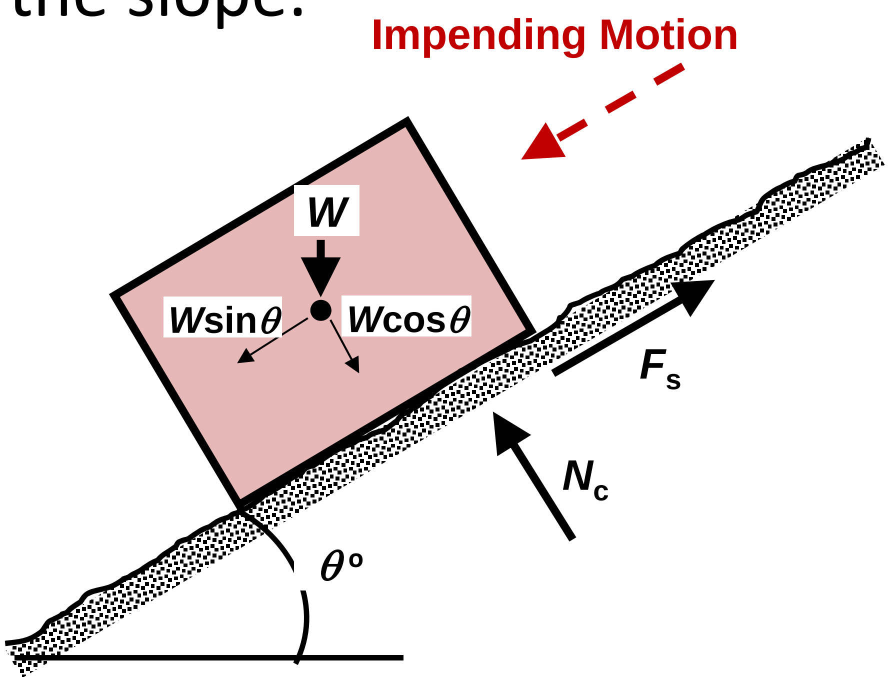
$$F_s = \mu_s N_c = 0.3(245.25) = 73.58N$$

$$F_s > P$$

- Since the maximum static frictional force, F_s = 73.58 N which is higher than the push force, P = 50 N, no slip will occur.
- The box will remain in equilibrium.

Equilibrium on an Inclined Plane

- Consider a box placed on an inclined plane as shown in the figure.
- The self-weight of the box will cause vertical and horizontal forces to the slope.

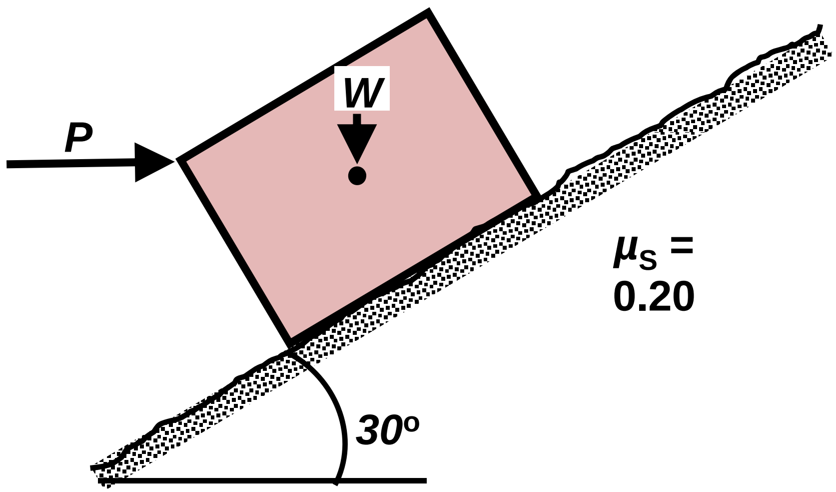


Equilibrium on an Inclined Plane

- The vertical force ($W\cos\theta$) will be balanced by Normal reaction (N_c).
- The horizontal force ($W\sin\theta$) will cause the box to slide down, thus the frictional force (F_s) is opposite to the direction of the impending motion as shown in the figure.
- If $F_s > W\sin\theta$, the box will remain in equilibrium.

Equilibrium on an Inclined Plane

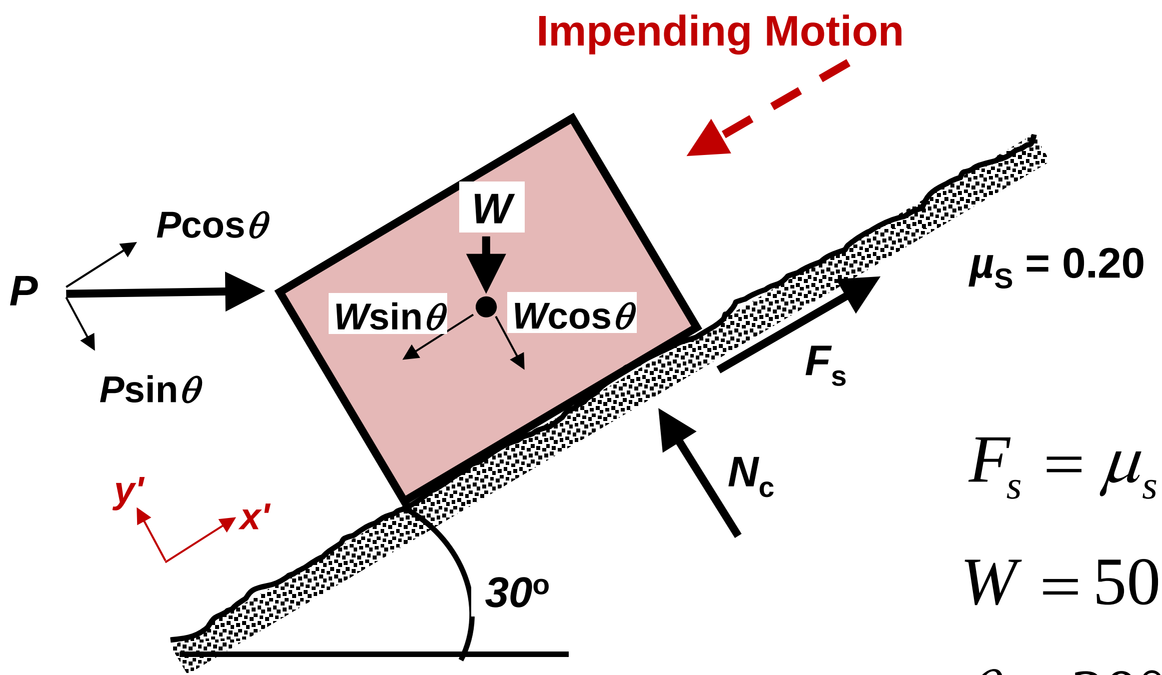
- A box is placed on a 30^o slope and held by hand with the force, *P*. The box has a mass of *W* = 50 kg and the coefficient of static friction between the crate and the ground is μ_s=0.20. Determine:
  a) The minimum force *P* to hold the box in equilibrium.
  b) The maximum force *P* before it can push the box upward.



Equilibrium on an Inclined Plane

a) The minimum force *P* to hold the box in equilibrium.

- Free body diagram (FBD):



$$F_s = \mu_s N_c$$

$$W = 50(9.81) = 490.5N$$

$$\theta = 30^\circ$$

Equilibrium on an Inclined Plane

a) The minimum force *P* to hold the box in equilibrium.

- Equilibrium Equation:
- +↖ $\Sigma F_y = 0$

$$N_c - P\sin 30° - 490.5\cos 30° = 0$$

$$N_c - 0.5P = 424.79 \qquad (1)$$

- + ↗ $\Sigma F_x = 0$

$$P\cos 30° + F_s - 490.5\sin 30° = 0$$

$$0.866P + 0.2N_c = 245.3 \qquad (2)$$

Equilibrium on an Inclined Plane

a) The minimum force *P* to hold the box in equilibrium.

- Equilibrium Equation:
- (1) into (2):

$$0.866P + 0.20(424.79 + 0.5P) = 245.3$$

$$0.866P + 0.1P + 84.96 = 245.3$$

$$P = \frac{245.3 - 84.96}{0.966} = 157.35N$$

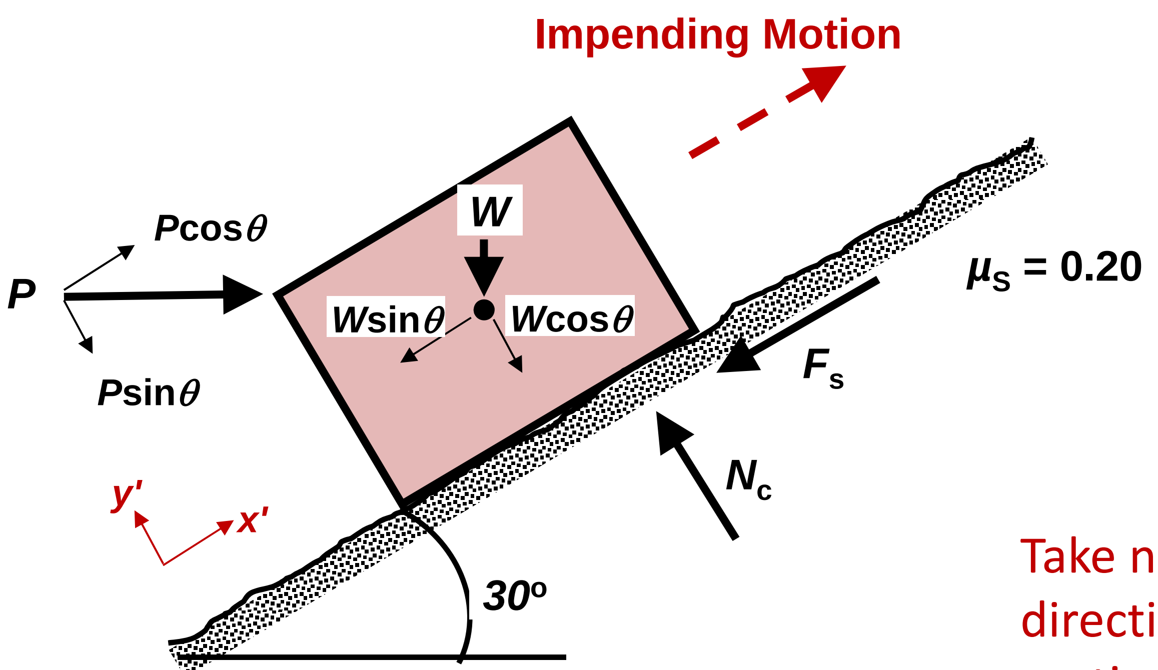
- ∴ Minimum horizontal force to hold the crate:

P = <u>157.35N</u>

Equilibrium on an Inclined Plane

b) The maximum force *P* before it can push the box upward.

- FBD:



Take note the changes of the direction for impending motion and F_s.

Equilibrium on an Inclined Plane

b) The maximum force *P* before it can push the box upward.

- Equilibrium Equation:
- $+\nwarrow \Sigma F_y = 0$

$$N_c - P\sin 30° - 490.5\cos 30° = 0$$

$$N_c - 0.5P = 424.79 \qquad (1)$$

- $+\nearrow \Sigma F_x = 0$

$$P\cos 30° - F_s - 490.5\sin 30° = 0$$

$$0.866P - 0.2N_c = 245.3 \qquad (2)$$

Equilibrium on an Inclined Plane

b) The maximum force *P* before it can push the box upward.

- Equilibrium Equation:
- (1) into (2):

$$0.866P - 0.20\left(424.79 + 0.5P\right) = 245.3$$

$$0.866P - 0.1P - 84.96 = 245.3$$

$$P = \frac{245.3 + 84.96}{0.766} = 431.15N$$

- ∴ Maximum horizontal force before pushing the box upward:

P = <u>431.15N</u>

Problem 1

- A crate of mass *m* kg is to be moved by applying a force *P* as shown in Figure P1. The crate is on a rough surface while the force *P* is applied incrementally starting from zero and is increased gradually until the crate is in motion.

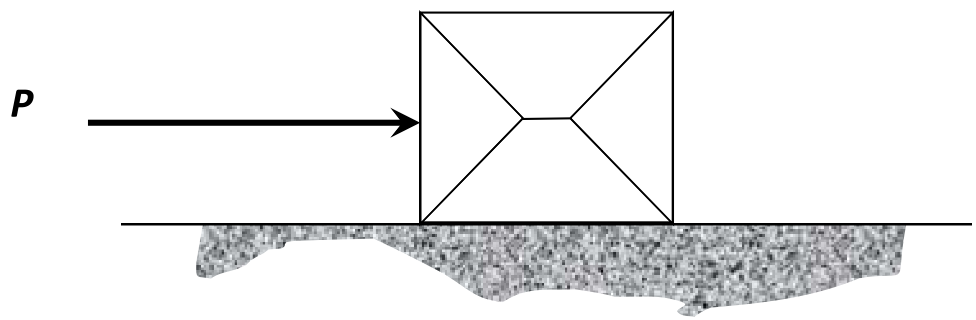


FIGURE P1

Problem 1 (cont.)

a) Draw a free body diagram (FBD) of the crate showing all forces acting on the crate.

b) Based on the properties of dry friction, plot a graph of the applied force, *P* versus frictional force, *F* between the crate and the rough surface from the start of application of force *P* until the crate is in motion.

c) Indicate the value of static frictional force, F_s and the kinetic friction force, F_k on the graph as in part b).

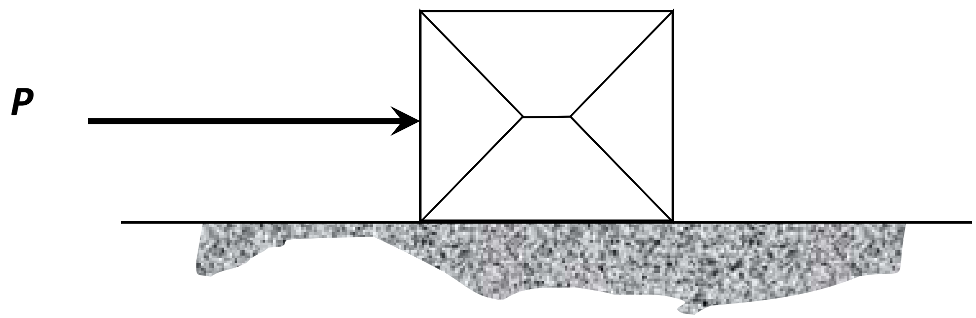


FIGURE P1

Problem 2

- A four meter long of mass 10 kg ladder AB is resting against a wall as shown in Figure P2. The coefficient of static friction (μ_s) between the ladder with the floor and the wall is 0.25. Determine the minimum value of θ so that the ladder remains in equilibrium.

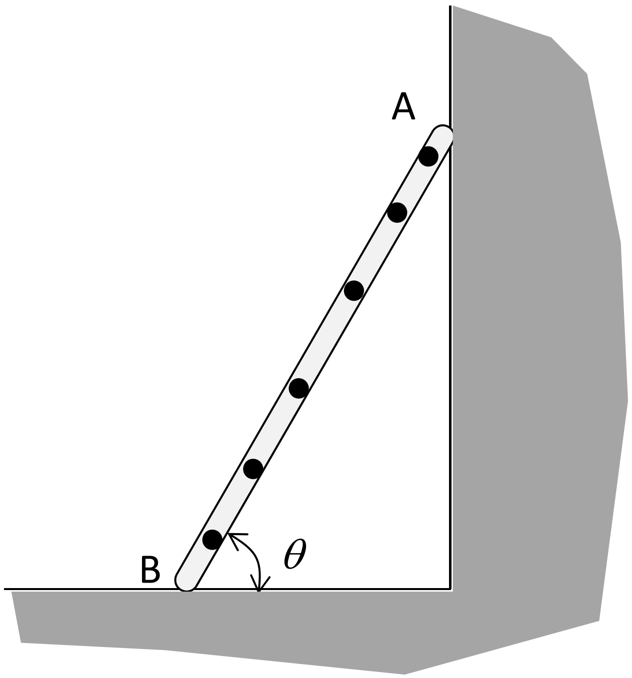


FIGURE P2

Problem 3

- The wooden box has a weight of 800 N and rest on the floor with coefficient of static friction, μ_s = 0.20. The mass of the man is 55 kg and the coefficient of static friction between the floor and his shoes (both legs) is μ_s = 0.50.
  a) If the man pushes horizontally on the wooden box, determine if he can move the box before he slipped.
  b) If he can move the box, does the box slip or tip?

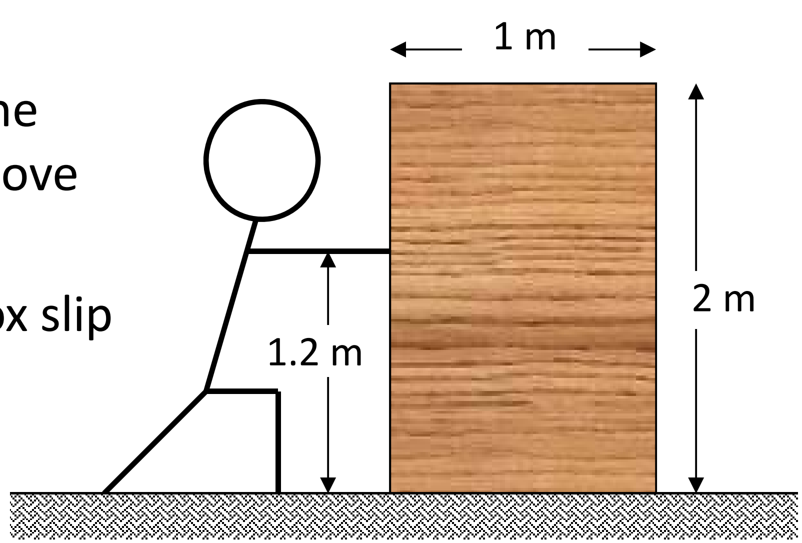


FIGURE P3

Problem 4

- Figure P4 shows two boxes on a sloped floor, weighing 50 kg and 80 kg respectively, and being pulled by a force *P*. The coefficient of static friction between the floor and the boxes is $\mu_s = 0.15$.
  - Determine the minimum force *P* to prevent the boxes from slipping down.
  - Find the maximum force *P* before it starts to pull the boxes upward.

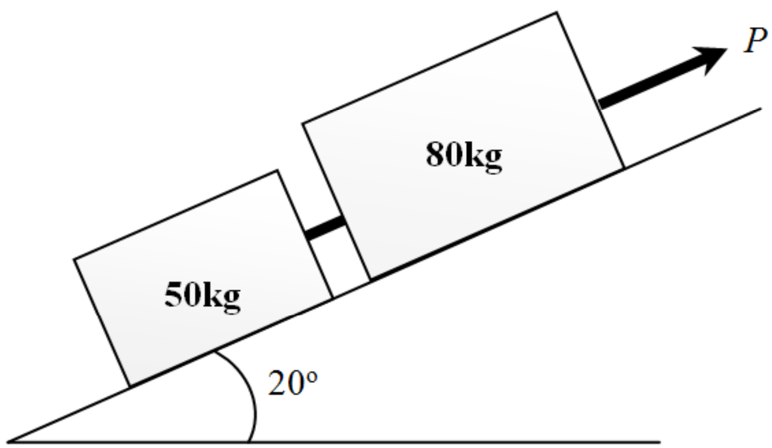


FIGURE P4

Problem 5

- A 200 kg block is connected to a weight *W* through a frictionless pulley as shown in Figure P5. The coefficient of static friction between block and the ramp is $\mu_s = 0.15$. The slope of the ramp is 30°.
  a) Determine the minimum mass of *W* (in kg) for the block not to slide down the ramp.
  b) If *W* = 100 kg, calculate the friction force between the block and the ramp.

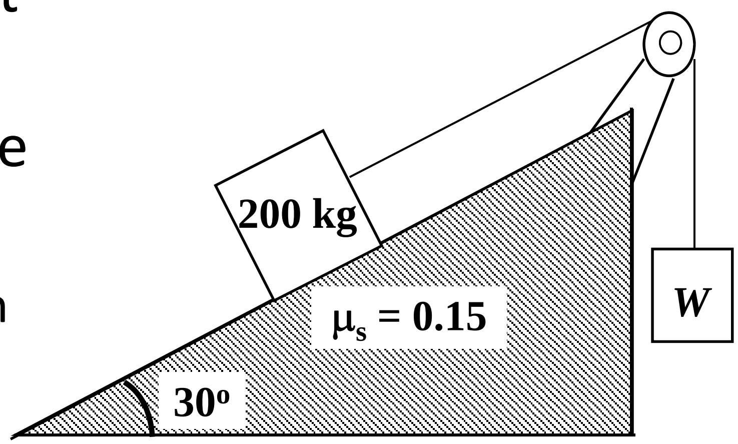


FIGURE P5

The End of Topic 7