SKAA 1213 - Engineering Mechanics

TOPIC 5

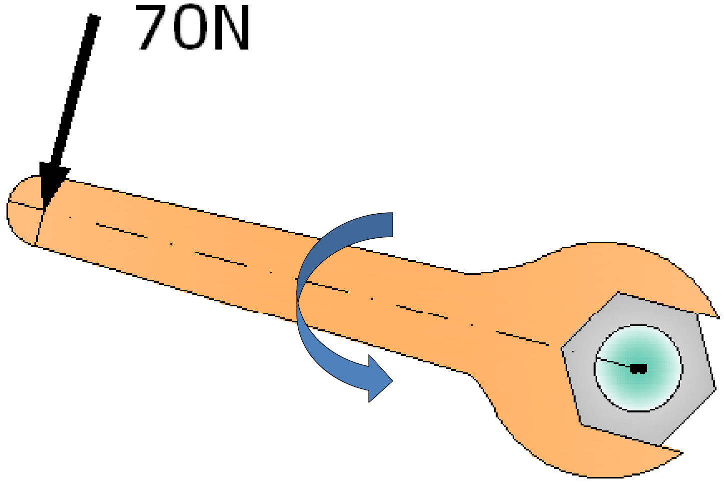
Moment and Couple

Lecturers:
Rosli Anang
Dr. Mohd Yunus Ishak
Dr. Tan Cher Siang

Moment of a Force

- Moment of a force about a point/axis the tendency of the force to cause the body to rotate about the point/axis.

- Moment is a **vector quantity**

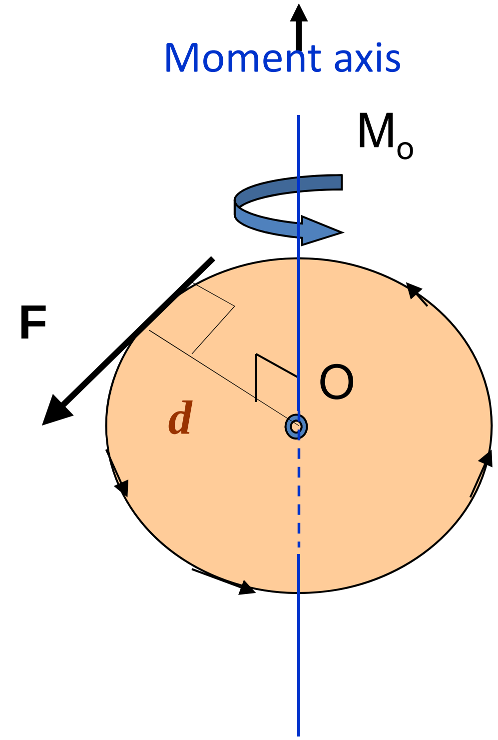


Moment of a Force

1. Scalar Formulation of Moment

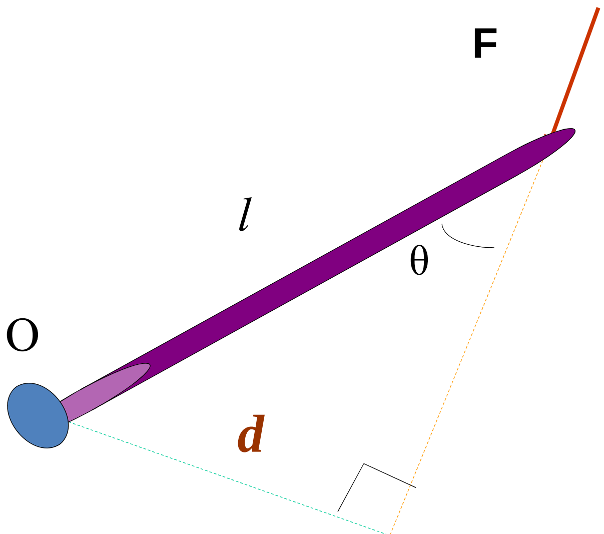
$M_o = Fd$

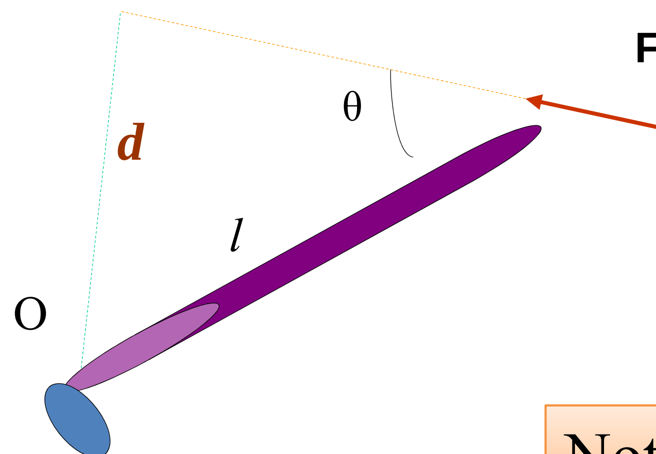
Where d is the perpendicular distance from the axis of point O to the action of the force F).



Direction of force : specified by using the ***right hand rule*.**

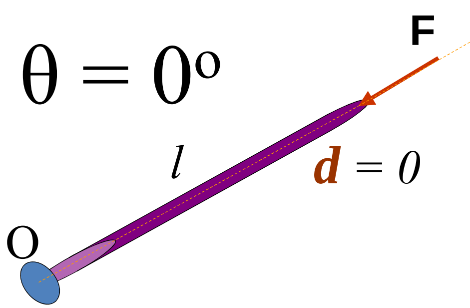
Moment Arm

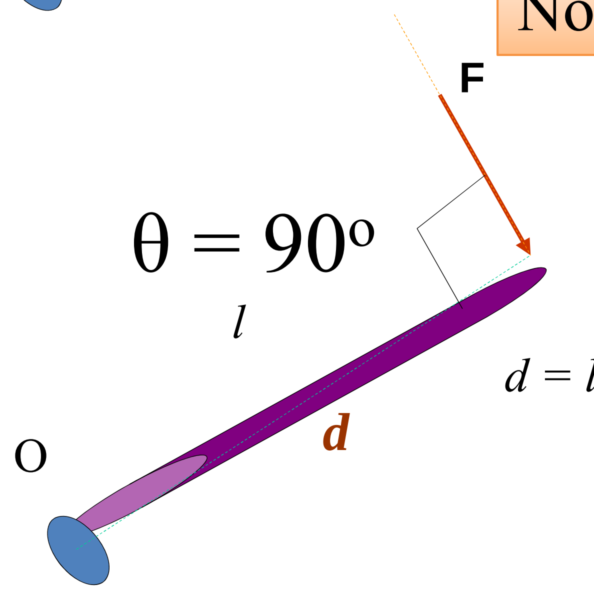




$M_o = Fd$

Note: $\boldsymbol{d} = l \sin \theta$

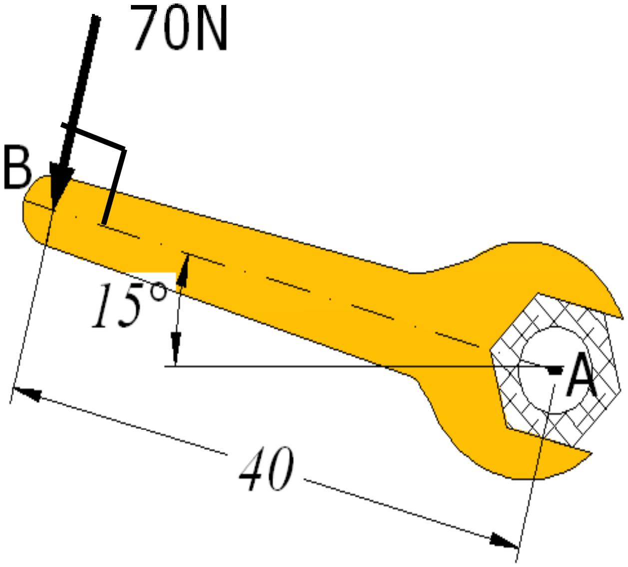




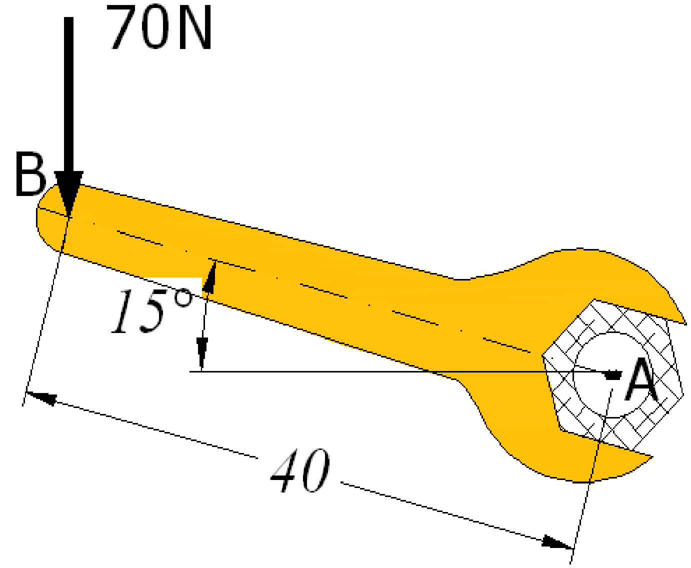
Example 1

Determine the moment of the 70N force about point A. *[Answer : (a)M_A 2800Nmm ↺ (b) M_A =2704.6Nmm ↺]*

(a)

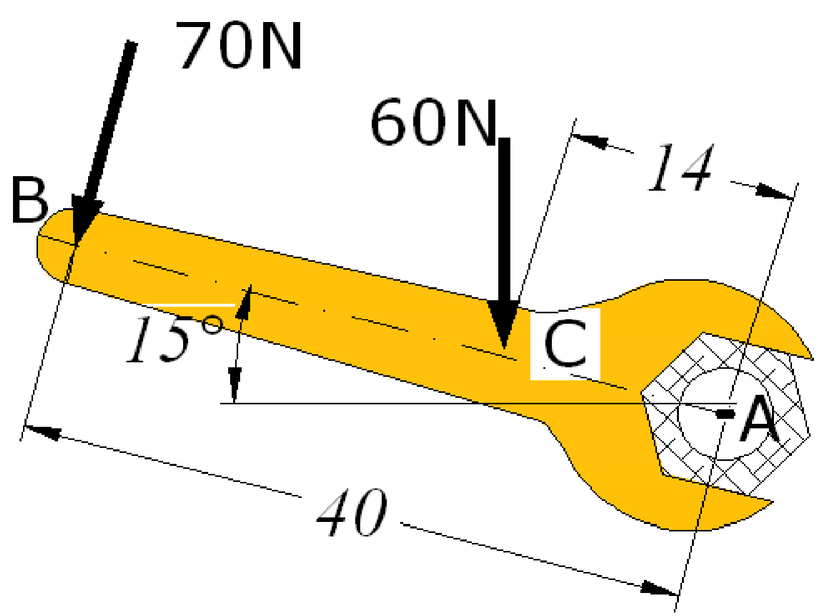


(b)



Example 2

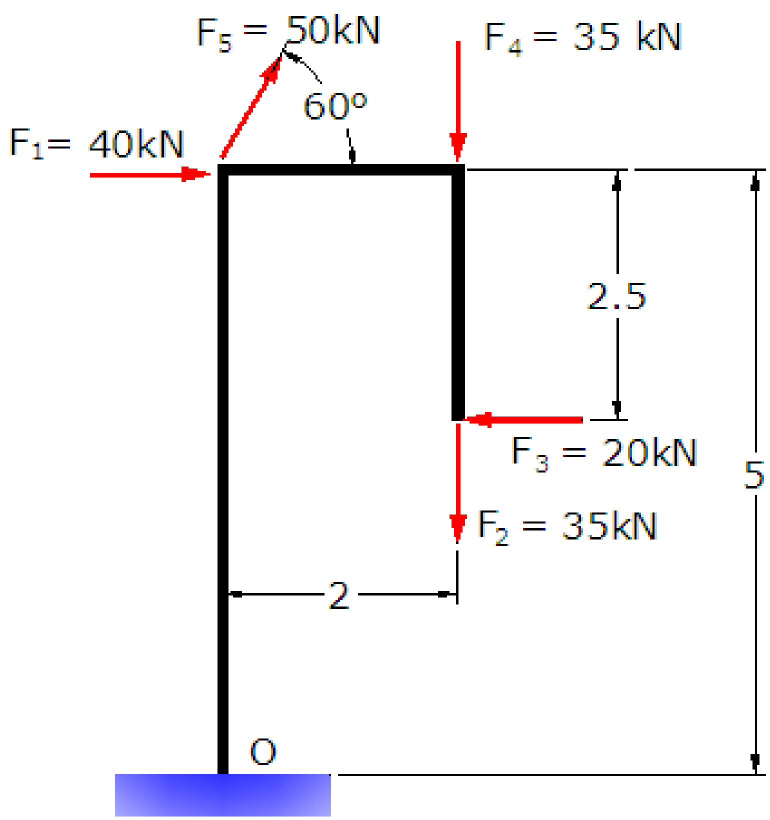
Determine the moment of the 70N and 60N forces about point A. *[Answer : M_A= 3611.4 Nm ↺]*



Example 3

Determine the moment of each the force about point O.

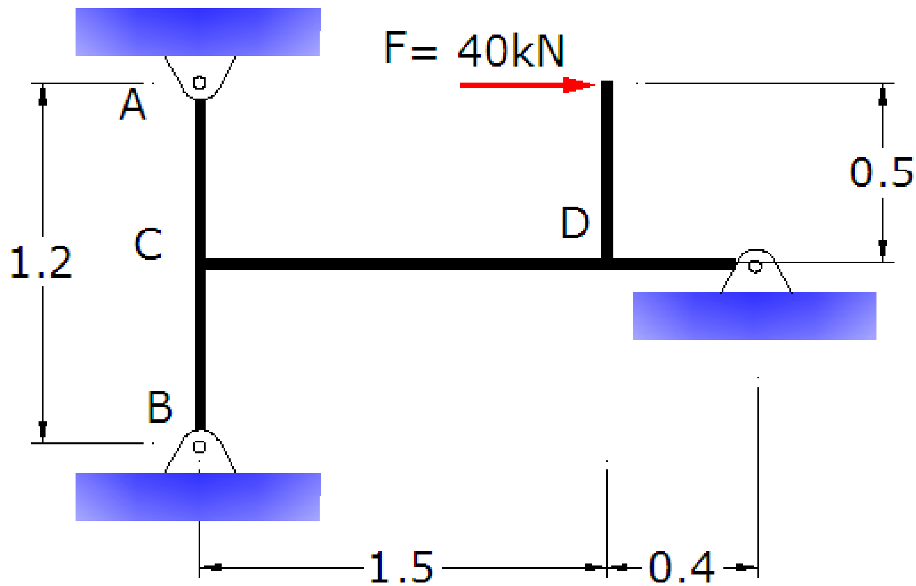
[Answer : M_{O1} = 200kNm ↻, M_{O2} =70kNm ↻, M_{O3} = 50kNm ↺, M_{O4} = 70kNm ↻, M_{O5} = 125kNm ↻]



Example 4

Determine the moments of the 40kN force about points A, B, C and D.

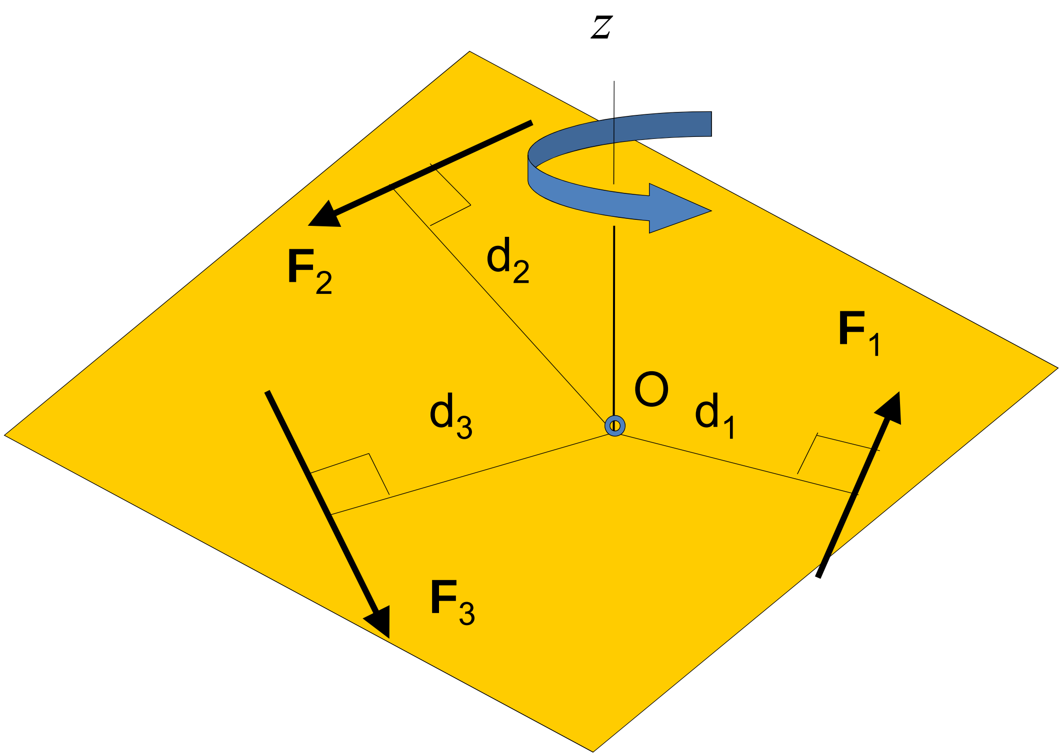
[Answer : $M_A = 0$ $M_B = 48kNm$ ↻ $M_C = 20kNm$ ↻ $M_D = 20kNm$ ↻]



Resultant Moment of Coplanar Forces

- determined by total up the moments of all the forces algebraically.

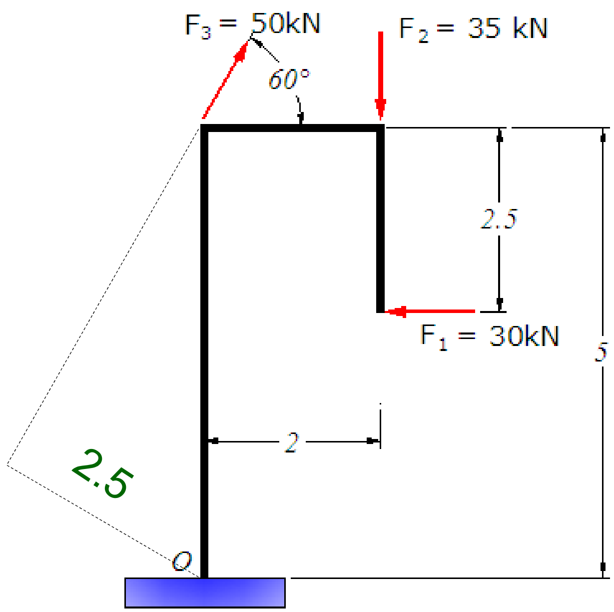
$$\circlearrowleft + M_{RO} = \Sigma Fd$$



The counterclockwise curl written along the equation indicates that, the moment of any force will be positive if it is directed along the +z axis.

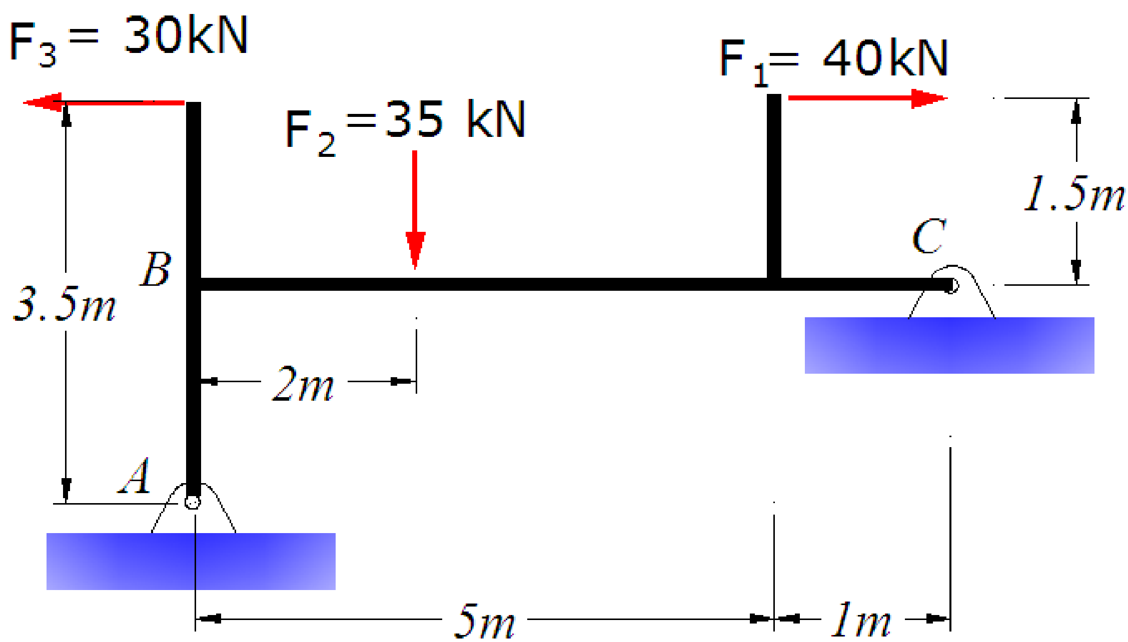
Example 5

Determine the moment of the three forces about point O. *[Answer : $\circlearrowleft M_O$ = -120kNm]*



Example 6

Determine the moments of the three forces about point B and C. *[Answer : ↺M_B = -85kNm , ↺M_C = 125Nm]*

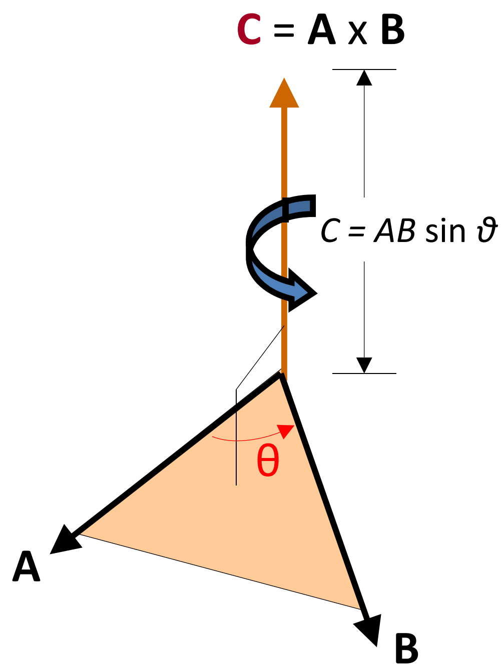


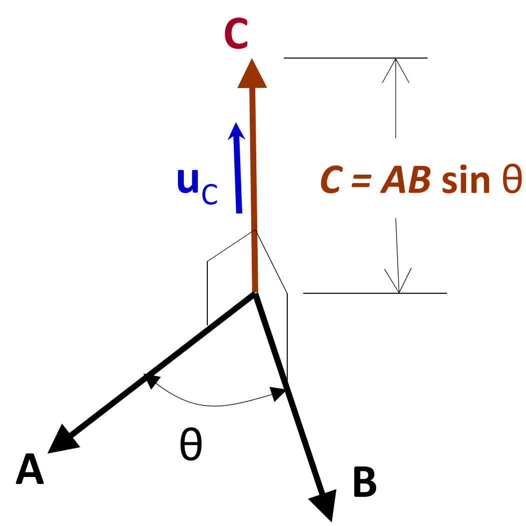
Cross Product

cross product of two vectors **A** and is written as **C** = **A x B**

Magnitude of *C* = *AB* sin θ

The direction of vector **C** is perpendicular to the plane **A** & **B** such that **C** is specified by the *right-hand rule.*





Cross Product - Laws of Operations

***Commutative law*:**

$$\mathbf{A} \times \mathbf{B} \neq \mathbf{B} \times \mathbf{A}$$

$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$$

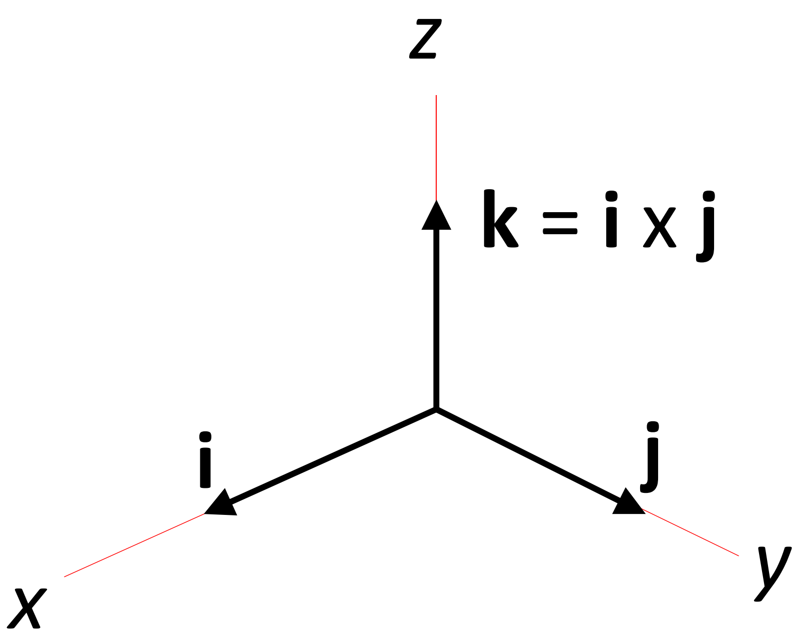
Multiplication by a scalar:

$$a\,(\mathbf{A} \times \mathbf{B}) = (a\mathbf{A}) \times \mathbf{B} = \mathbf{A} \times (a\mathbf{B}) = (\mathbf{A} \times \mathbf{B})a$$

Distributive law:

$$\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D})$$

Cross Product of the Cartesian unit vectors

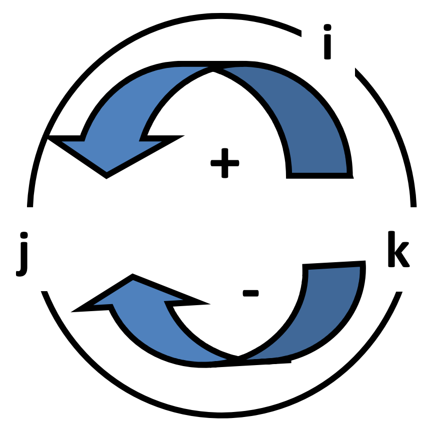


In a similar manner,

i x j = k **i x k = -j** **i x i = 0**

j x k = i **j x i = -k** **j x j = 0**

k x i = j **k x j = -i** **k x k = 0**



This diagram is helpful for obtaining the result of cross products of unit vectors

Tips : Apply right hand rule

Cross product of two vectors **A** and **B.**

Determinant form:

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

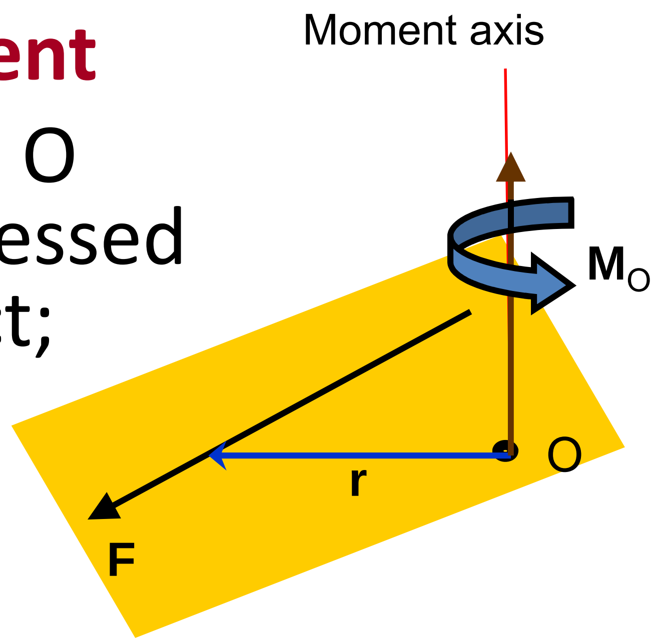
Moment of a Force

2. Vector Formulation of Moment

- The moment of **F** about point O discussed earlier, can be expressed using the vector cross-product;

$$\mathbf{M_O = r \times F}$$

Where r represent the position vector drawn from O to any point lying on the line of action of F.

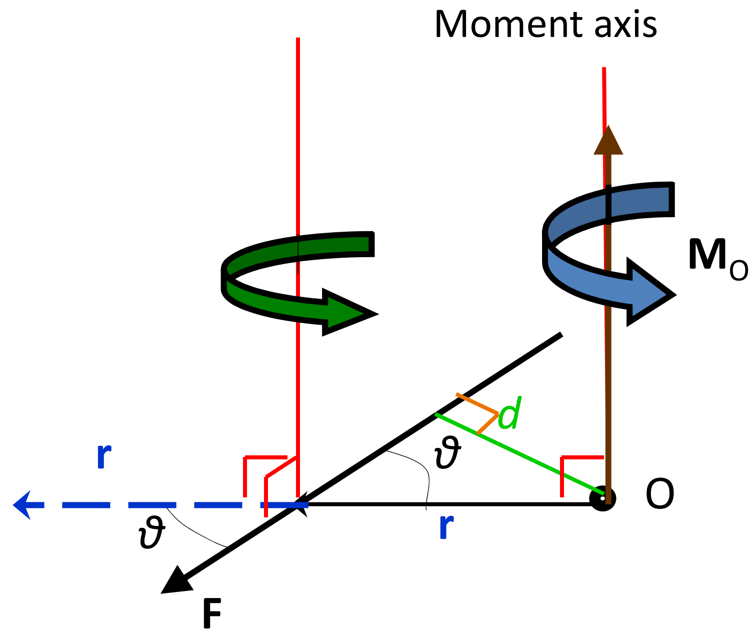


Magnitude:

$$M_O = rF \sin\theta = F(r \sin\theta) = Fd$$

Direction: Apply right-hand rule at the intersection point of the tails of extended ***r*** and ***F***

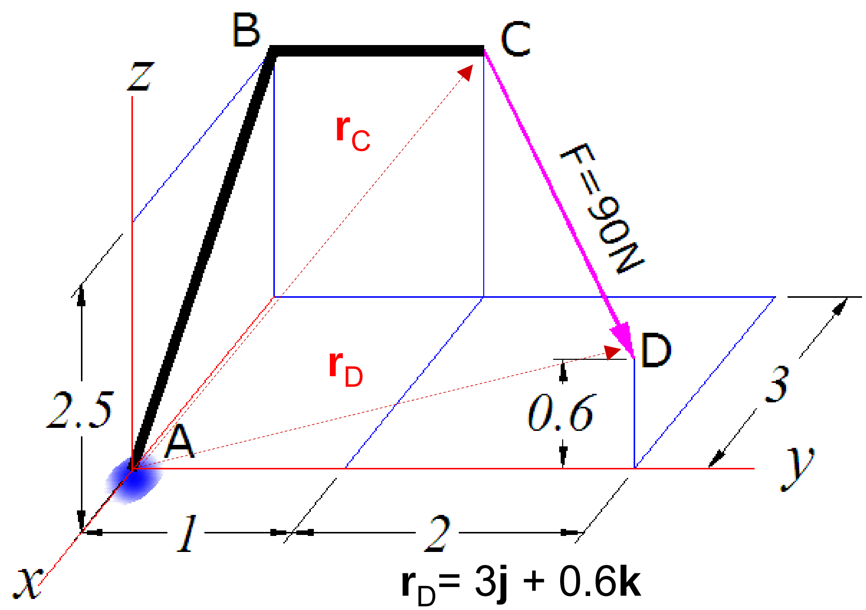
***Note that the moment axis is perpendicular to the plane containing* r *and* F**



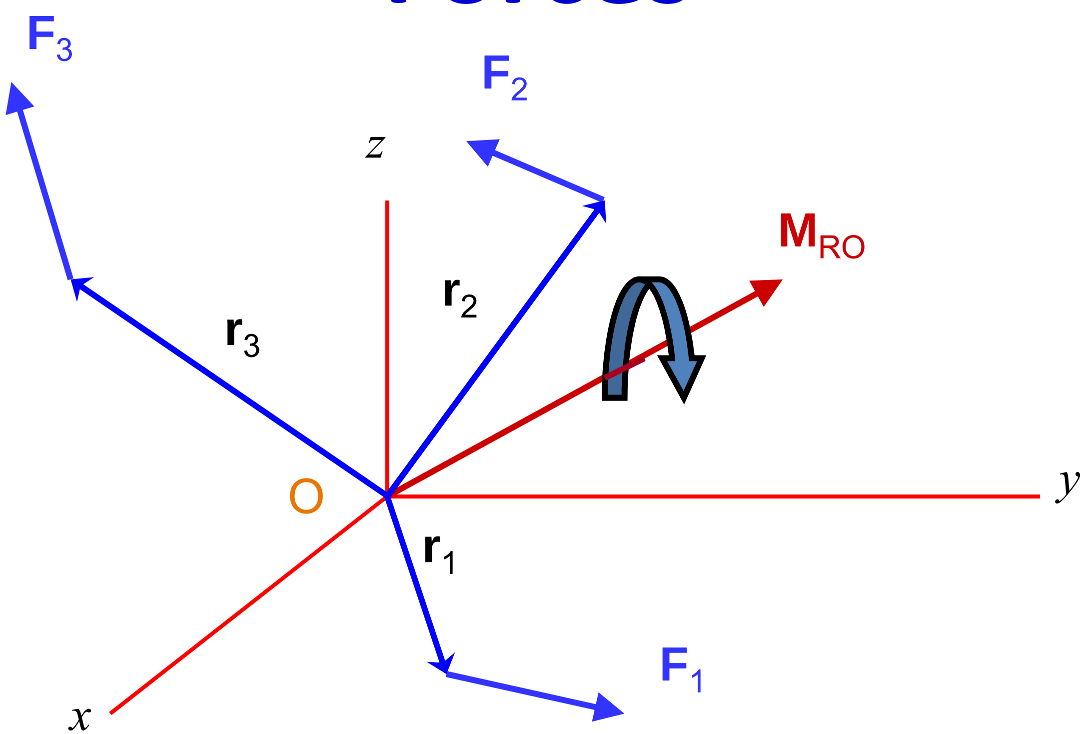
r is treated as a sliding vector

Example 7

A force F=90N directed from C to D. Determine the magnitude of the moment created about the support at point A, and their coordinate direction angles. *[Answer :* $M_A = 253.4Nm$ $\alpha = 126.9^o$ $\beta = 81^o$ $\gamma = 141.6^o$ *]*



Resultant moment of a System of Forces

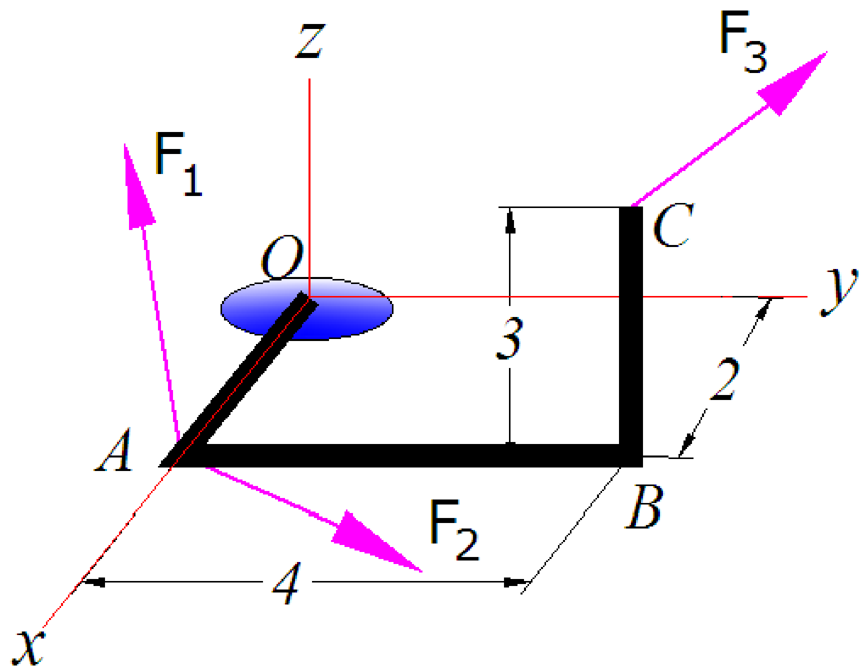


$$M_{RO} = \sum (\mathbf{r} \times \mathbf{F})$$

Example

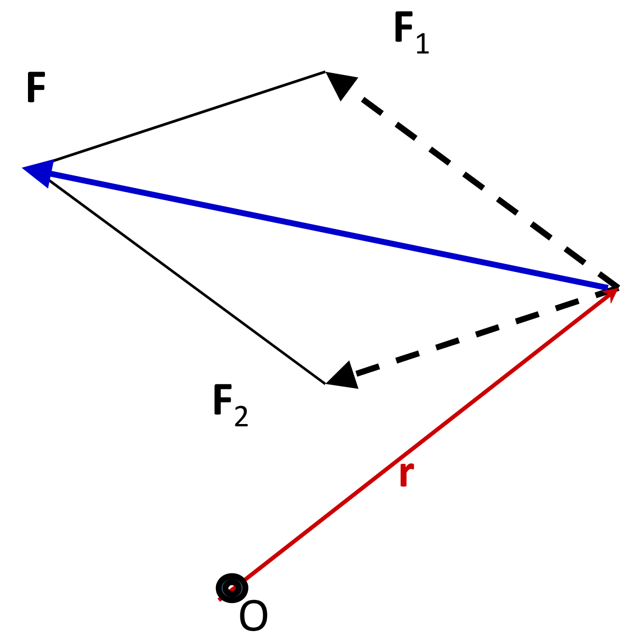
Three forces acting on the rod. Determine the resultant moment about O and the coordinate direction angles. Given $\mathbf{F}_1 = 20\mathbf{i} + 80\mathbf{k}$, $\mathbf{F}_2 = 40\mathbf{i} + 30\mathbf{j} - 25\mathbf{k}$ and $\mathbf{F}_3 = -35\mathbf{i} + 50\mathbf{j} - 15\mathbf{k}$.

[Answer : $M_{RO} = 410\text{Nm}$ $\alpha = 120.8^o$ $\beta = 117^o$ $\gamma = 43^o$]



Principle of Moments
(Varignon's theorem)

The moment of a force about a point is equal to the summation of the moments of the force's components about the point.



$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$

Proof: $\mathbf{M}_o = \mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2$

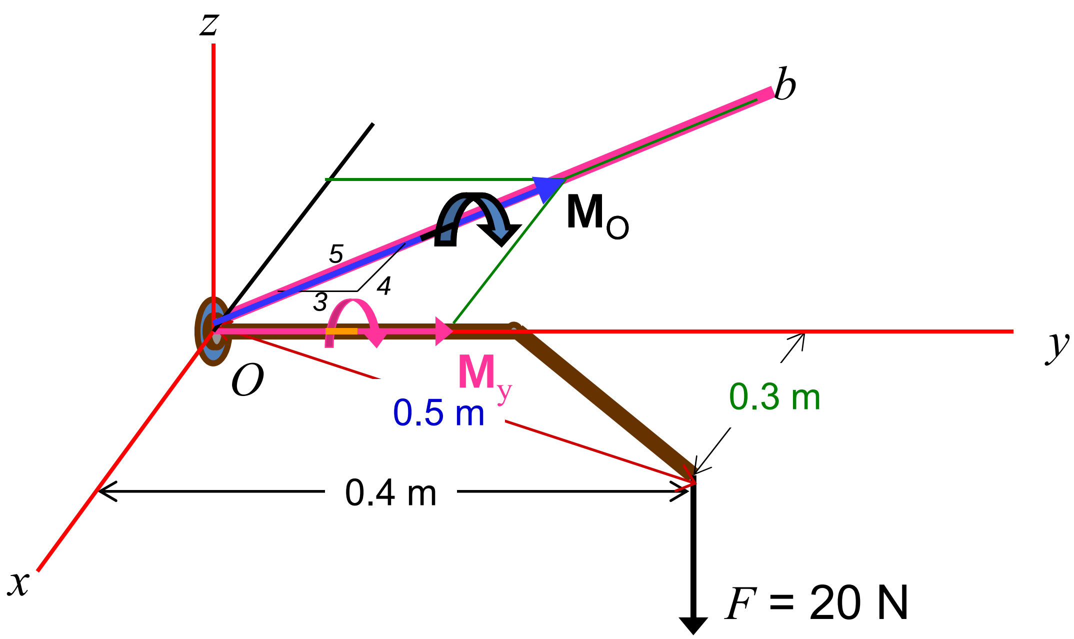
$= \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2)$

$= \mathbf{r} \times \mathbf{F}$

Useful to determine the moment arms of the force's components than the moment arm of the force itself.

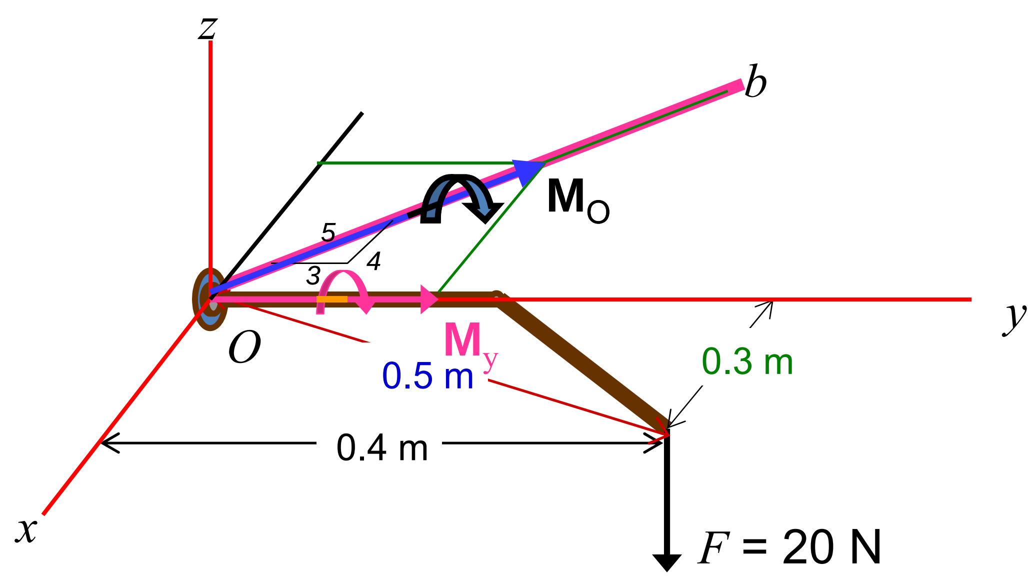
Moment of a Force about a Specified Axis

- Can be solved by **scalar** or **vector** analysis.



In some situations we need the component of the moment along a specified axis that passes through the point. Let say component of M_O about y axis, M_y .

1. Scalar Analysis



a) $M_o = 20(0.5) = 10$ Nm (direction defined by RH Rule about the *Ob* axis)

$\mathbf{M}_y = (3/5)(10) = 6$ Nm *(component method)*

b) $\mathbf{M}_y = 20(0.3) = 6$ Nm *(direct method)*

- If the line of action of a force **F** is perpendicular to any specified axis *aa*, then;

$$M_a = Fd_a$$

where d_a is the perpendicular distance from the force line of action to the axis.

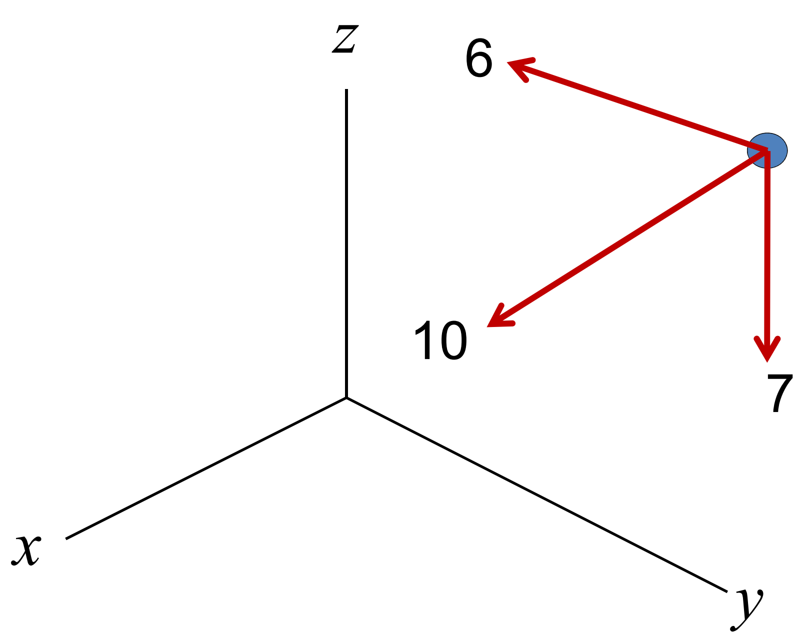
- The direction is determined from the thumb of the Right Hand when the fingers are curled in accordance with the direction of rotation.

- *A force will **NOT** contribute a moment about a specified axis if the force **line of action is parallel** to the axis or its line of action **passes through** the axis.*

Example

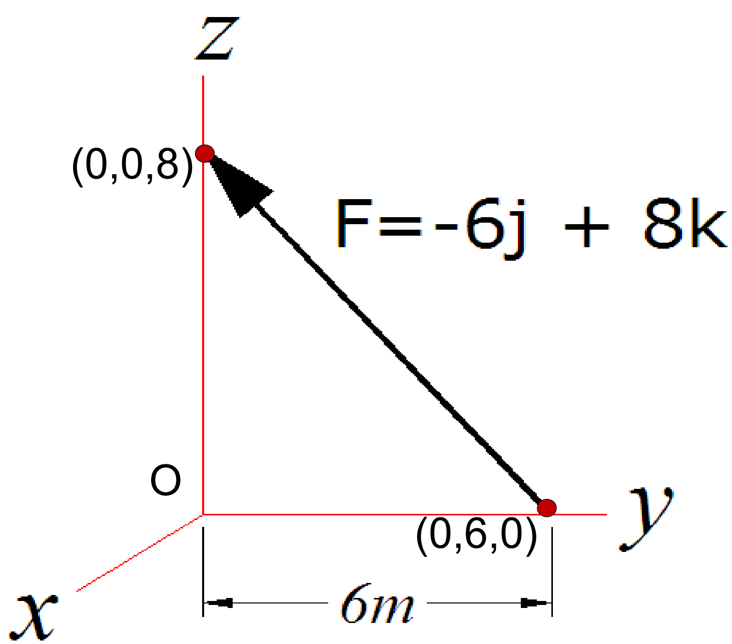
What are the values moment about the *x,y, z* axes.

[Answer : M_{Ox}=13 Nm M_{Oy}== 59 Nm M_{Oz}= -32 Nm]



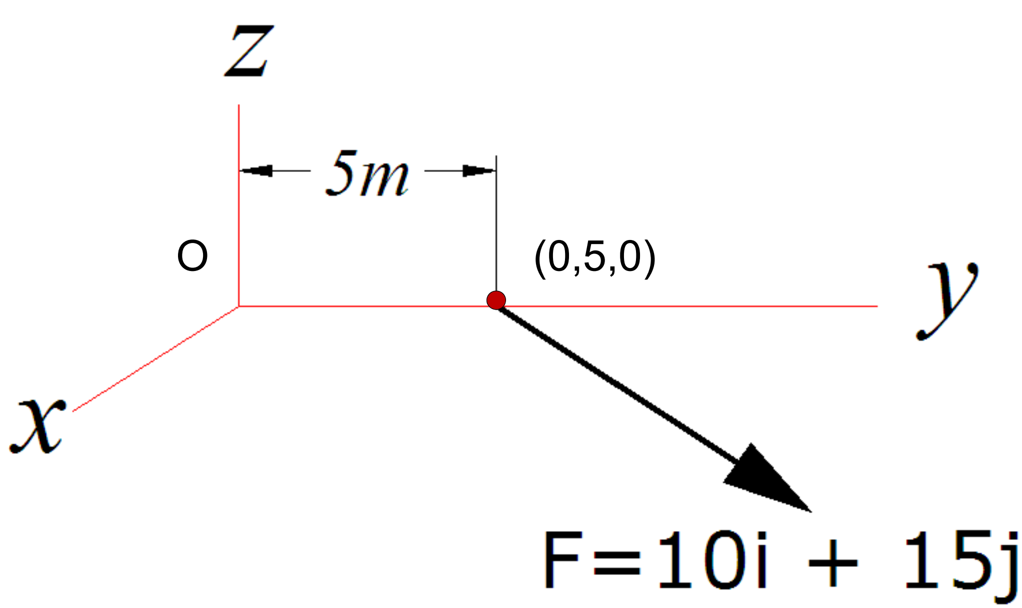
Example

Find the Moment about a specified axis using Scalar notation method. *[Answer : M_{Ox}= 48Nm M_{oy} = 0Nm , M_{oz} = 0Nm]*

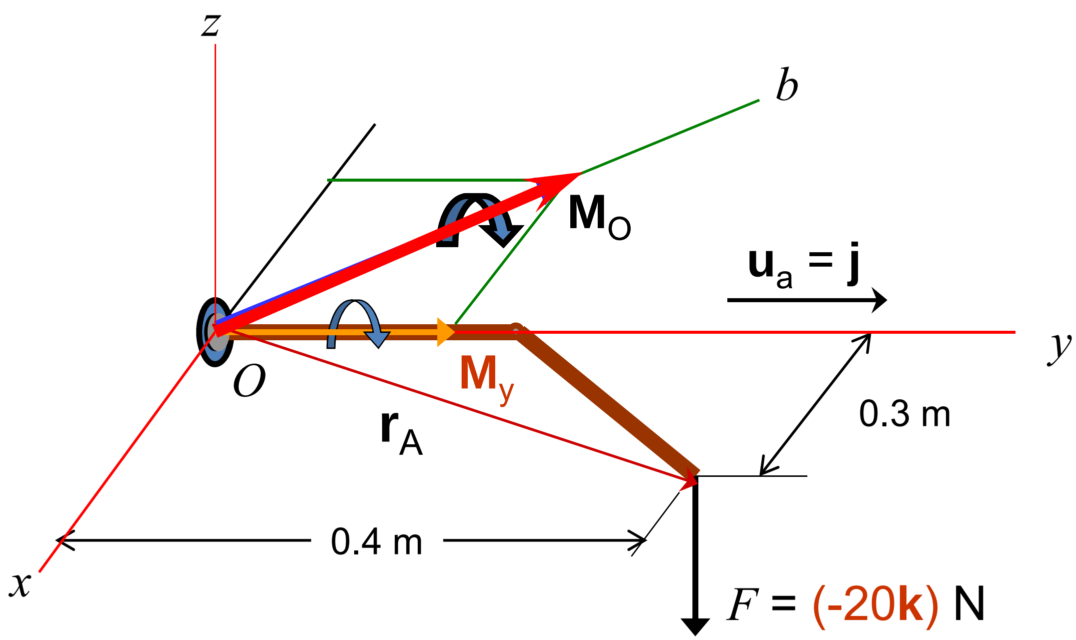


Example

Find the Moment about a specified axis using Scalar notation method. *[Answer : M_{Ox} = 0Nm , M_{oy} = 0Nm, M_{oz} = 50 Nm]*



2. Vector Analysis



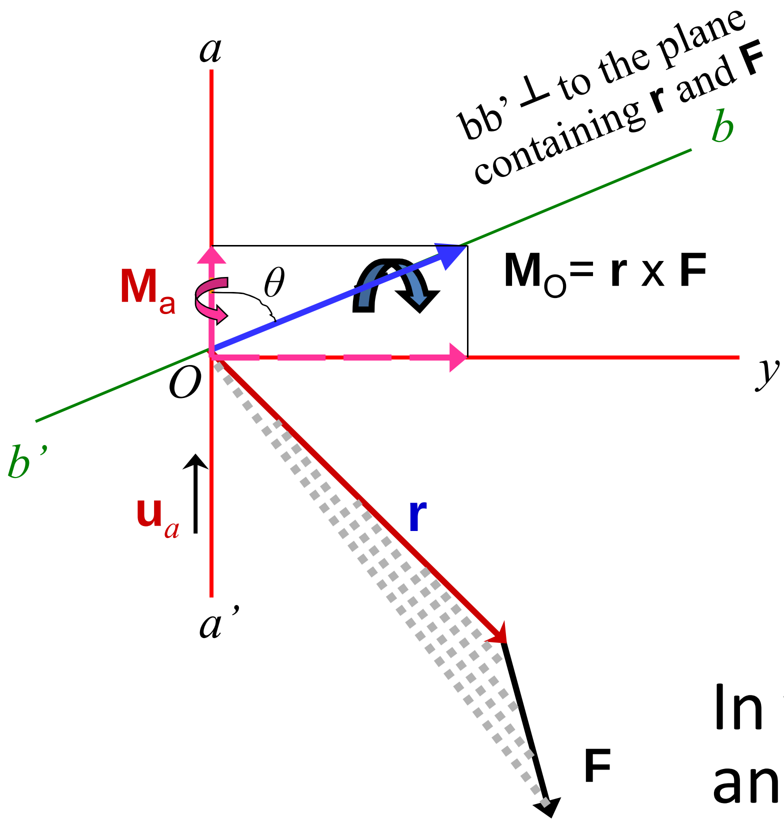
$$\mathbf{M}_O = \mathbf{r}_A \times \mathbf{F}$$
$$= (0.3\mathbf{i} + 0.4\mathbf{j}) \times (-20\mathbf{k})$$
$$= (-8\mathbf{i} + 6\mathbf{j}) \text{ Nm}$$

The component of this moment along the y axis is then determined from the dot product

Since the unit vector of this axis is $\mathbf{u}_a = \mathbf{j}$, then

$$M_y = \mathbf{M}_O \cdot \mathbf{u}_a = (-8\mathbf{i} + 6\mathbf{j}) \cdot \mathbf{j} = 6 \text{ Nm}$$

Vector analysis is advantages to find moment of force about an axis when the force components or the moment arms are difficult to determine.



How to get M_a ?

1. Find $M_o = r \times F$
2. $M_a = M_o \cos\theta = M_O \cdot u_a$

$M_a = (r \times F) \cdot u_a = u_a \cdot (r \times F)$

In vector algebra , combination of dot and cross product yielding the scalar M_a is called the ***triple scalar product***.

The triple scalar product :

$$M_a = (u_{ax}\,\mathbf{i} + u_{ay}\,\mathbf{j} + u_{az}\,\mathbf{k}) \cdot \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} = \begin{vmatrix} u_{ax} & u_{ay} & u_{az} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Once M_a is determined, $\mathbf{M}_a$ as a Cartesian vector ;

$$\mathbf{M}_a = M_a \mathbf{u}_a = [\mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F})]\mathbf{u}_a$$

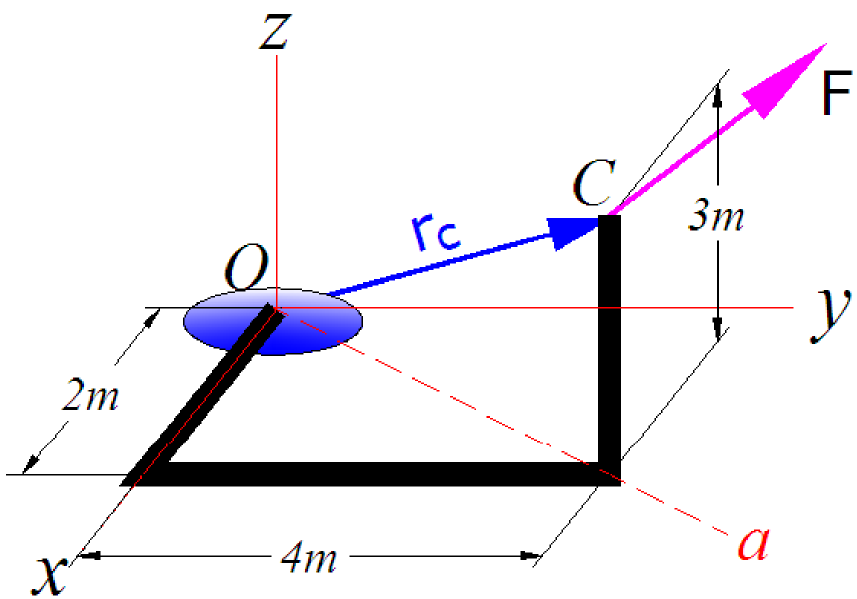
To find the **resultant** of a series of forces about the axis aa', the moment components of each force are added together algebraically, since the component lies along the same axis.

$$M_a = \sum[\mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F})] = \mathbf{u}_a \cdot \sum(\mathbf{r} \times \mathbf{F})$$

Example

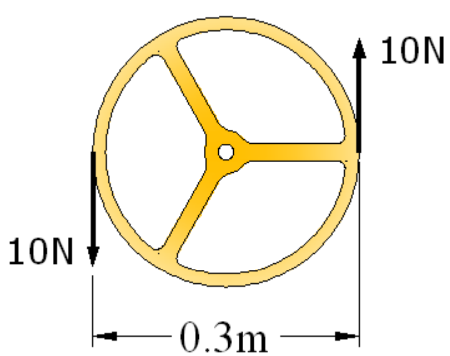
The force F= -35**i** + 50**j** - 15**k** acts at C. Determine the moment of this force about *x* and *a* axes.

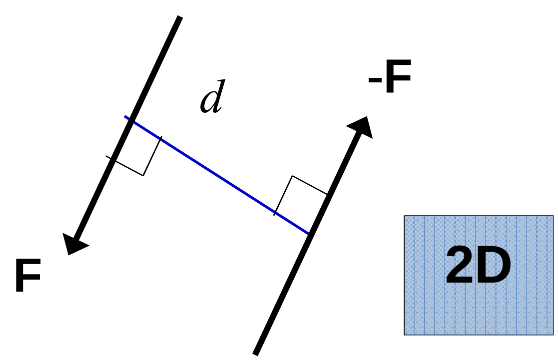
[Answer : M_x= -210Nm M_a = -161Nm]

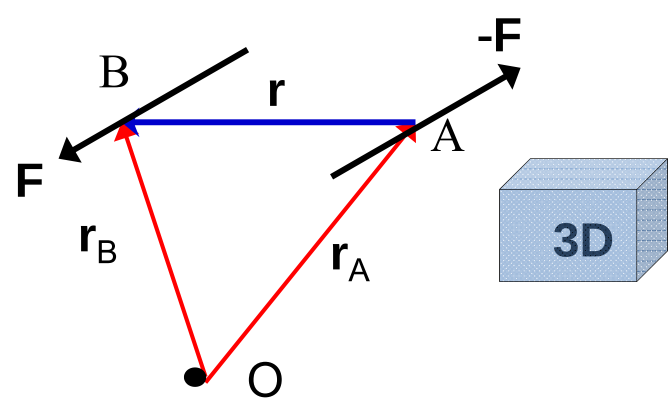


Couples and Couple Moments

- **Definition :** **two parallel forces** that have the **same magnitude** but **opposite direction**, and separated by a perpendicular distance, *d*.
- The moment produced is called **couple moment**.
- ∑F = 0, the only effect is tendency of rotation.

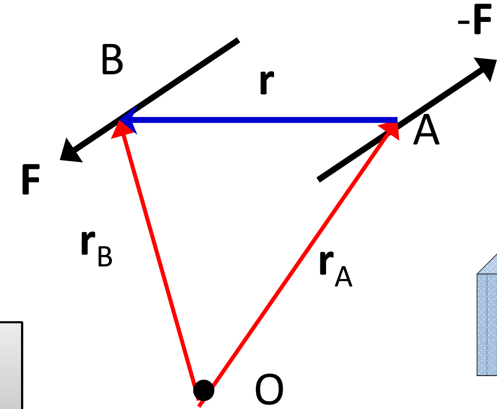


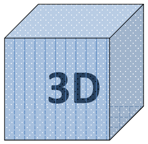




Moment of a Couple

Determination of moments of couple forces about any point :



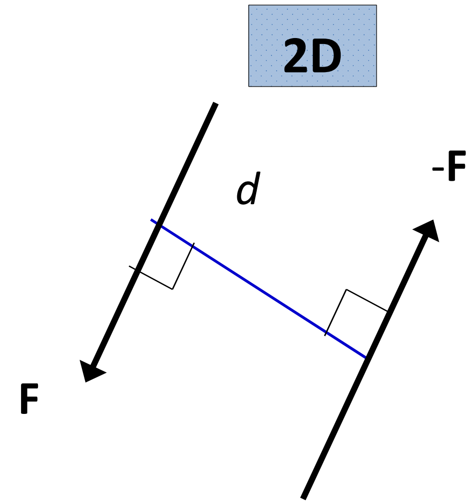


about A: **M = r X F**

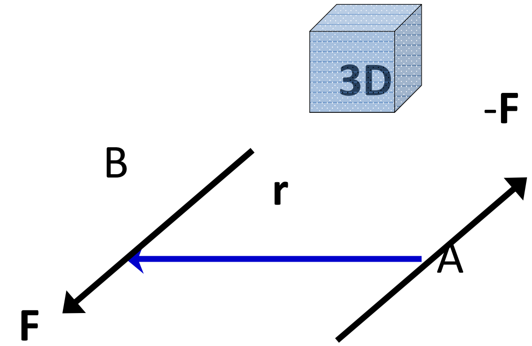
about O: **M =** r_B **X (F) +** r_A **X (-F)**

This indicates that a couple moment is a free vector. It can act at any point since M only depends upon the position vector r, not r_A ***and*** r_B***.***

Scalar Formulation: $M = Fd$



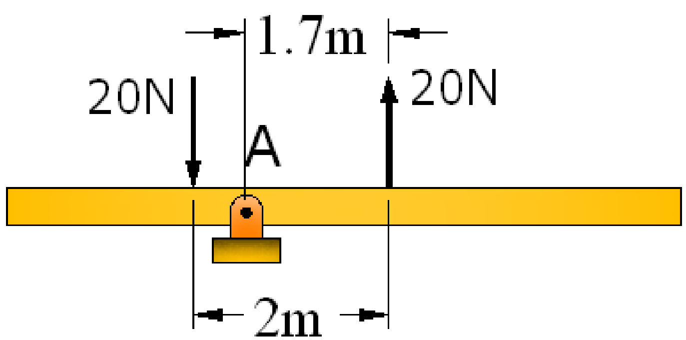
Vector Formulation: $\mathbf{M} = \mathbf{r} \times \mathbf{F}$

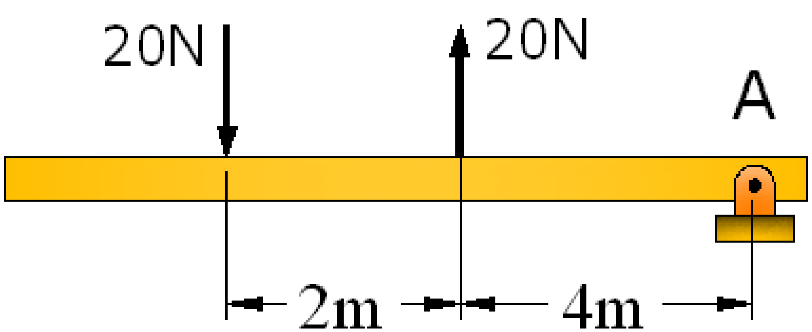


Properties of Moment of a Couple

1. The couple moment is unaffected by the pivot location

*Couples at the same position for example below.





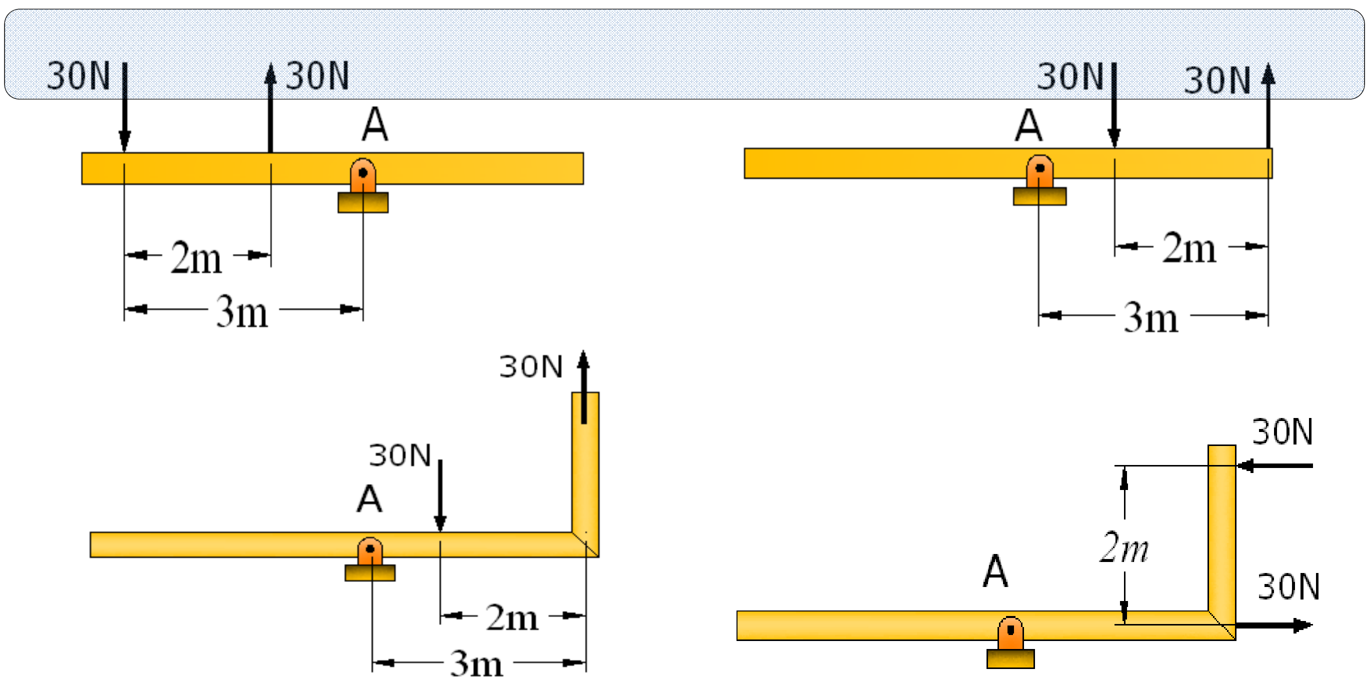
M_A=20(0.3)+20(1.7)

= 40Nm

M_A=20(6)-20(4)

= 40Nm

2. A couple can be shifted and still have the same moment about a given point.



Couple at different position & moments calculated at the same point.

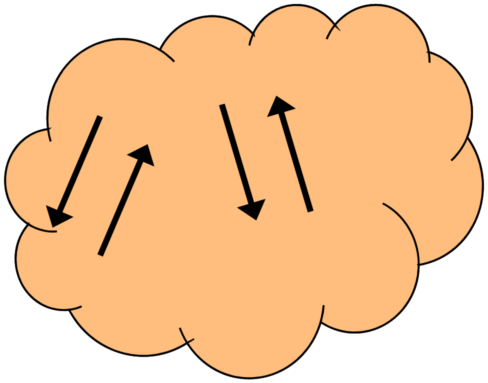
$M_A = 30(\mathbf{2}) = 60\text{Nm}$

- **Equivalent Couples**

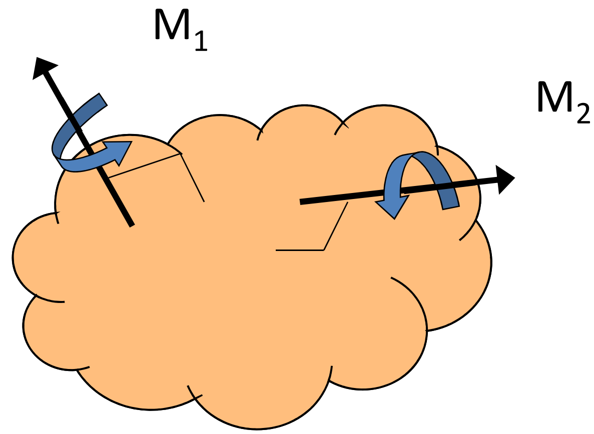
  Two couples which produce the **same moment** lie either in the **same plane** or in planes **parallel** to each other. The direction of the couple moments is the same and is perpendicular to the parallel planes.

- **Resultant Couple Moment**

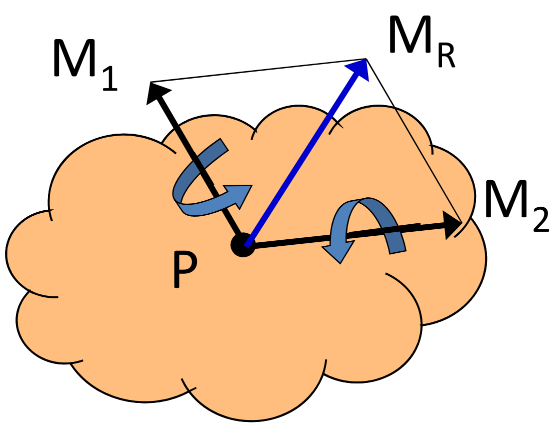
  Since couple moments are **free vectors** they can be applied at **any point** on a body and added vertically.



Two set of couple forces



Two couple moments

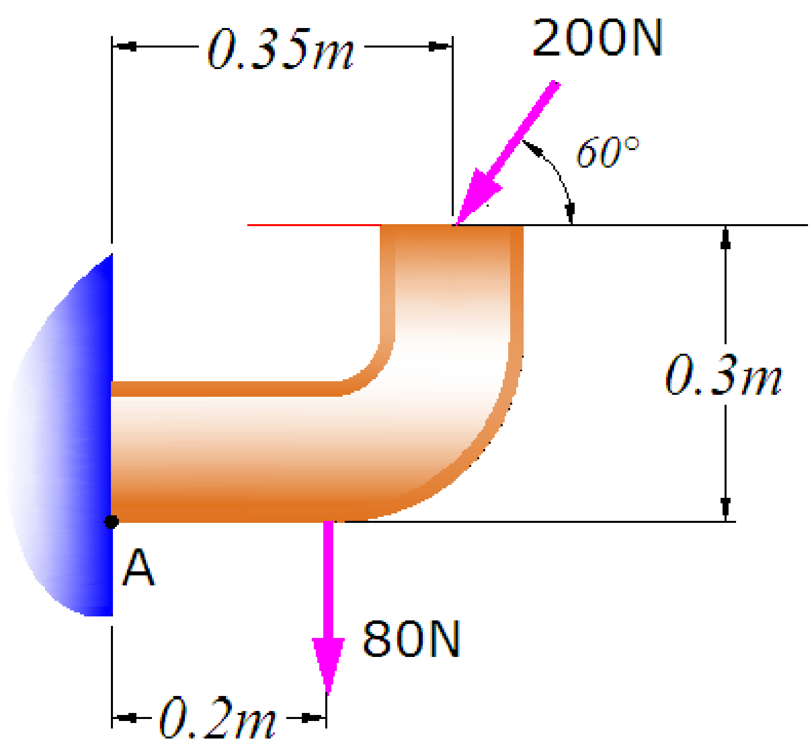


Moved to any arbitrary point and added to obtain resultant couple moment

$$M_R = M_1 + M_2$$

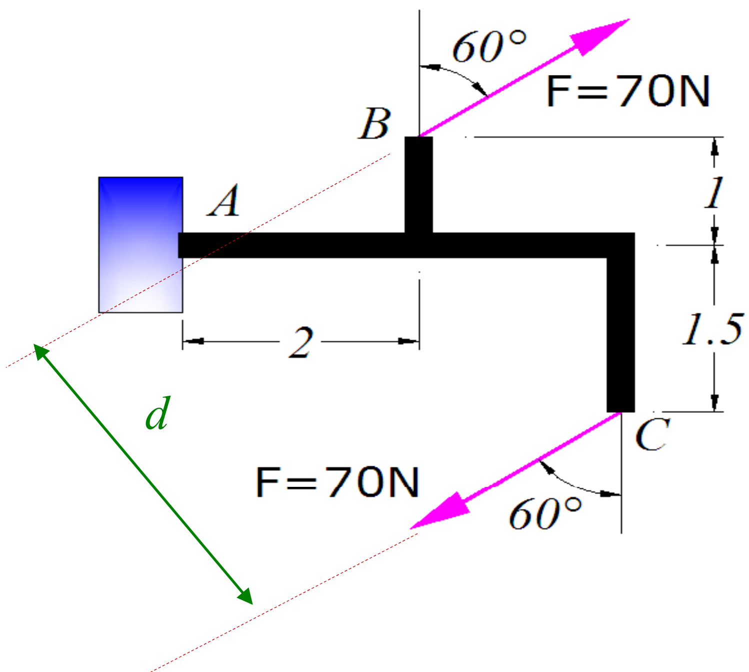
Example

Replace the forces acting on the structure by an equivalent resultant force and couple moment at A. *[Answer: $\circlearrowleft M_{RA}$ = = $\underline{-46.6Nm}$ ($\circlearrowright$)]*



Example

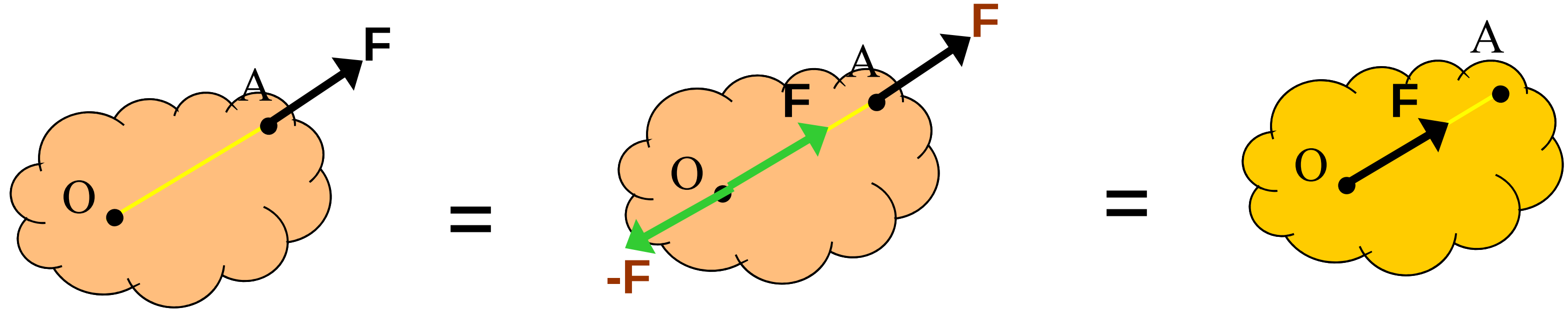
Determine the moment of the couple on the member shown. *[Answer : ↺M= -221.5Nm]*



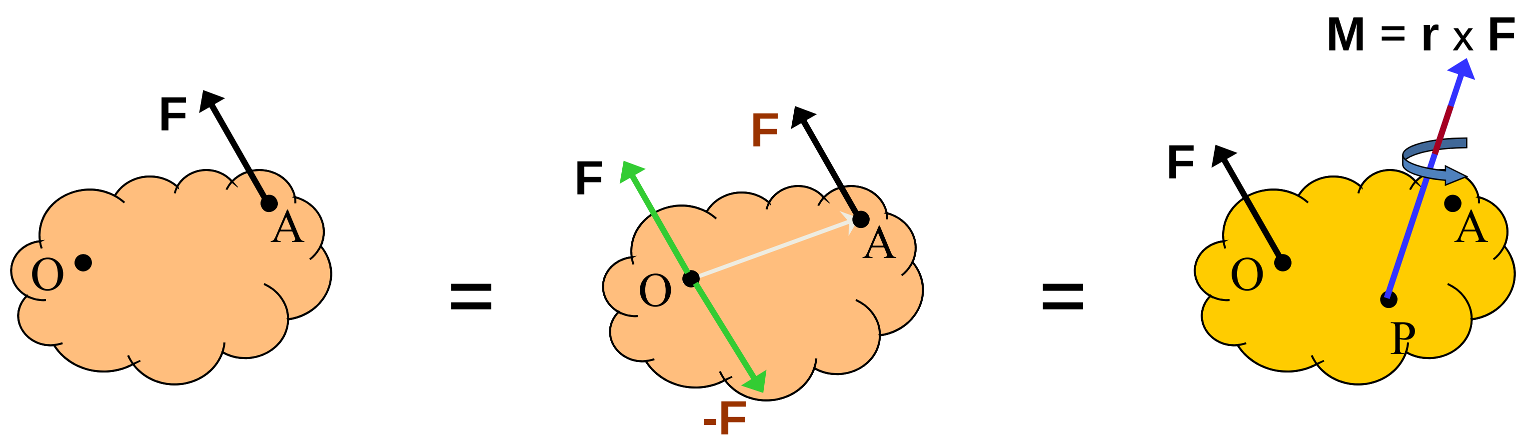
Equivalent System

Replacing system of forces and couple moments acting on a body by a **single force and couple** acting on a specified point O that produce the **same external effects** of translation and rotation.

Case 1: *Point O Is On the Line of Action*



Case 2: *Point O Is Not On the Line of Action*

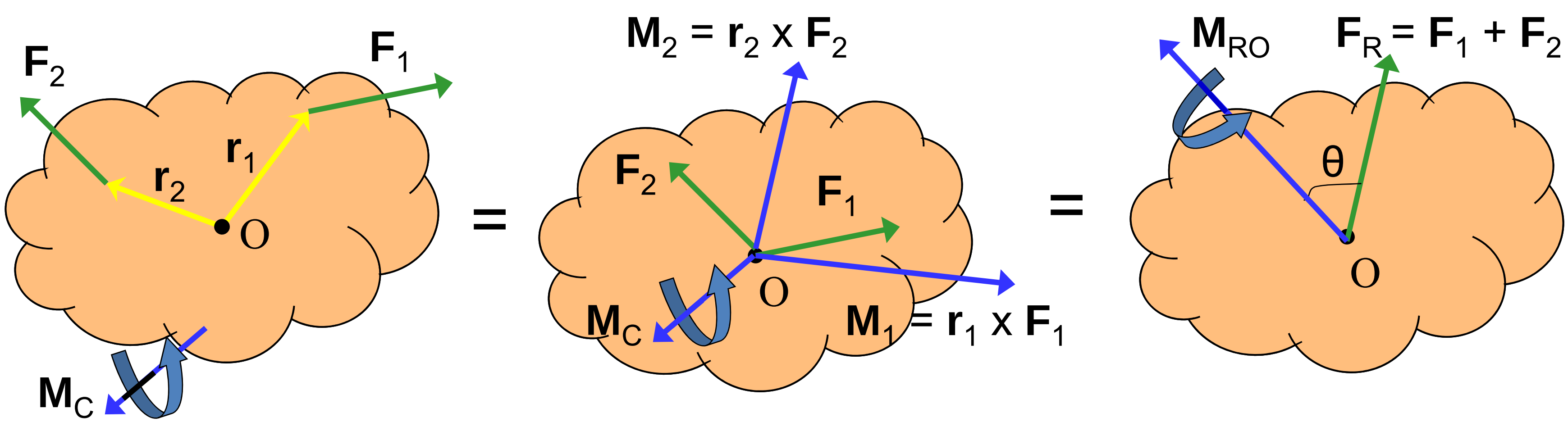


*Note: Since couple is a free vector , it may be applied at any point

Resultants of a Force System

$$M_{RO} = \sum M_c + \sum M_o$$

$$F_R = \sum F$$

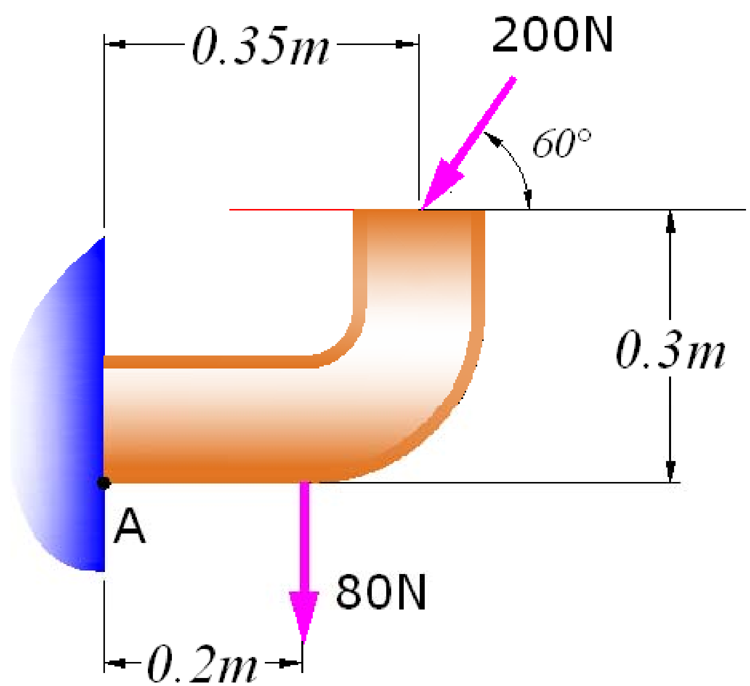


NOTE:

- Both the magnitude & direction of F_R are independent of the location of O, however,
- M_{RO} depends on the location of O since the moment M_1 & M_2 are determined by using the position vectors r_1 & r_2.
- M_{RO} is a free vector and can act at any point on the body.

Example

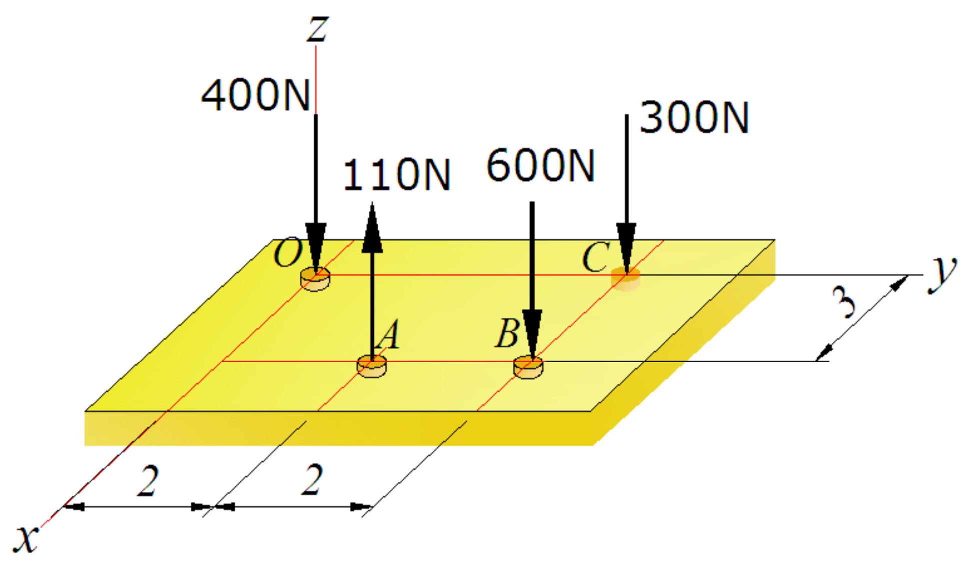
Determine the magnitude, direction and location of a resultant force which is equivalent to the given system of forces measured horizontally from A. *[Answer : $F_R = 272N(\swarrow)$ $\theta = 68.4^o$ $d = 0.18m$]*



Example

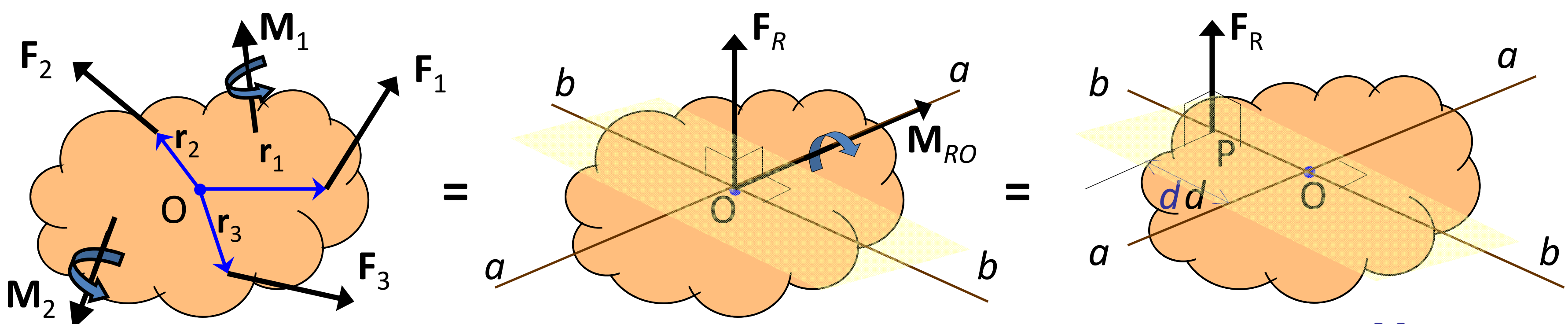
Determine the magnitude and direction of a resultant force equivalent force system and locate its point of application.

[Answer : $\mathbf{F}_R$ = -1190N (↓) ,y = 2.84m x = 1.24m]



Simplification to a Single Force System

Consider a **special case** for which the system of forces and couple moments reduces at point O of the resultant force $\mathbf{F}_R$ and couple $\mathbf{M}_R$ which are **perpendicular** to each other.

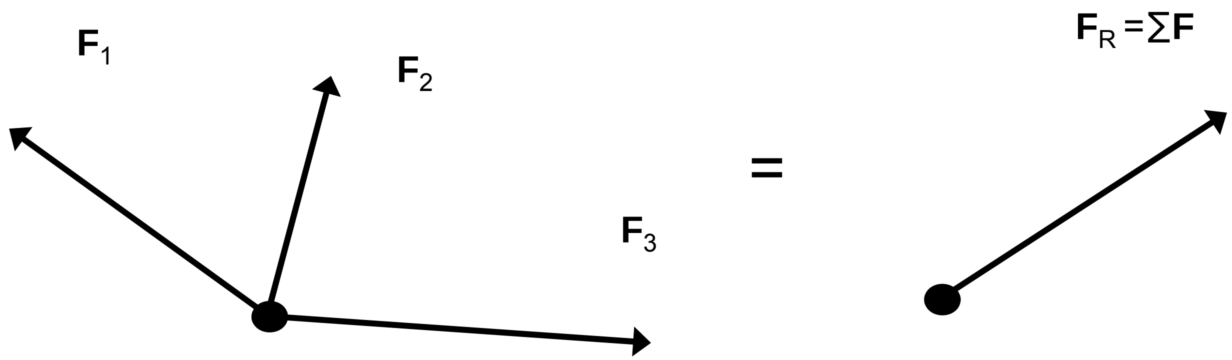


$$d = \frac{\mathbf{M}_{RO}}{\mathbf{F}_R}$$

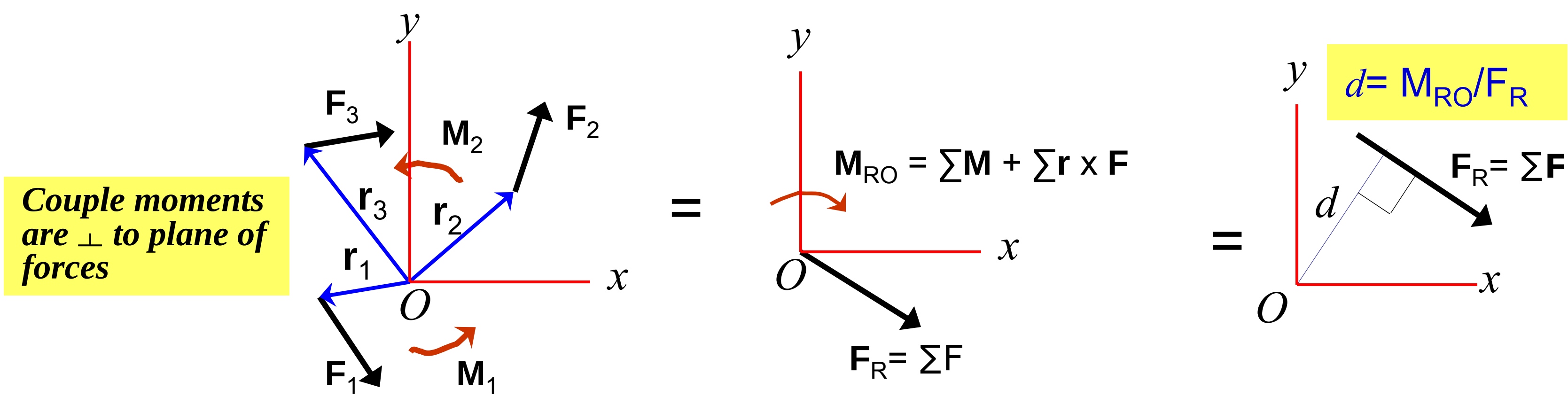
If the system of forces is either concurrent, coplanar, or parallel, it can be reduced (as in the above case), to a single resultant force $\mathbf{F}_R$.

This is because in each of these cases $\mathbf{F}_R$ and $\mathbf{M}_R$ will always be perpendicular to each other when the force system is simplified at ***any*** point.

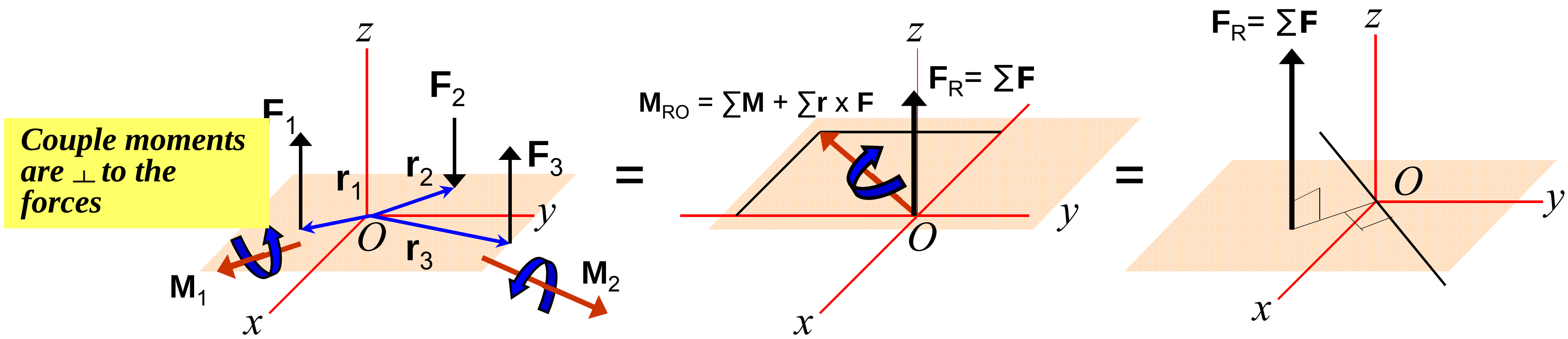
1.Concurrent Force System



2.Coplanar Force System



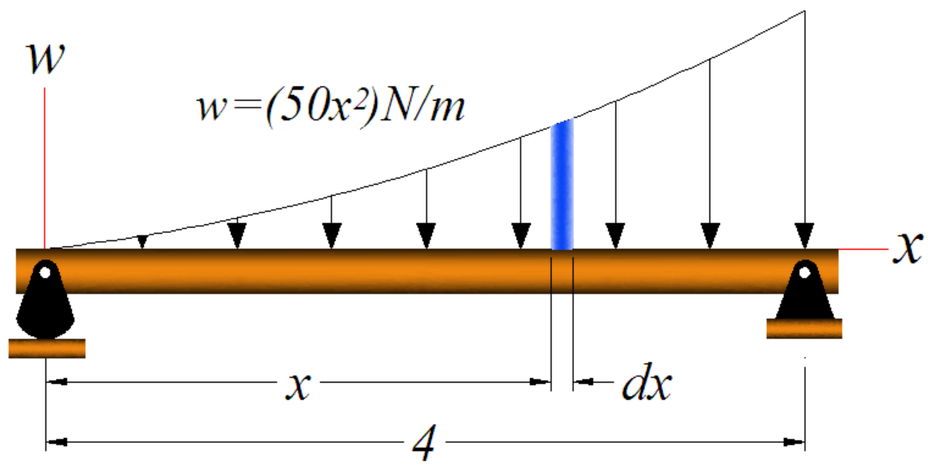
3.Parallel Force System



Example

Determine the magnitude and location of the equivalent resultant force acting on the beam.

[Answer : $F_R = -1190N$ (↓) , $\bar{x} = 3m$]

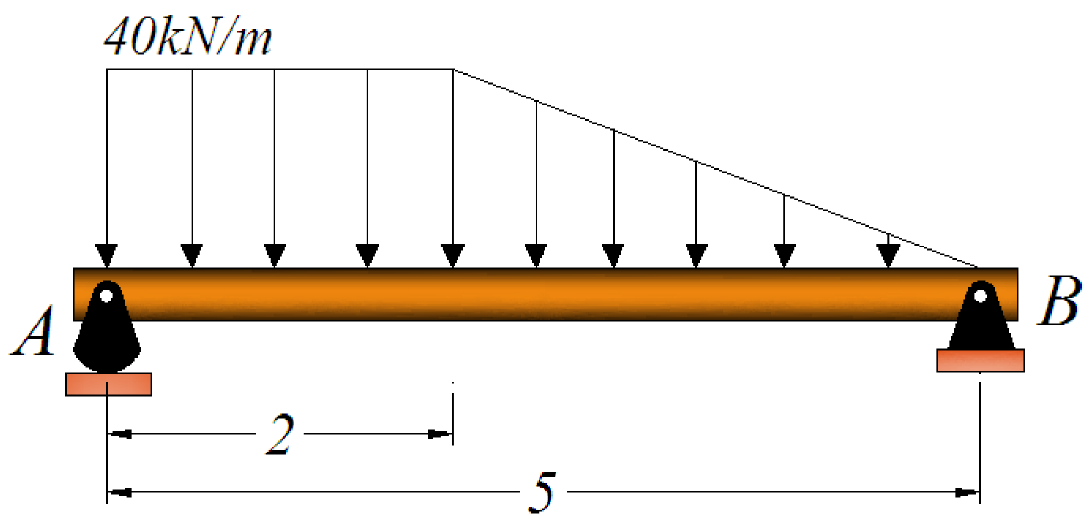


$$dA = w\,dx = 50x^2\,dx$$

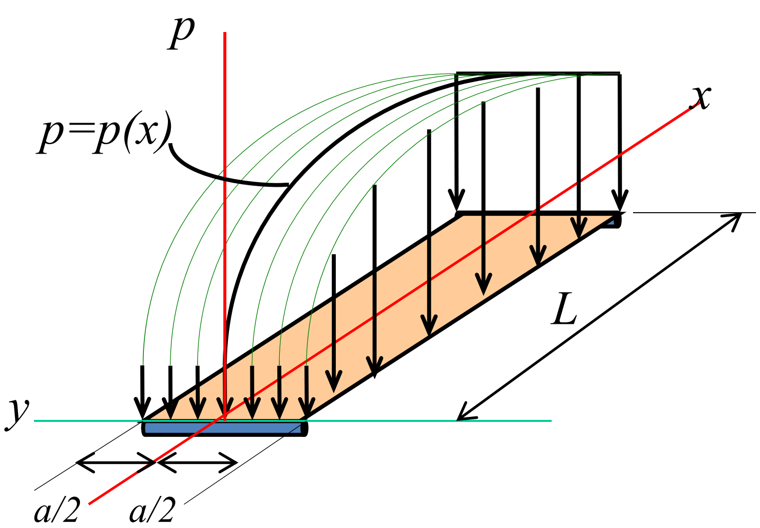
Example

Determine the magnitude and location of the equivalent resultant force acting on the beam.

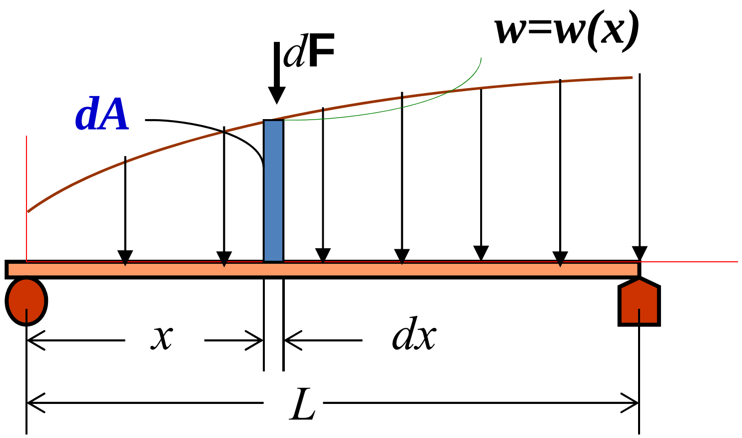
[Answer : F_R = 140KN , $\bar{x}$ = 1.86m]



Reduction of a simple Distributed Loading

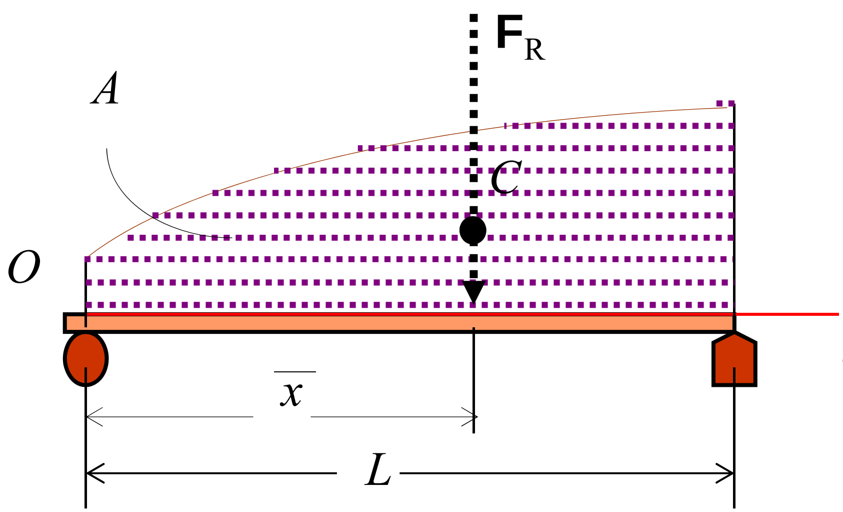


Uniform pressure along one axis on a flat rectangular surface. The load intensity is of the load represented by the arrows form a system of parallel forces, infinite in numbers, each acting on a separate differential area.



Load function, ***p = p(x)***
[pressure uniform in y axis]
Multiply ***p=p(x)*** with the width ***a***, we obtain;
w= p(x) a = w x

This loading function is a measure of load distribution along the line y=0 which is the plane of symmetry of the loading. Note: it is load per unit length.



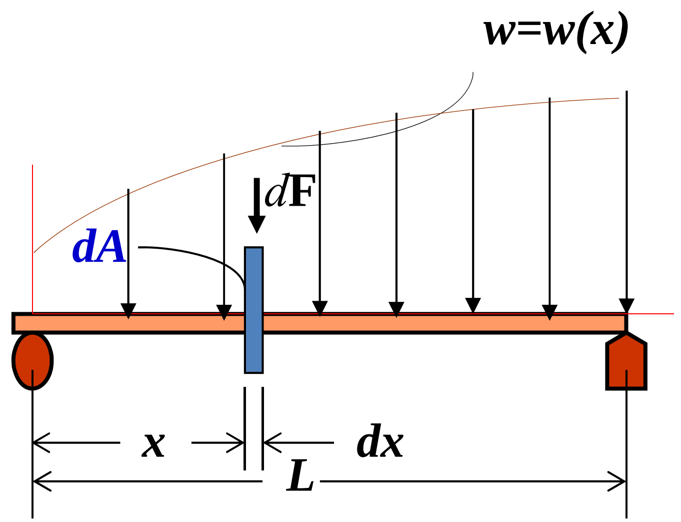
In a system of coplanar parallel forces, the load intensity can be represented by ***w = w(x)***

This system of forces can be simplified to a single force $\mathbf{F}_R$ and its location $\bar{x}$ can be specified.

Magnitude of Resultant Force

For an elemental length dx as shown in the diagram, the force acting is;

dF = $w(x)\ dx$ = dA *[shaded area]*



For entire length;

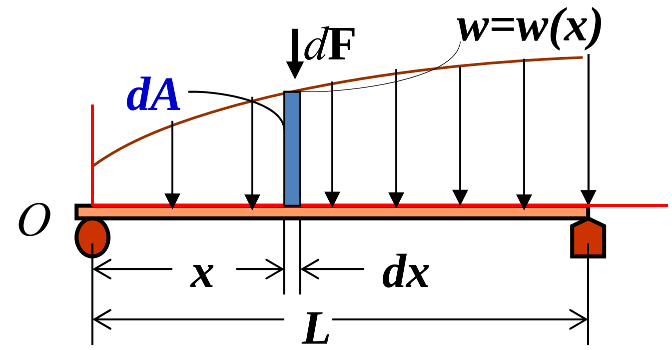
$+\downarrow F_R = \sum F$: $F_R = \int w(x)\ dx = \int dA = A$

Hence, the magnitude of the resultant force is equal to the total area A under the loading diagram w = *w(x)*.

Location of Resultant Force

$$M_{RO} = \Sigma M_O$$

Equating the moment of the F_R and the force distribution about O.

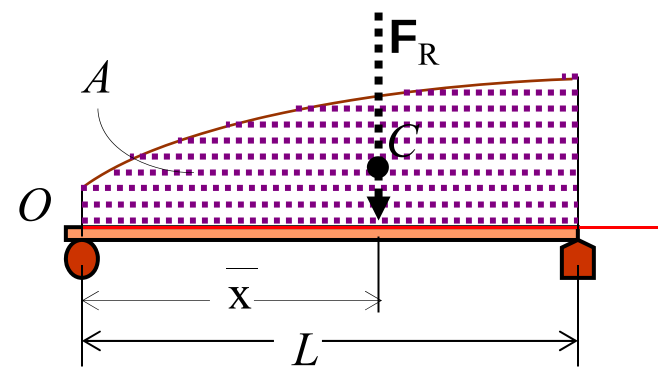


dF produces a moment of **x dF =x w(x) dx** *about O.*

$$\circlearrowright + M_{RO} = SM_O: \quad \bar{x} F_R = \int_L x\, w(x)\, dx$$

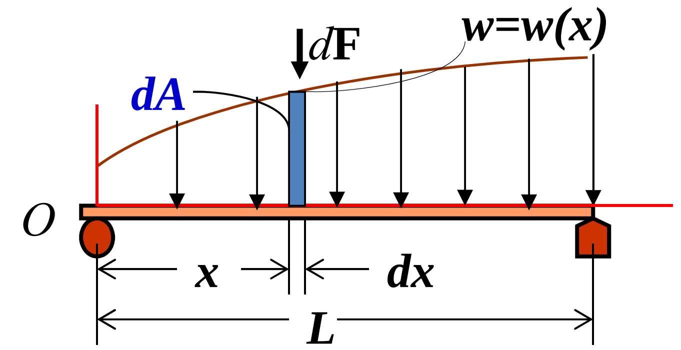
Solving for $\bar{x}$;

$$\bar{x} = \frac{\int_L x\, w(x)\, dx}{\int_L w(x)\, dx} = \frac{\int_A x\, dA}{\int_A dA}$$



Location of Resultant Force

This eqn represents the $\bar{x}$ coordinate for the geometric center (centroid) of the area under the distributed loading diagram *w(x)*.



*d*F produces a moment of
x dF =x w(x) dx *about O.*

The resultant force has a line of action which passes through the centroid C fo the area defined by the distributed loading diagram *w(x).*