SKAA 1213 - Engineering Mechanics

TOPIC 2

3D Forces

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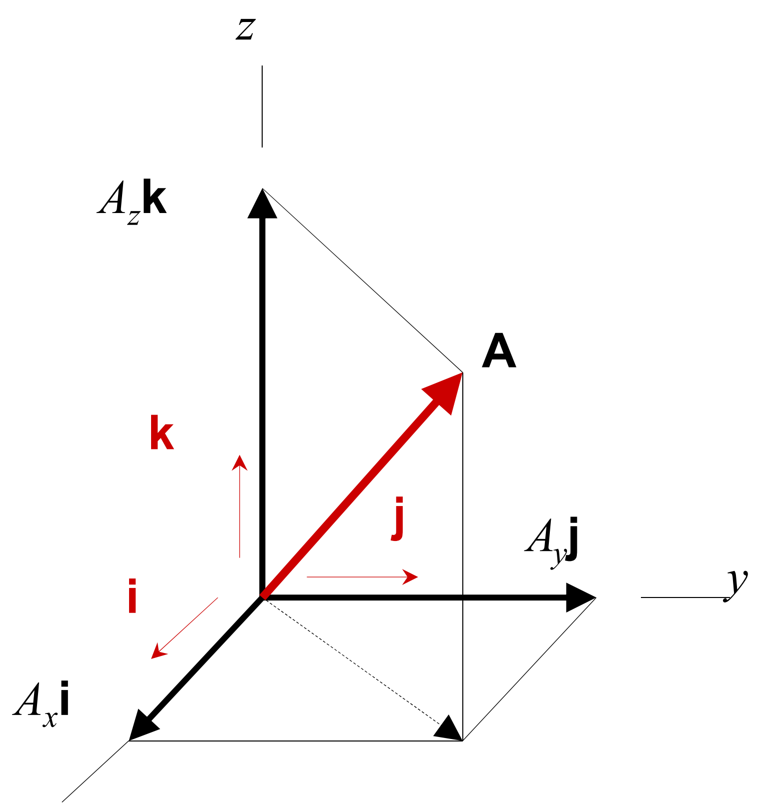
Cartesian Vector Form in 3D

Given A = $A_x + A_y + A_z$

Cartesian unit vectors i, j, k are used for the directions of *x, y, z* .

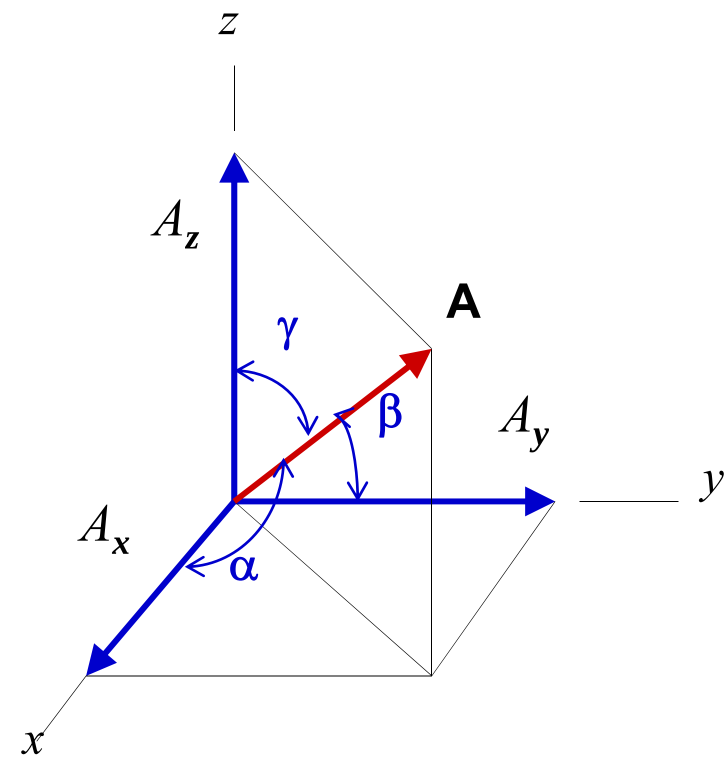


A = A_xi + A_yj + A_zk

The magnitude of vector $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

The orientation of vector A is determined by the coordinate direction angles α, β, γ measured between the tail of A and the positive *x, y, z* axes.

Note : (All angles are between 0 and 180 degrees)



$$\cos\alpha = \frac{A_x}{A} \qquad \cos\beta = \frac{A_y}{A} \qquad \cos\gamma = \frac{A_z}{A}$$

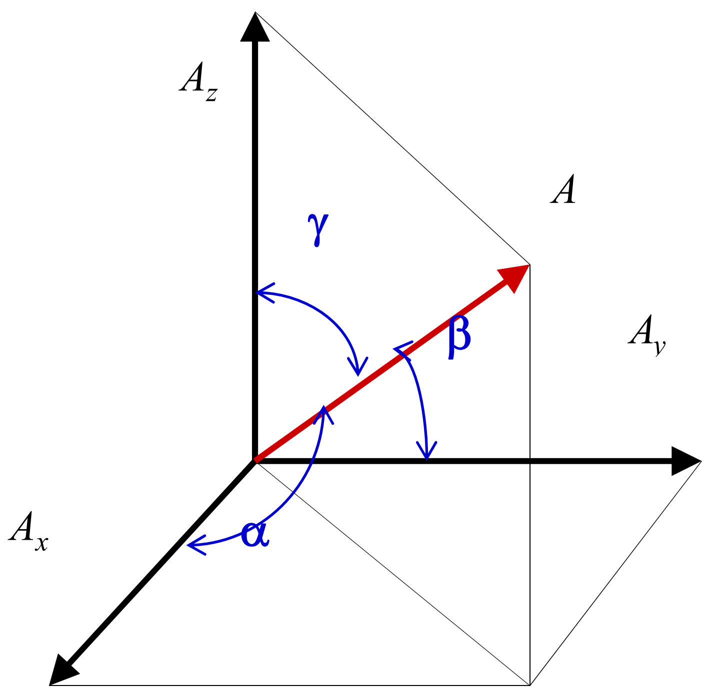
These numbers are known as direction cosines

Unit vector, $\mathbf{u}_A = \dfrac{\mathbf{A}}{A}$ Where , $A = A_x i + A_y j + A_z k$

$$= \frac{A_x}{A} i + \frac{A_y}{A} j + \frac{A_z}{A} k$$

$$= \cos\alpha\, i + \cos\beta\, j + \cos\gamma\, k$$

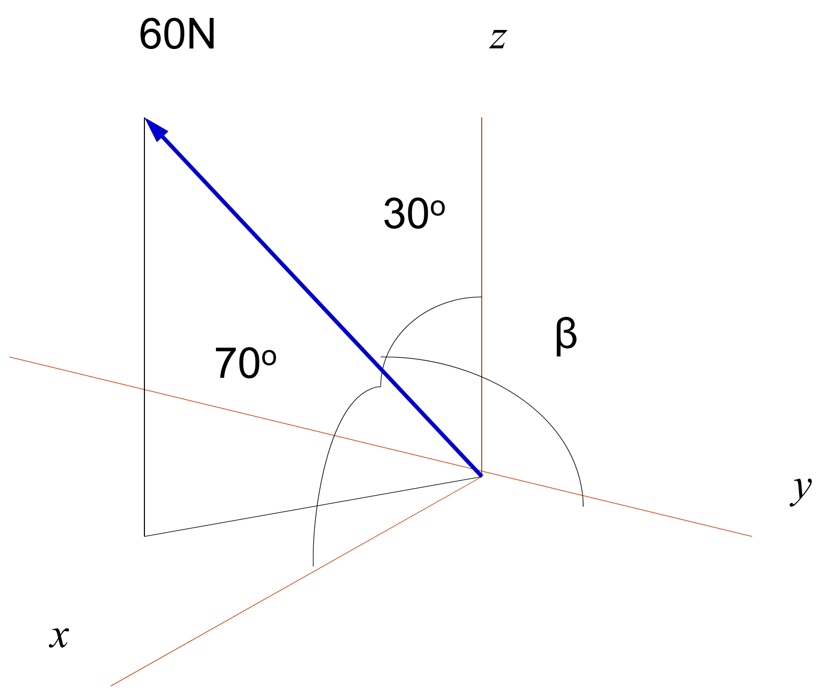
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$



Example 1

Determine the coordinate direction angle β and express the force as a Cartesian vector.

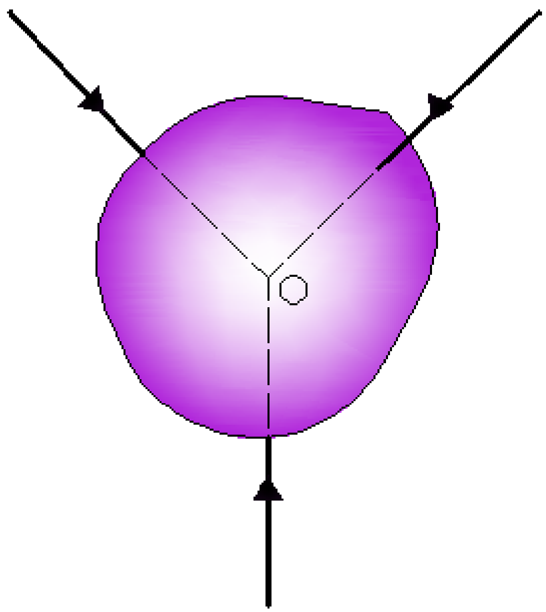
[Answer : $\beta = 111.4^o$, $F = \{20.5\, i - 21.9\, j + 52.0\, k\}$ N]



Concurrent Force System

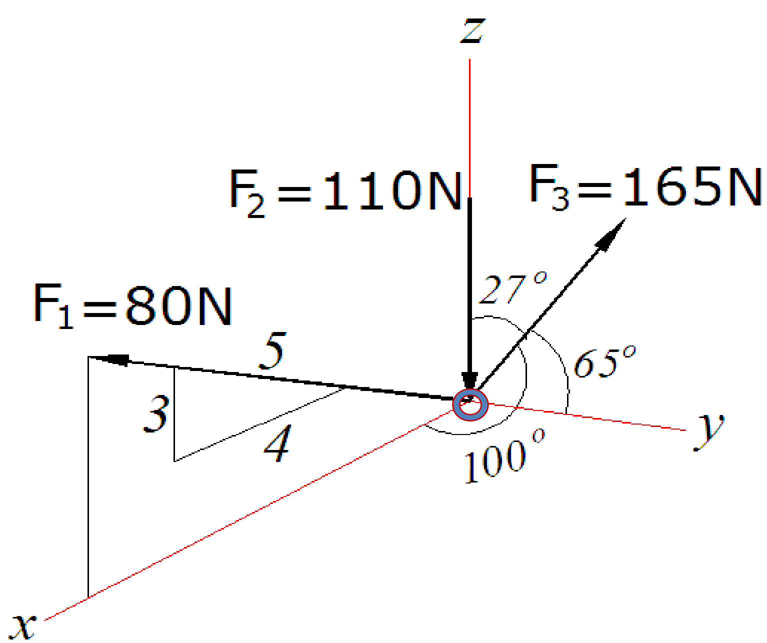
- The resultant force is the vector sum of all the forces in the system.

$$\mathbf{F}_R = \Sigma\mathbf{F} = \Sigma F_x\mathbf{i} + \Sigma F_y\mathbf{j} + \Sigma F_z\mathbf{k}$$



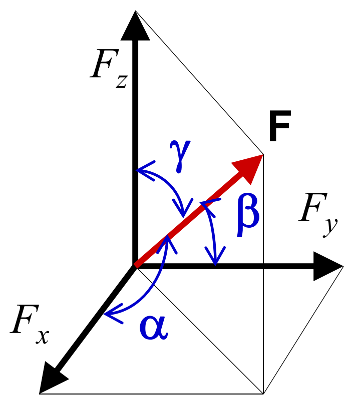
Example 2

Three forces acting on a hook. Determine the resultant force F_R and the coordinate direction angles. *[Answer : F_R = 115.5 N , α = 72.2° β = 52.9° γ = 42.6°]*



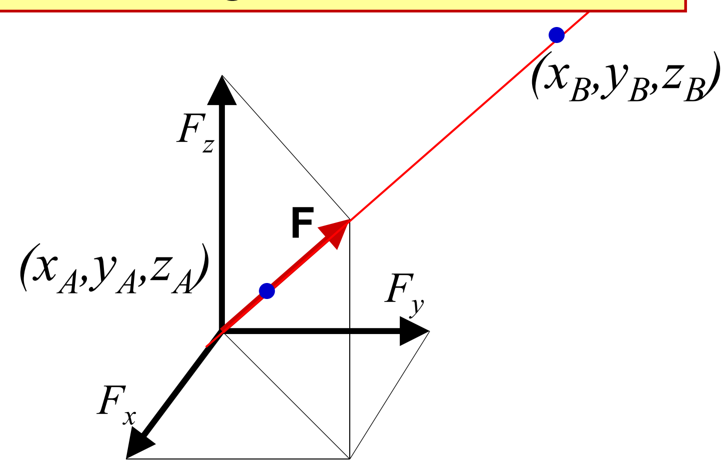
If the coordinate direction angles of a force is not given but two points through which it passes, we can use **position vector** & **unit vector** to deduce the force in Cartesian vector form.

Direction Angles known



$$\mathbf{F} = F\cos\alpha\,\mathbf{i} + F\cos\beta\,\mathbf{j} + F\cos\gamma\,\mathbf{k}$$
$$= F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$$

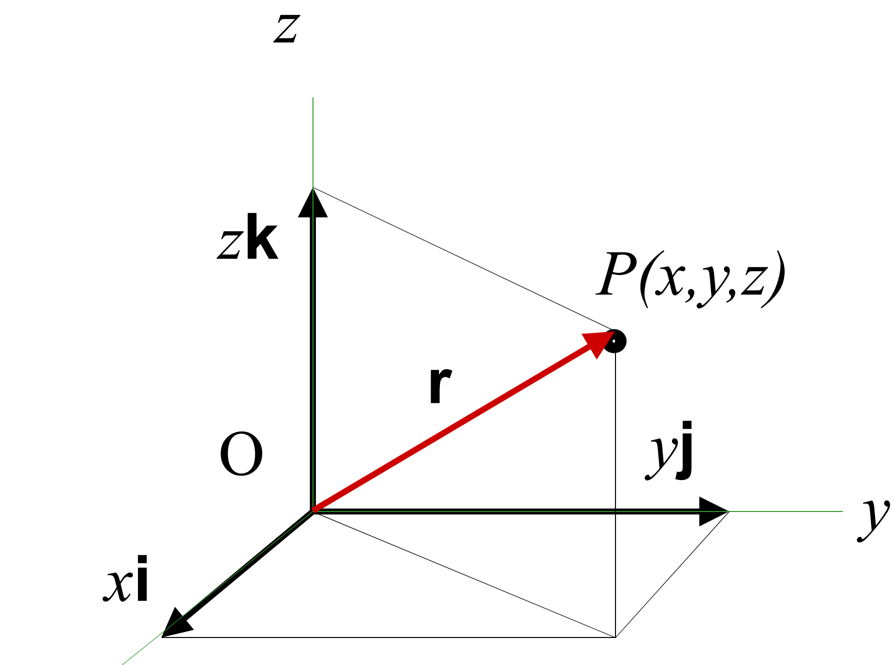
Passing Points known



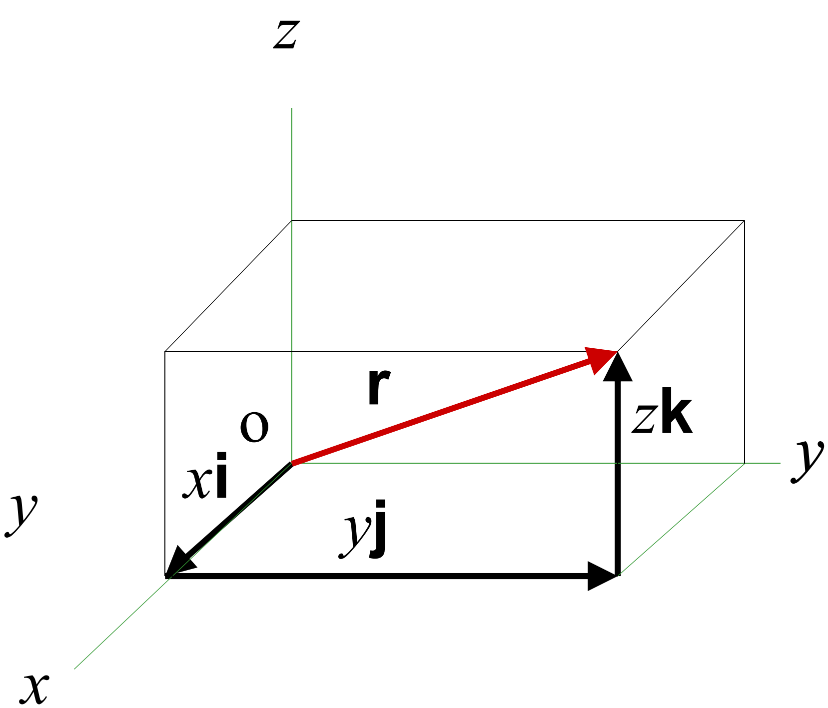
$$\mathbf{F} = F\mathbf{u}$$
$$= F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$$

Position Vector

Definition : location of a fix vector in space relative to a reference origin.



$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

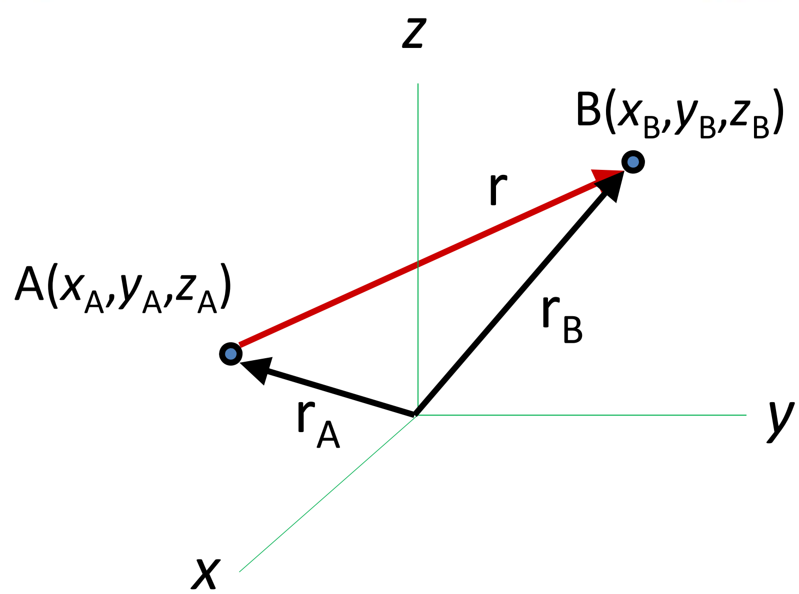


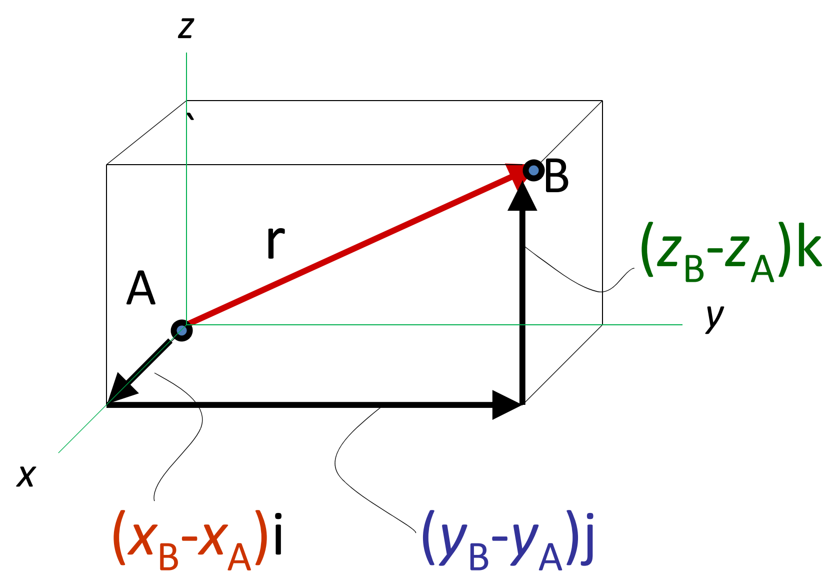
Note the head-to-tail vector addition of the three components yields vector r.

$$\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A$$

$$= (x_B i + y_B j + z_B k) - (x_A i + y_A j + z_A k)$$

$$\mathbf{r} = (x_B - x_A)i + (y_B - y_A)j + (z_B - z_A)k$$

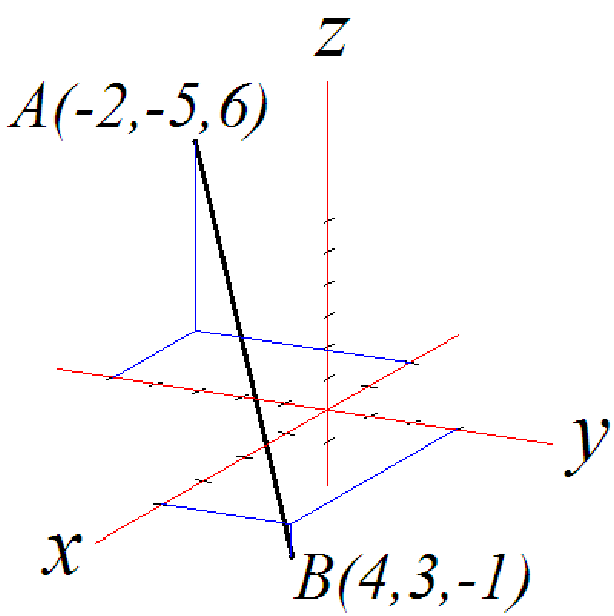




Note the head-to tail of the three components yields r.

Example 3

The position vector **r** acting from point A(-2,-5,6) to B(4,3,-1) in Cartesian vector form are shown below. Determine the distance between points A and B and the coordinate direction angles. *[Answer : $r = 12.2$ m , $\alpha = 60.5^o$, $\beta = 49.0^o$, $\gamma = 125.0^o$]*

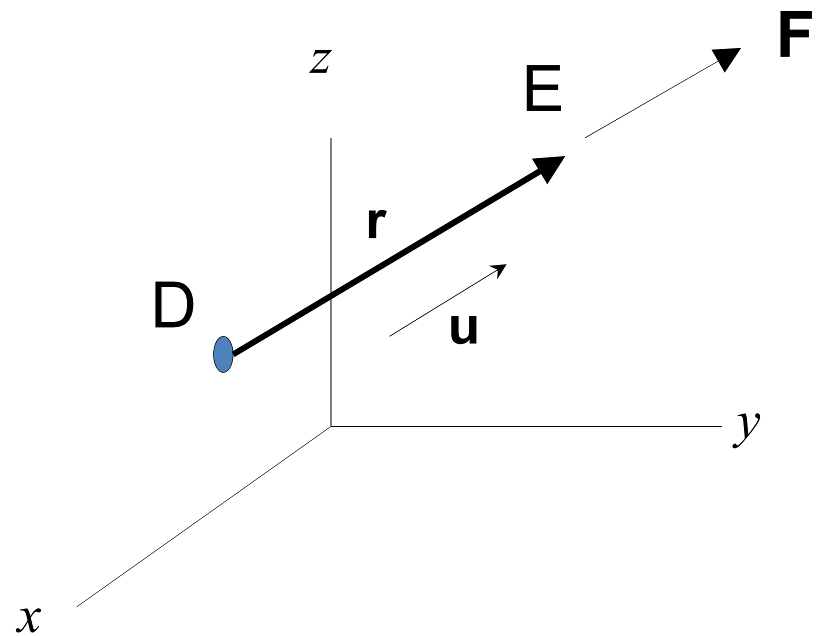


3D Static Problems

The direction of a force in 3D can be identified by 2 points which its **line of action passes**.

Method :

Formulated F as a Cartesian vector by using unit vector u = **r**/r since it has the **same direction** and sense as the position vector r directed from point D to point E.



$$\mathbf{F} = F\,\mathbf{u} = F\left(\frac{\mathbf{r}}{r}\right)$$

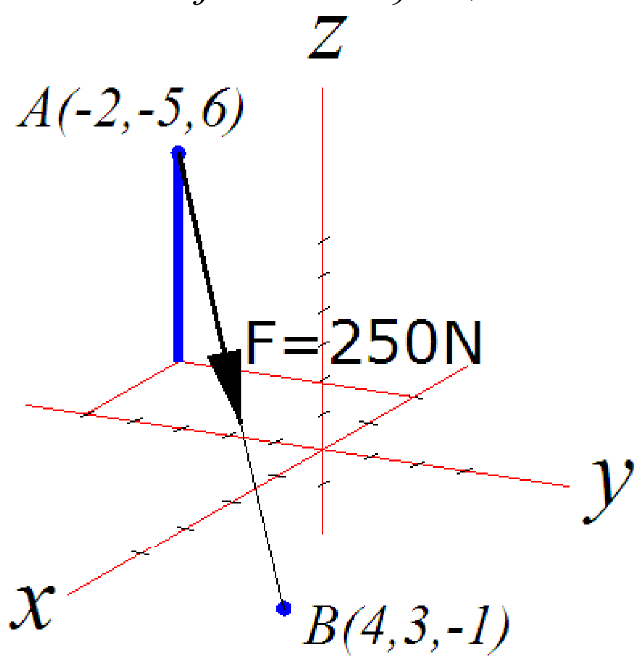
Procedure to Analyze a Force Vector Along a Line

1. Compute the position vector **r** directed from A to B its magnitude.
2. Compute the unit vector **u** = **r**/*r* which defines the direction and sense of both **r** and **F.**
3. Determine **F** by combining its magnitude *F* and direction **u**. i.e, **F**=*F* **u.**

Example 4

The diagram below shows a force F= 250N is applied to the chord which is attached to the top of a 6m pole. Express force **F** as a Cartesian vector; and determine its coordinate direction angles.

[Answer : F = {123 i + 163.9 j - 143.4 k} N , α = 60.5^o β = 49.0^o , γ = 125.0^o]

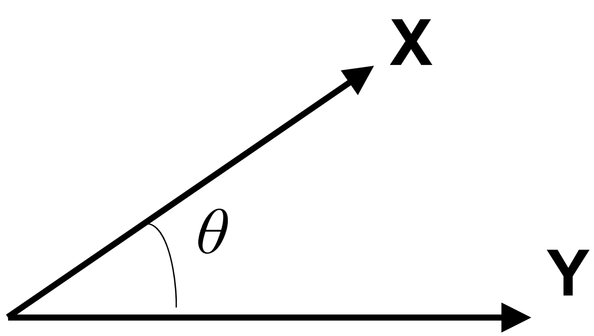


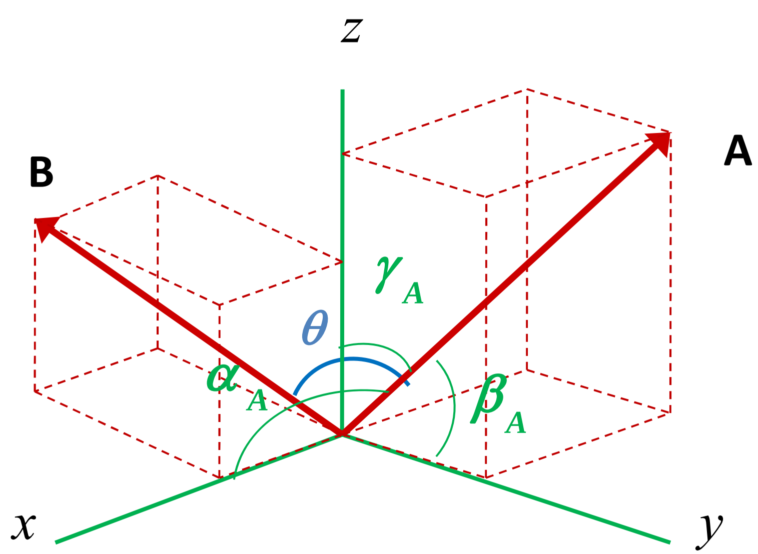
Dot Product

Definition : X·Y=|X||Y|cos θ, where, (0° ≤ θ ≤ 180°)

Properties : scalar product of vectors

Usage : widely used to solve 3D problems



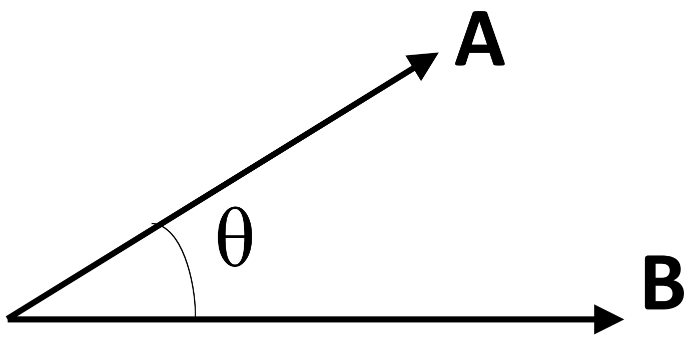


θ = ?

Note: If X is perpendicular to Y (θ =0), X·Y =0 since cos90 = 0

3D

Dot Product Laws of Operations



Commutative law:

$$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$$

Multiplication by a scalar:

$$a\,(\mathbf{A} \cdot \mathbf{B}) = (a\mathbf{A}) \cdot \mathbf{B} = \mathbf{A} \cdot (a\mathbf{B}) = (\mathbf{A} \cdot \mathbf{B})a$$

Distributive law:

$$\mathbf{A} \cdot (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \cdot \mathbf{B}) + (\mathbf{A} \cdot \mathbf{D})$$

Dot Product of the Cartesian Unit Vectors

$\mathbf{i} \cdot \mathbf{i} = (1)(1)\cos 0^o = 1$

$\mathbf{i} \cdot \mathbf{j} = (1)(1)\cos 90^o = 0$

$\mathbf{j} \cdot \mathbf{j} = 1, \ \mathbf{k} \cdot \mathbf{k} = 1, \ \mathbf{i} \cdot \mathbf{k} = 0, \ \mathbf{j} \cdot \mathbf{k} = 0$

Dot product of two general vectors **A** and **B.**

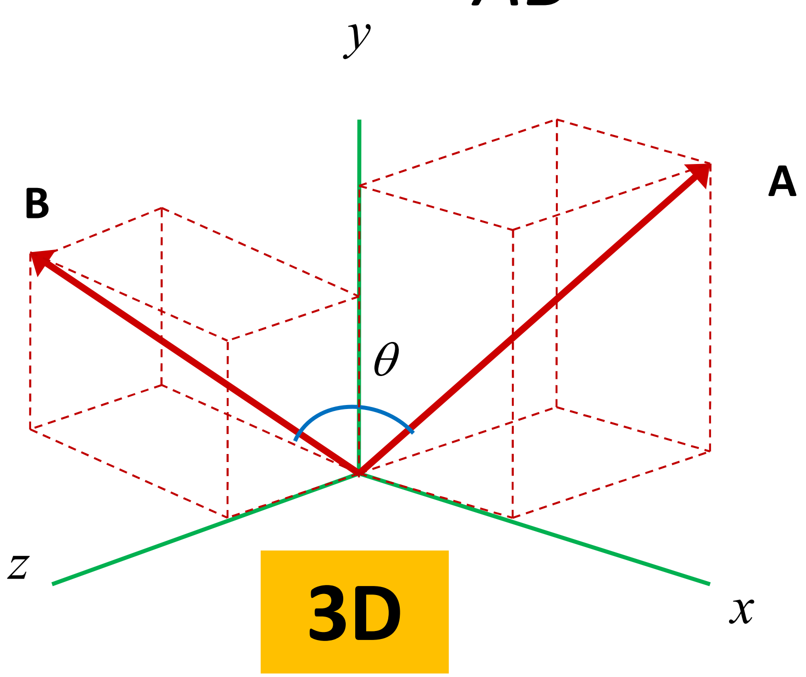
$$\begin{aligned}\mathbf{A} \cdot \mathbf{B} &= (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) \cdot (B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}) \\ &= A_x B_x (\mathbf{i}\cdot\mathbf{i}) + A_x B_y (\mathbf{i}\cdot\mathbf{j}) + A_x B_z (\mathbf{i}\cdot\mathbf{k}) \\ &\quad + A_y B_x (\mathbf{j}\cdot\mathbf{i}) + A_y B_y (\mathbf{j}\cdot\mathbf{j}) + A_y B_z (\mathbf{j}\cdot\mathbf{k}) \\ &\quad + A_z B_x (\mathbf{k}\cdot\mathbf{i}) + A_z B_y (\mathbf{k}\cdot\mathbf{j}) + A_z B_z (\mathbf{k}\cdot\mathbf{k})\end{aligned}$$

$$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$$

Application of Dot Product in Force Vectors

1. The **angle formed between two vectors** or intersecting lines.

$$\theta = \cos^{-1}\left(\frac{\mathbf{A}\cdot\mathbf{B}}{AB}\right)$$ *where,* $(0^o \le \theta \le 180^o)$

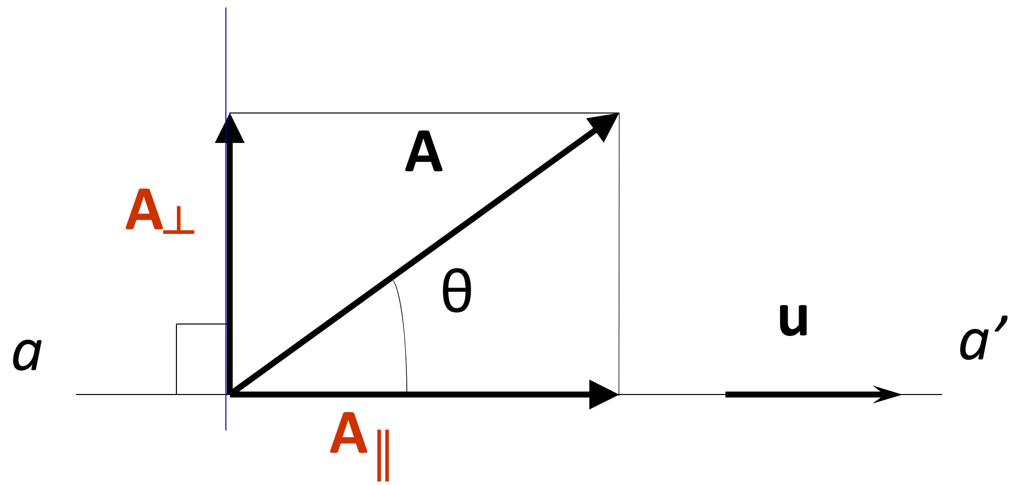


$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$$

θ = ?

2. The components of the vector **parallel** and **perpendicular** to a line.



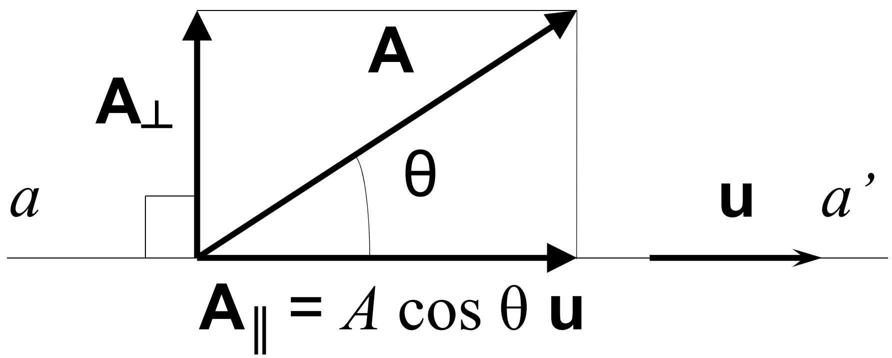
Let $\mathbf{A}_{\parallel}$ (*projection* of A on the line) be the component of vector **A** which collinear with the line aa'.

$A_{\parallel} = A \cos\theta = \mathbf{A}\cdot\mathbf{u}$ *(since, u=1)*

Hence, the scalar projection of A along a line is determined from the dot product of A and the unit vector u which defines the direction of the line.

The component $\mathbf{A}_{\parallel}$ can be expressed as

$$\mathbf{A}_{\parallel} = A \cos\theta \, \mathbf{u} = (\mathbf{A}\cdot\mathbf{u})\mathbf{u}$$



$$\mathbf{A} = \mathbf{A}_{\parallel} + \mathbf{A}_{\perp}$$

$$\mathbf{A}_{\perp} = \mathbf{A} - \mathbf{A}_{\parallel}$$

There are 2 ways of obtaining $A_{\perp}$.

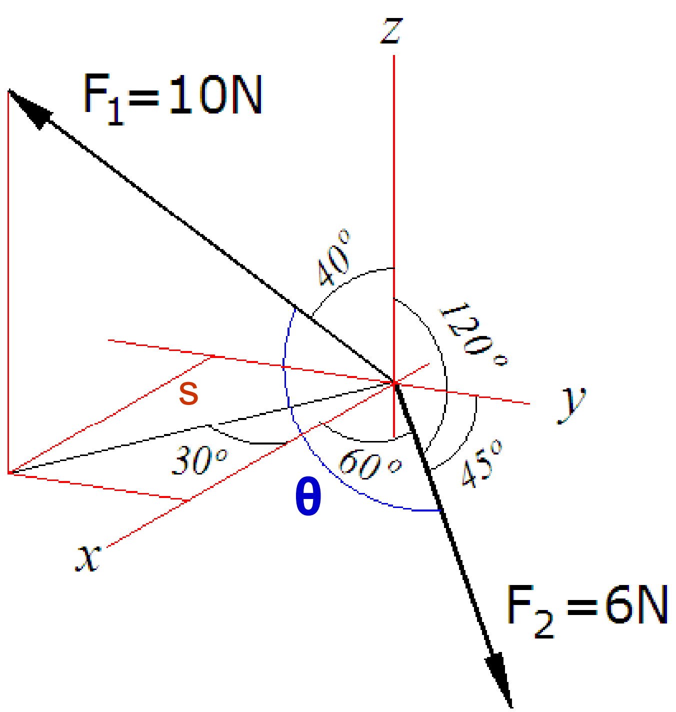
a. Obtain $\theta = \cos^{-1}(\mathbf{A}\cdot\mathbf{u}/A)$, then $A_{\perp} = A \sin\theta$

b. If $A_{\parallel}$ is known, $A_{\perp} = \sqrt{A^2 - A_{\parallel}^2}$

Example 5

(a) Determine the angle θ between vector $\mathbf{F}_1$ and $\mathbf{F}_2$.

(b) Find the magnitude of the projected component of $\mathbf{F}_1$ along $\mathbf{F}_2$, and the projection of $\mathbf{F}_2$ along $\mathbf{F}_1$.



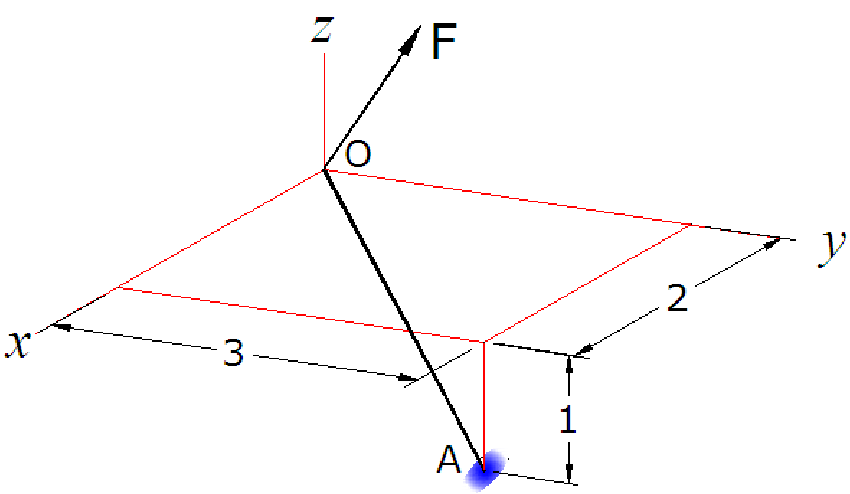
[Answer : (a) F_1 *=[5.57i - 3.22j + 7.66k]N*

F_2 *= [3.0i + 4.24j - 3.0k]N* $\theta = 109.4^o$

(b) $F_{1\|2} = -3.32\ N$, $F_{2\|1} = -2.0\ N$ *]*

Example 6

The force **F**= (20**i** + 40**j** + 90**k**) N acts at the end of a pole as shown in the figure below. Find the magnitude of the $\mathbf{F}_1$ and $\mathbf{F}_2$ which is along and perpendicular to the pole.



[Answer : F_1= 18.75 F_2 = 97.7 N , θ = 79.2^o]