SEL4223 Digital Signal Processing

Discrete-Time Signal

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Introduction

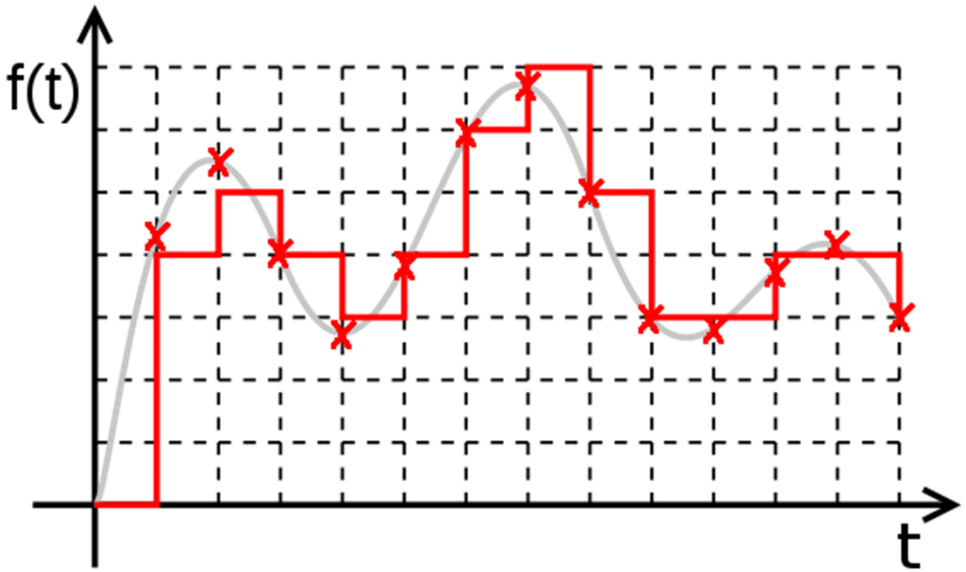
- Digital Signal Processing (DSP) has developed rapidly over the past 40 years due to significant advances in digital computer technology and integrated-circuit fabrication (MSI→LSI→VLSI).
- In telecommunication:
  - ADC - samples an analog signal and encodes the signal into pulse code modulation (PCM) format.
  - Filter noise and interference from signal, transcode, compress, and encrypt the signal prior to placing it on a carrier of an electrically based digital circuit for transmission.
  - Sending the signal to DAC for decoding back into real world analog form, perhaps with improved clarity, a shifted frequency, or demodulated for transmission.

Introduction (cont.)

- **Signal** is defined as any physical quantity that varies with time, space or any other independent variable or variables.
- **A discrete-time signal** is a series consisting of a sequence of quantities. In other words, it is a series that is a function over a domain of discrete integers. Each value in the sequence is called a sample.
- As most of the signals encountered in science and engineering are analog in nature, a process on converting the analog signal to discrete signal is needed. This process is called digitization, which consists of Sampling and Quantization process.

Digitization

- Referring to figure below,
  - converting continuous value to discrete value on horizontal-axis is called sampling
  - converting the continuous value to discrete value on vertical-axis is called quantization.



Digitization (cont.)

- In the previous figure, the continuous signal is first sampled where values of $f(t)$ at only selected t are kept. This is shown by the red cross. So far only values on the horizontal have been digitized.
- To digitize the vertical axis, values on the red cross are quantized where in this example these values are round toward nearest values of 3 bit integer. For example the first red cross value is quantized from 4.2 to 4 (or 100 in binary).

Classification of Signal

- Continuous-time and discrete time – Continuous time signal take on values in the continuous interval while discrete time signal take samples only at certain values of time.
- Periodic and aperiodic – Periodic signal is consistently repeated at certain time period from $-\infty$ to ∞.
- Deterministic and random – Deterministic signal can be represented with mathematical description while random signal is not where the signal evolves in time in an unpredictable manner.

Sampling

- The sampling process can be viewed as

$$t = nT_s$$

t – continuous time (s)

n – discrete time (integer). Also called sample

T_s – sampling time (s)

Sampling Theorem

- To ensure the digitized discrete-time signal represents its original continuous-time signal sufficiently and correctly, the sampling process need to follow Nyquist theorem as below:

$$F_s \geq 2F_N$$

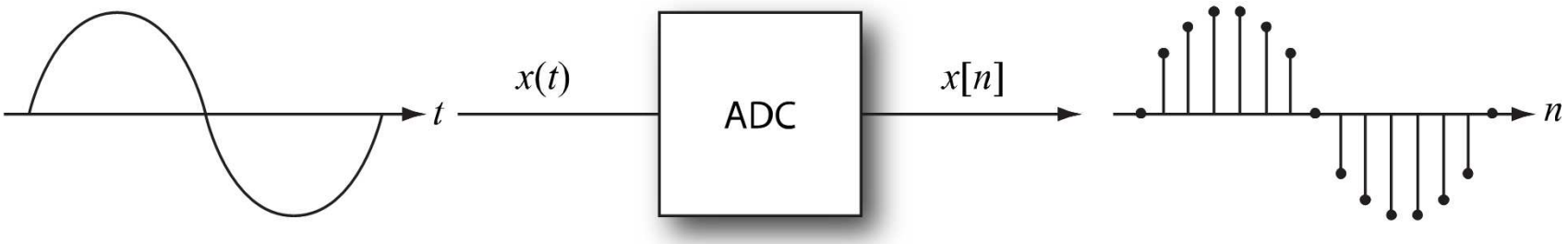
- Where:

F_s – $Frquency\ sampling$

F_N – $Highest\ frequency\ component\ in\ the\ signal$

Signal representation

- $x(t)$ – analog/continuous
- $x[n]$ – digital/discrete

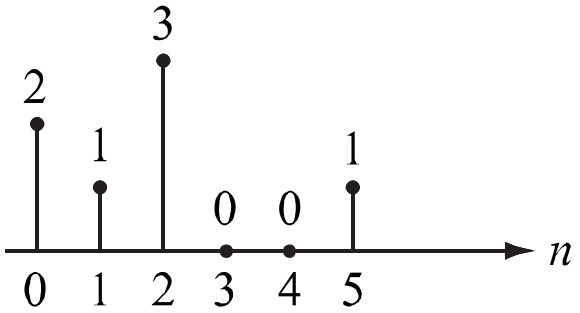


Discrete Signal Representation

Numerical method:

- $x[n] = [2,1,3,0,0,1]$, where the arrow shows the position at $n = 0$.
  $\uparrow$
- If no arrow showed, the first sample holds the position at $n = 0$

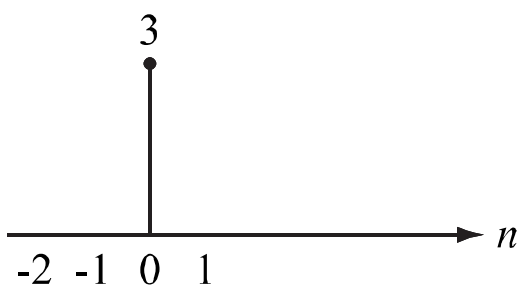
Graphical method:

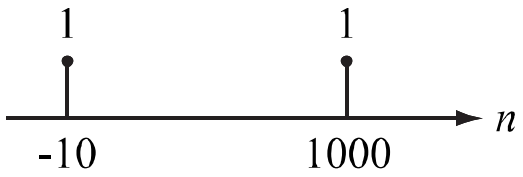


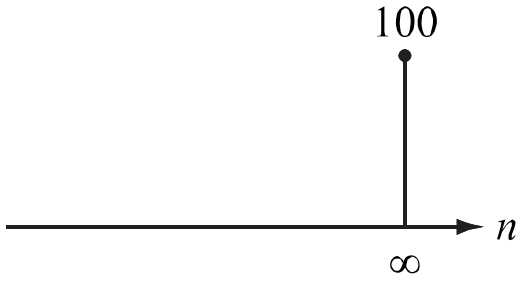
- Discrete signal length (also called size): Number of samples in the signal. Count from the lowest n to the highest n that has a nonzero value

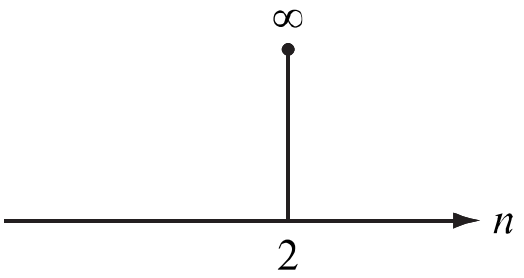
Quiz 1

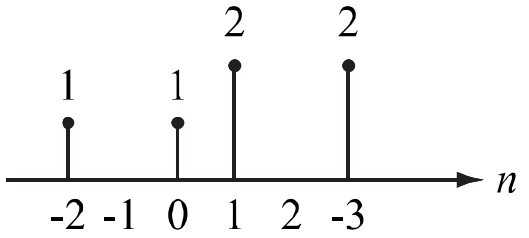
1. What is the length of these signals?

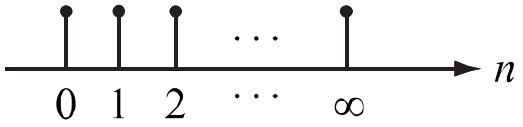








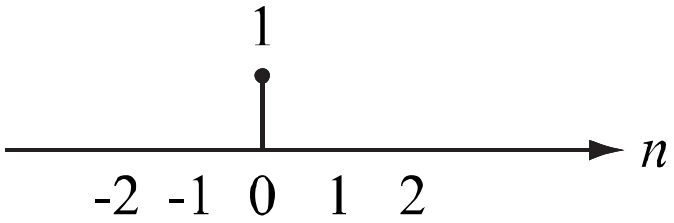




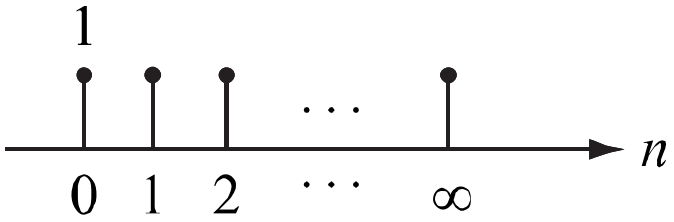
2. What is the size of a continuous signal?

Types of Discrete Signal

- There are 2 basic types of discrete signals: impulse signal and step signal
- $\delta[n]$ - impulse signal

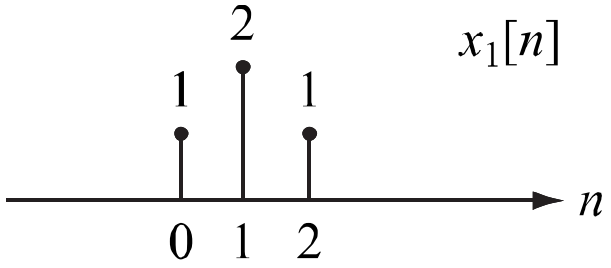


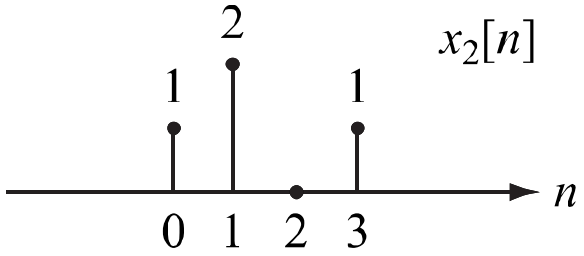
- $u[n]$ - step signal

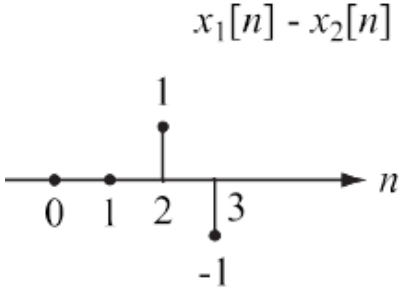


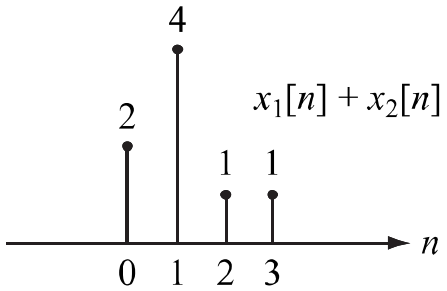
Arithmetic Operations

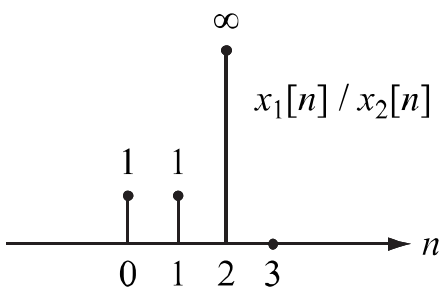
- For addition, subtraction, multiplication, division operations, the operations are made on each of the sample of similar n

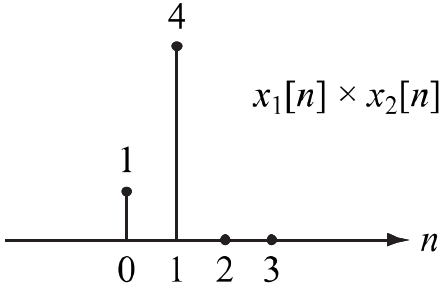






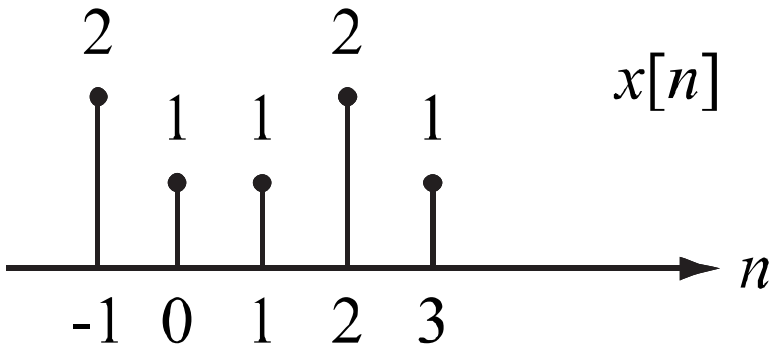


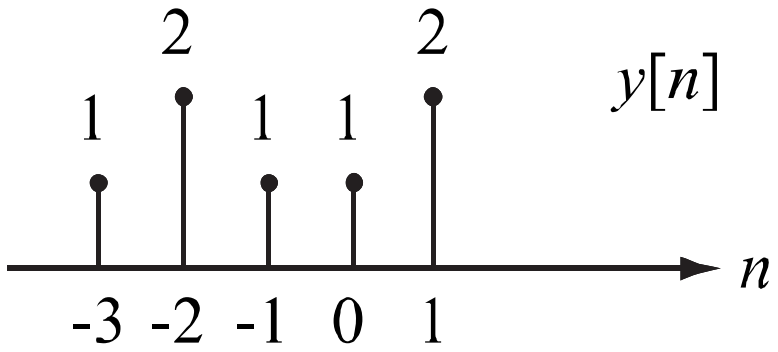




Folding Operation

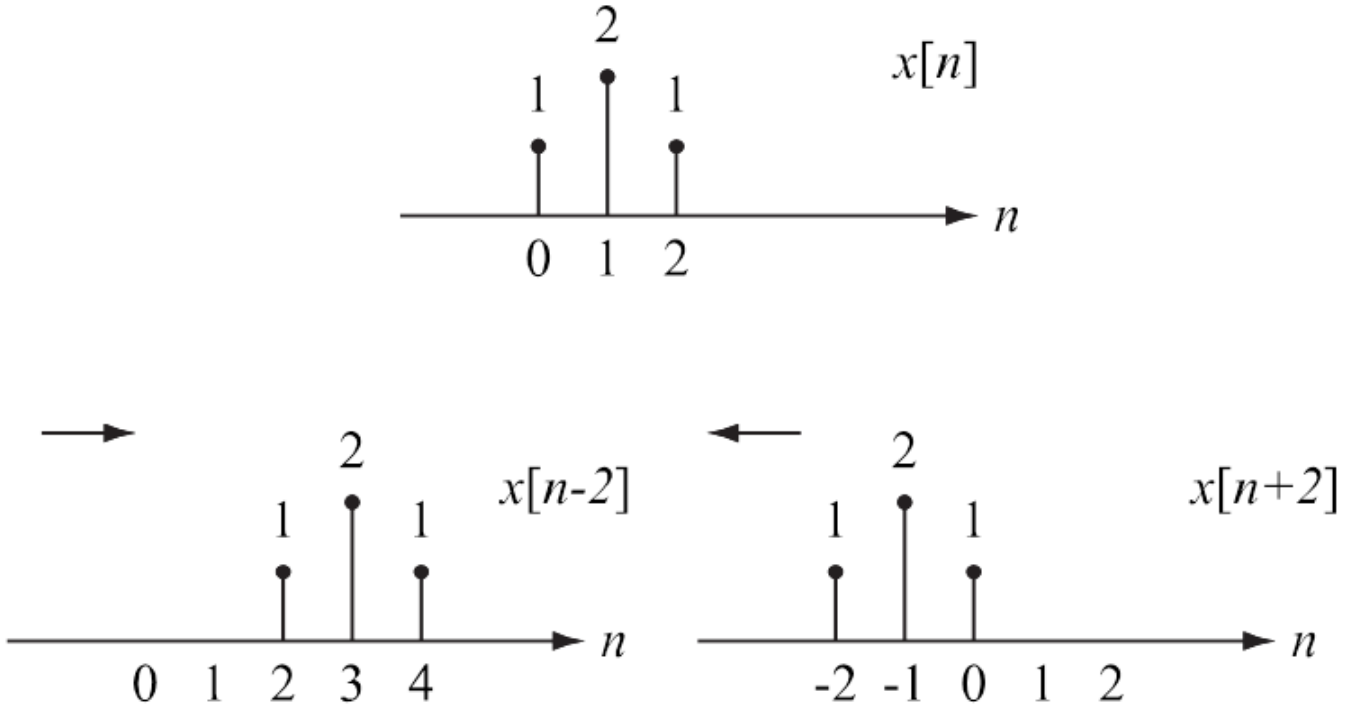
- Fold signal at $n = 0$ where $y[n] = x[-n]$



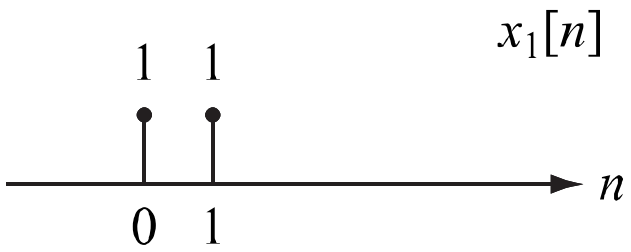


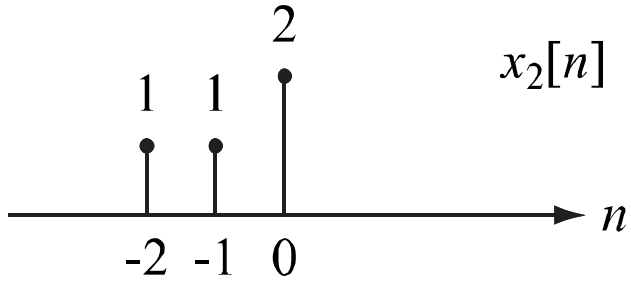
Delay Operation

- Shift the signal to the right or left by n_d samples $y[n] = x[n - n_d]$



Quiz 2





Find:

1) $y[n] = x_1[-n] + x_2[n]$
2) $y[n] = x_1[n] + x_2[n-3]$
3) $y[n] = x_2[-n-2]$
4) $y[n] = x_1[n] + x_2[-2n]$
5) $y[n] = x_2[n] - x_1[n]$
6) $y[n] = u[-n] + x_1[n]$
7) $y[n] = \delta[n-3] + x_2[-n] + x_1[n+2]$
8) $y[n] = \delta[n] \times x_2[n]$
9) $y[n] = u[n] \times x_2[n]$
10) $y[n] = u[-n+2] - x_1[n]$
11) $y[n] = x_2[n] + x_2[n-1]$

References

1) John G. Proakis, Dimitris K Manolakis, “Digital Signal Processing: Principle, Algorithm and Applications”, Prentice-Hall, 4th edition (2006).

2) Sanjit K. Mitra, “Digital Signal Processing-A Computer Based Approach”, McGraw-Hill Companies, 3rd edition (2005).

3) Alan V. Oppenheim, Ronald W. Schafer, “Discrete-Time Signal Processing”, Prentice-Hall, 3rd edition (2009).